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Volume VI

ANSWERS TO QUESTIONS

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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN

VOLS. I, II, AND III,

EXCEPT THE

EXAMPLES FOR PRACTICE.

It will be noticed that the Key is divided into sections corresponding to the various sections of the questions and examples at the end of each volume. The answers are so numbered as to be similar to the numbers before the questions to which they refer.

CONTENTS.

ARITHMETIC.

(QUESTIONS 1-75.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four,

9,006,042 = Nine million, six thousand and forty-two.

850,317,002 = Eight hundred fifty million, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million, four thousand and seven = 5,004,007.

One hundred and eight million, ten thousand and one = 108,010,001.

Eighteen million and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column, and reserve 1 ten for the column

$$\begin{array}{r} 3290 \\ 504 \\ 865403 \\ 2074 \\ 81 \\ 7 \\ \hline 871359 \end{array} \quad \text{Ans.}$$

of tens. $1+8+7+9=25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2=13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3=11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1+6=7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure 8 in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0=19$; write the nine and reserve the 1. $1+8+7+0+0+9=25$; write the 5 and reserve the 2. $2+0+4+5+2=13$; write the 3 and reserve the 1. $1+2+5+3=11$; write the 1 and reserve 1. $1+6=7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{r} 709 \\ 8304725 \\ 391 \\ 100302 \\ 300 \\ 909 \\ \hline 8407336 \end{array} \text{ Ans.}$$

(9) (*a*) In subtracting whole numbers, place the subtrahend or smaller number under the minuend or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. As 1 *ten* $=10$ *units*, we have 10 units $+$ 2 units $=12$ units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so

only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r} 50962 \\ 3338 \\ \hline 47624 \end{array} \text{ Ans.}$$

(*b*)
$$\begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array} \text{ Ans.}$$

(10) (*a*)
$$\begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array} \text{ Ans.}$$

(*b*)
$$\begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array} \text{ Ans.}$$

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction we have $\begin{array}{r} 1004 \\ \text{——} \\ \hline 49 \end{array}$ in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl} 1004 & \textit{larger} \\ 955 & \textit{smaller} \\ \hline 1959 & \textit{sum.} \end{array} \text{ Ans.}$$

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004)

and the difference (49); or, 1,004 – 49 = 955, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959 sum. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

```
  5962            3874
  8471            2039
  9023            ----
 -----            5913 sum.
 23456 sum.
```

Subtracting the smaller number (5,913) from the greater (23,456) we have

```
 23456
  5913
 -----
 17543 difference.   Ans.
```

(13)
```
 $44675 = amount willed to his son.
  26380 = amount willed to his daughter.
 ------
 $71055 = amount willed to his two children.
$125000 = amount willed to his wife and two children.
  71055 = amount willed to his two children.
 ------
 $53945 = amount willed to his wife.   Ans.
```

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)
```
 526387
      7
-------
3684709  Ans.
```

7 × 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 tens + 4 tens reserved = 60 tens or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve

the 2 thousands. 7×6 thousands $= 42$ thousands; $42 + 2$ thousands reserved $= 44$ thousands or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7×2 ten-thousands $= 14$ ten-thousands; $14 + 4$ ten-thousands reserved $= 18$ ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7×5 hundred-thousands $= 35$ hundred-thousands; $35 + 1$ hundred-thousand reserved $= 36$ hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times $7 = 49$; write the 9 and reserve the 4. 7 times $8 = 56$; $56 + 4$ reserved $= 60$; write the 0 and reserve the 6. 7 times $3 = 21$; $21 + 6$ reserved $= 27$; write the 7 and reserve the 2. $7 \times 6 = 42$; $42 + 2$ reserved $= 44$; write the 4 and reserve 4. $7 \times 2 = 14$; $14 + 4$ reserved $= 18$; write the 8 and reserve the 1. $7 \times 5 = 35$; $35 + 1$ reserved $= 36$; write the 36.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, $7 \times 700{,}298$ and $10 \times 700{,}298$. The actual operation is performed as follows:

$$
(b)\qquad
\begin{array}{r}
700298 \\
\underline{17} \\
4902086 \\
\underline{700298} \\
11905066
\end{array}
\quad \text{Ans.}
$$

7 times $8 = 56$; write the 6 and reserve the 5. 7 times $9 = 63$; $63 + 5$ reserved $= 68$; write the 8 and reserve the 6. 7 times $2 = 14$; $14 + 6$ reserved $= 20$; write the 0 and reserve the 2. 7 times $0 = 0$; $0 + 2$ reserved $= 2$; write the 2. 7 times $0 = 0$; $0 + 0$ reserved $= 0$; write the 0. 7 times $7 = 49$; $49 + 0$ reserved $= 49$; write the 49.

To multiply by the 1 ten we say 1 times $700298 = 700298$, and write 700298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*)

$$\begin{array}{r} 217 \\ \underline{103} \\ 651 \\ \underline{2170} \\ 22351 \\ \underline{67} \\ 156457 \\ \underline{134106} \\ 1497517 \end{array}$$

Multiply any two of the numbers together and multiply their product by the third number.

1497517 Ans.

(15) If your watch ticks every second, then to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

$$\begin{array}{rl} 60 & \text{seconds} = 1 \text{ minute.} \\ \underline{60} & \text{minutes} = 1 \text{ hour.} \\ 3600 & \text{seconds} = 1 \text{ hour.} \\ \underline{24} & \text{hours} = 1 \text{ day.} \\ 14400 & \\ \underline{7200} & \\ 86400 & \text{seconds} = 1 \text{ day.} \\ \underline{7} & \text{days} = 1 \text{ week.} \\ 604800 & \text{seconds in 1 week or the number of times that} \\ \text{Ans.} & \text{your watch ticks in 1 week.} \end{array}$$

(16) If a monthly publication contains 24 pages, a yearly volume will contain 12×24 or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

$$\begin{array}{r} 24 \\ \underline{12} \\ 288 \\ \underline{8} \\ 2304 \end{array}$$

2304 Ans.

(17) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much, plus \$1,200, then the building is worth

$$\begin{array}{rr} & \$3246 \\ & \underline{3} \\ & 9738 \\ \text{plus} & \underline{1200} \\ & \$10938 \end{array} = \text{value of building.}$$

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$\begin{array}{rr} & \$10938 \\ & 2 \\ \hline & 21876 \\ \text{plus} & 1875 \\ \hline & \$23751 \end{array} = \text{value of tools.}$$

$$\begin{array}{lr} \text{Value of building} = & \$10938 \\ \text{Value of tools} \quad = & 23751 \\ \hline & \$34689 \end{array} = \text{value of the building and tools. } (a) \text{ Ans.}$$

$$\begin{array}{lr} \text{Value of engine and boiler} = & \$\ \ 3246 \\ \text{Value of building and tools} = & 34689 \\ \hline & \$37935 \end{array} = \text{value of the whole plant. } (b) \text{ Ans.}$$

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the *dividend* and 15 in the *divisor* are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{96 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

The 5 and 15 are *not* to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and 15 *never* existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the *dividend* and 96 in the *divisor* are *divisible* by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 96 and write the 8 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times 1}{\underset{8}{\cancel{96}} \times 3 \times 7 \times 6} =$$

The 72 and 96 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 *never* existed. Thus,

$$\frac{6 \times 48 \times 28 \times 1}{8 \times 3 \times 7 \times 6} =$$

Again, 28 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{6 \times 48 \times \overset{4}{\cancel{28}} \times 1}{8 \times 3 \times \underset{1}{\cancel{7}} \times 6} =$$

The 28 and 7 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 *never* existed. Thus,

$$\frac{6 \times 48 \times 4 \times 1}{8 \times 3 \times 1 \times 6} =$$

Again, 48 in the *dividend* and 6 in the *divisor* are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, cross off the 6 and write the 1 *under* it. Thus,

$$\frac{6 \times \overset{8}{\cancel{48}} \times 4 \times 1}{8 \times 3 \times 1 \times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 *never* existed. Thus,

$$\frac{6 \times 8 \times 4 \times 1}{8 \times 3 \times 1 \times 1} =$$

Again, 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, cross off the 3 and write the 1 *under* it. Thus,

$$\frac{\overset{2}{\cancel{6}} \times 8 \times 4 \times 1}{8 \times \underset{1}{\cancel{3}} \times 1 \times 1} =$$

The 6 and 3 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{2 \times 8 \times 4 \times 1}{8 \times 1 \times 1 \times 1} =$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2 \times \overset{1}{\cancel{8}} \times 4 \times 1}{\underset{1}{\cancel{8}} \times 1 \times 1 \times 1} = \frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1}.$$

Since there are *no two remaining numbers* (one in the dividend and one in the divisor) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the *dividend* together, and divide their *product* by the *product* of all the *uncanceled* numbers in the divisor. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the *dividend* equals $2 \times 1 \times 4 \times 1 = 8$; the product of all the *uncanceled* numbers in the *divisor* equals $1 \times 1 \times 1 \times 1 = 1$.

Hence, $\frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1} = \frac{8}{1} = 8.$ Ans.

Or, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{\cancel{8}}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{\underset{1}{\cancel{8}}}{\cancel{96}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \cancel{7} \times \underset{1}{\cancel{6}}} = \frac{8}{1} = 8.$$ Ans.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the *dividend* and 70 in the *divisor* are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 *never* existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the *dividend* and 20 in the *divisor* are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are *not* to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 1} =$$

Again, 16 in the *dividend* and 24 in the *divisor* are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also cross off the 24 and write the 3 *under* it. Thus,

$$\frac{4 \times 60 \times 5 \times \overset{2}{\cancel{16}} \times 14}{7 \times 50 \times \underset{3}{\cancel{24}} \times 1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 2 \times 14}{7 \times 50 \times 3 \times 1} =$$

Again, 60 in the *dividend* and 50 in the *divisor* are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{4 \times \overset{6}{\cancel{60}} \times 5 \times 2 \times 14}{7 \times \underset{5}{\cancel{50}} \times 3 \times 1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 14}{7 \times 5 \times 3 \times 1} =$$

The 14 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times 5 \times 3 \times 1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 2}{1 \times 5 \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{4\times 6\times \overset{1}{\cancel{5}}\times 2\times 2}{1\times \underset{1}{\cancel{5}}\times 3\times 1}=$$

The 5 in the *dividend* and 5 in the *divisor* are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 *never* existed. Thus,

$$\frac{4\times 6\times 1\times 2\times 2}{1\times 1\times 3\times 1}=$$

The 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{4\times \overset{2}{\cancel{6}}\times 1\times 2\times 2}{1\times 1\times \underset{1}{\cancel{3}}\times 1}=$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\frac{32}{1}=32.\quad \text{Ans.}$$

Hence, $$\frac{\overset{4}{\cancel{80}}\times \overset{\overset{2}{\cancel{6}}}{\cancel{60}}\times \overset{\overset{1}{\cancel{5}}}{\cancel{50}}\times \overset{2}{\cancel{16}}\times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}}\times \underset{\underset{1}{\cancel{5}}}{\cancel{50}}\times \underset{\underset{1}{\cancel{3}}}{\cancel{24}}\times \underset{1}{\cancel{20}}}=\frac{4\times 2\times 1\times 2\times 2}{1\times 1\times 1\times 1}=\frac{32}{1}=32.\ \text{Ans.}$$

(19) 28 acres of land at \$133 an acre would cost 28 × \$1 3 3 = \$3,724.

```
   2 8
 1 0 6 4
 2 6 6
$3 7 2 4
```

If a mechanic earns \$1,500 a year and his expenses are \$968 per year, then he would save \$1500—\$968, or \$532 per year.

$$\begin{array}{r} \$1500 \\ 968 \\ \hline \$532 \end{array}$$

If he saves \$532 in 1 year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{l} 532\,)\,3724\,(\,7 \text{ years. Ans.} \\ \underline{3724} \end{array}$$

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) — 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{rr} & 365 \\ & 3 \\ \hline & 1095 \\ & 246 \\ \hline \textit{difference} & 849 \text{ miles. Ans.} \end{array}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in one mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

$$\begin{array}{l} 16)1869120(116820 \text{ rev. Ans.} \\ 16 \\ \overline{26} \\ 16 \\ \overline{109} \\ 96 \\ \overline{131} \\ 128 \\ \overline{32} \\ 32 \\ \overline{0} \end{array}$$

(22) (*a*) 576) 589824 (1024 Ans.
576
1382
1152
2304
2304

(*b*) 43911) 369730620 (8420 Ans.
351288
184426
175644
87822
87822
0

(*c*) 505) 2527525 (5005 Ans.
2525
2525
2525

(*d*) 1234)4961794302(4020903 Ans.
4936
2579
2468
11143
11106
3702
3702

(23) The harness evidently cost the difference between \$444 and the amount which he paid for the horse and wagon. Since \$264 + \$153 = \$417, the amount paid for the horse and wagon, \$444 − \$417 = \$27, the cost of the harness.

\$264	\$444
153	417
\$417	\$27 Ans.

(24) (*a*)

$$\begin{array}{r} 1024 \\ 576 \\ \hline 6144 \\ 7168 \\ 5120 \\ \hline 589824 \end{array} \text{ Ans.}$$

(*b*)

$$\begin{array}{r} 5005 \\ 505 \\ \hline 25025 \\ 250250 \\ \hline 2527525 \end{array} \text{ Ans.}$$

(*c*)

$$\begin{array}{r} 43911 \\ 8420 \\ \hline 878220 \\ 175644 \\ 351288 \\ \hline 369730620 \end{array} \text{ Ans.}$$

(25) Since there are 12 months in a year, the number of days the man works is $25 \times 12 = 300$ days. As he works 10 hours each day, the number of hours that he works in one year is $300 \times 10 = 3,000$ hours. Hence, he receives for his work $3,000 \times 30 = 90,000$ cents, or $90,000 \div 100 = \$900$. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$$\begin{array}{r} \frac{18}{5}\text{)} \; 18 \; (3\frac{3}{5} \quad \text{Ans.} \\ 15 \\ \hline 3 \end{array}$$

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to sixteenths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms

of the fraction by some number which will make the denominator 16. This number evidently is 4; hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4+6+5}{16} = \frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers we have $\frac{10}{16} + \frac{7}{16} = \frac{10+7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

42	Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.
31	
9	
$1\frac{1}{16}$	
$83\frac{1}{16}$ Ans.	

(41) $29\frac{3}{4}+50\frac{5}{8}+41+69\frac{3}{16}=?$ $\frac{3}{4}=\frac{3\times4}{4\times4}=\frac{12}{16}$. $\frac{5}{8}=\frac{5\times2}{8\times2}=\frac{10}{16}$. $\frac{12}{16}+\frac{10}{16}+\frac{3}{16}=\frac{12+10+3}{16}=\frac{25}{16}=1\frac{9}{16}$

The problem now becomes $29+50+41+69+1\frac{9}{16}=?$

$$\begin{array}{rl}
29 & \text{square inches.}\\
50 & \text{square inches.}\\
41 & \text{square inches.}\\
69 & \text{square inches.}\\
1\frac{9}{16} & \text{square inches.}\\
\hline
190\frac{9}{16} & \text{square inches.} \quad \text{Ans.}
\end{array}$$

(42) (*a*) $\dfrac{7}{\frac{3}{16}}=7\div\frac{3}{16}=7\times\frac{16}{3}=\frac{7\times16}{3}=\frac{112}{3}=37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}}=\frac{15}{32}\div\frac{5}{8}=\frac{15}{32}\times\frac{8}{5}=\dfrac{\overset{3}{\cancel{15}}\times\cancel{8}}{\underset{4}{\cancel{32}}\times\cancel{5}}=\frac{3}{4}$. Ans.

(*c*) $\dfrac{\frac{4+3}{2+6}}{5}=\dfrac{\frac{7}{8}}{5}=\frac{7}{8\times5}=\frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32}=\frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8}-\frac{7}{16}=?$ When the *denominators* of fractions are *not alike* it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the*

fractions to fractions having a common denominator. Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \text{ Ans.}$$

(*b*) $13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16 - 7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}$. $7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}$.

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$ and

$$\frac{89}{16} = 16)89(5\frac{9}{16}$$
$$\begin{array}{r} 80 \\ \hline 9 \\ \hline 16 \end{array}$$

the same result that was obtained by the first method.

(*c*) $312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18 - 5}{32} = \frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array} \quad 83 + \frac{13}{32} = 83\frac{13}{32}. \text{ Ans.}$$

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles.

Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|l} 5 & 12,\ 5,\ 35 \\ \hline 7 & 12,\ 1,\ 7 \\ \hline & 12,\ 1,\ 1, \end{array} \text{ or } 5\times7\times12=420.$$

(46)

$$\begin{array}{rl} & 573\frac{4}{5} \text{ tons.} \\ & 216\frac{5}{8} \text{ tons.} \\ \hline \textit{difference} & 357\frac{7}{40} \text{ tons. Ans.} \end{array} \qquad \begin{array}{l} \frac{4}{5}=\frac{32}{40} \\ \frac{5}{8}=\frac{25}{40} \\ \hline \frac{7}{40}=\textit{difference.} \end{array}$$

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans

(48) Referring to Arts. **114** and **116**,

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45 =

$$\frac{\cancel{2}\times\cancel{3}\times7\times19\times\cancel{11}\times7\times5\times\overset{\overset{3}{\cancel{15}}}{\cancel{45}}}{\cancel{3}\times4\times\cancel{11}\times\underset{4}{\cancel{20}}\times1\times8\times\underset{\cancel{3}}{\cancel{6}}\times1}=\frac{7\times19\times7\times5\times3}{4\times4\times8}=\frac{13{,}965}{128}=$$

$109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.\quad 12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers

to improper fractions. $\frac{845}{4} \times \frac{15}{8} = \frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently $\frac{12,675}{32} \div 2\frac{1}{2} = \frac{\overset{2,535}{\cancel{12,675}}}{\underset{16}{\cancel{32}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{2,535}{16} = 158\frac{7}{16}$ pounds. The amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

	tenths.	hundredths.	
.	0	8	= *Eight* **hundredths.**

	tenths.	hundredths.	thousandths.	
.	1	3	1	= *One hundred thirty-one* **thousandths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
0	0	0	1	= *One* **ten-thousandth.**

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
0	0	0	0	2	7	= *Twenty-seven* **millionths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
0	1	0	8	= *One hundred eight* **ten-thousandths.**

tenths. hundredths. thousandths. ten-thousandths.

93.0 1 0 1 = Ninety-three, and *one hundred one* **ten-thousandths**

In reading decimals, read the number just as you would i there were no ciphers before it. Then count from the decima point towards the right, beginning with tenths, to as man places as there are figures, and the *name* of the last figur must be annexed to the previous reading of the figures t give the decimal reading. Thus, in the first example above the simple reading of the figure is *eight*, and the name of it position in the decimal scale is **hundredths,** so that th decimal reading is *eight* **hundredths.** Similarly, the fig ures in the fourth example are ordinarily read *twenty-seven* the name of the position of the figure 7 in the decimal scal is **millionths,** giving, therefore, the decimal reading a *twenty-seven* **millionths.**

If there should be a whole number before the decima point, read it as you would read any whole number, an read the decimal as you would if the whole number wer not there; or, read the whole number and then say, "and so many hundredths, thousandths, or whatever it may be as "ninety-three, *and* one hundred one ten thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of unit, and is expressed by a numerator and a denominator while a decimal fraction is a number of *tenths*, *hundredths* *thousandths*, etc., of a unit, and is expressed by placing period (.), called a decimal point, to the left of the figure of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator. Ans.

$$\frac{7}{8}\quad 8)\underline{7.000}\quad .875 \text{ Ans.}$$

$$\frac{5}{32}\quad 32)5.00000(.15625 \text{ Ans.}$$

```
32)5.00000(.15625  Ans.
   32
   ---
   180
   160
   ---
    200
    192
    ---
     80
     64
     ---
     160
     160
     ---
```

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator $(32.5 + .29 + 1.5)$ is to be divided by the denominator $(4.7 + 9)$. The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

```
          32.5
        +   .29
        +  1.5
 13.7 ) 34.29000 ( 2.5029  Ans.
        274
  4.7   ---
+ 9.0    689
 ----    685
 13.7    ---
          400
          274
          ---
          1260
          1233
          ----
            27
```

Since there are 5 decimal places in the dividend and 1 in the divisor, there are $5 - 1$ or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283 \times \overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283 \times \overline{8+5}}{2.63} = ? \quad \overline{8+5} = 13.$$

$$\begin{array}{r} 1.283 \\ \times 13 \\ \hline 3849 \\ 1283 \end{array}$$

$$\begin{array}{l} 2.63)\overline{16.679000}(6.3418 \quad \text{Ans.} \\ \underline{1578} \\ 899 \\ \underline{789} \\ 1100 \\ \underline{1052} \\ 480 \end{array} \qquad \begin{array}{r} 480 \\ \underline{263} \\ 2170 \\ \underline{2104} \\ 66 \end{array}$$

(*c*) $$\frac{\overline{589+27} \times \overline{163-8}}{25+39} = ?$$

$$\begin{array}{r} 589 \\ +27 \\ \hline 616 \end{array} \qquad \begin{array}{r} 163 \\ -8 \\ \hline 155 \\ \times 616 \\ \hline 930 \\ 155 \\ 930 \end{array}$$

$$\begin{array}{r} 25 \\ +39 \\ \hline 64 \end{array}$$

$$\begin{array}{l} 64)\overline{95480.000}(1491.875 \quad \text{Ans.} \\ \underline{64} \\ 314 \\ \underline{256} \\ 588 \\ \underline{576} \\ 120 \\ \underline{64} \\ 560 \\ \underline{512} \\ 480 \\ \underline{448} \\ 320 \\ \underline{320} \end{array}$$

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$

$$\begin{array}{r} 40.6 \\ +\ \ 7.1 \\ \hline 47.7 \end{array} \qquad \begin{array}{r} 3.029 \\ -\ 1.874 \\ \hline 1.155 \\ \times\ \ 47.7 \\ \hline 8085 \\ 8085 \\ 4620 \\ \hline 55.093500 \end{array}$$

$$\begin{array}{r} 6.27 \\ +\ 8.53 \\ \hline 14.80 \\ -\ \ 8.01 \\ \hline 6.79 \end{array}$$

$$6.79)55.093500(8.1139. \text{ Ans}$$

$$\begin{array}{l} 5432 \\ \hline 773 \\ 679 \\ \hline 945 \\ 679 \\ \hline 2660 \\ 2037 \\ \hline 6230 \\ 6111 \\ \hline 119 \end{array}$$

6 decimal places in the dividend — 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875=\dfrac{875}{1{,}000}=\dfrac{175}{200}=\dfrac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\cancel{8}_{\,2}}\times\frac{\cancel{12}^{\,3}}{1}=\frac{21}{2}=10\frac{1}{2} \text{ inches. Ans}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16}\div 12=\frac{\cancel{3}}{16}\times\frac{1}{\cancel{12}_{\,4}}=\frac{1}{64} \text{ of a foot.}$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

... coal at a profit of $1.65 per ton, ... must be as many tons of coal in ... contained times in 36.30, or 22 tons.

1.65) 36.30 (22 tons. Ans.
330

330
330

(63)

```
231 ) 17892.00000 ( 77.45454, or 77.4545 to
      1617               four decimal places.  Ans.
      ----
       1722
       1617
       ----
        1050
         924
        ----
         1260
         1155
         ----
          1050
           924
          ----
          1260
          1155
          ----
          1050
```

(64) $\dfrac{\overset{37.13}{\cancel{74.26}} \times \overset{2}{\cancel{24}} \times \overset{.0952}{\cancel{3.1416}} \times 19 \times 19 \times 350}{\underset{1,000}{\cancel{33,000}} \times \cancel{12} \times \underset{2}{\cancel{4}}} =$

$$\frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \frac{446,618.947600}{1,000} =$$

446.619 to three decimal places. Ans.

```
  37.13        19        361          3.534776
  .0952        19        350            126350
  -----       ---      -----        ----------
   7426       171      18050         176738800
  18565       19       1083         10604328
 33417        ---      ------      21208656
 --------     361      126350      7069552
 3.534776                         3534776
                                  ------------
                                  446618.947600
```

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

$$
\begin{array}{lr}
(a) & 709.6300 \\
 & .8514 \\
\hline
 & 708.7786 \text{ Ans.}
\end{array}
$$

In the above example we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and, as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths;* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

$$
\begin{array}{lr}
(b) & 81.963 \\
 & 1.700 \\
\hline
 & 80.263 \text{ Ans.}
\end{array}
\qquad
\begin{array}{lr}
(c) & 18.00 \\
 & .18 \\
\hline
 & 17.82 \text{ Ans.}
\end{array}
\qquad
\begin{array}{lr}
(d) & 1.000 \\
 & .001 \\
\hline
 & .999 \text{ Ans.}
\end{array}
$$

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .008 \\ \hline .8801 \end{array} \textit{ sum.}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r} 872.1000 \\ .8801 \\ \hline 871.2199 \end{array} \text{ Ans.}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r} 5.0280 \\ .0073 \\ \hline 5.0353 \end{array} \textit{ sum.}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

$$\begin{array}{r} 6.704 \\ 2.380 \\ \hline 4.324 \end{array} \textit{ difference.}$$

The sign between the parentheses indicates that the quantities obtained by performing the above operations, are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation we obtain .7113 as the final result.

$$\begin{array}{r} 5.0353 \\ 4.324 \\ \hline .7113 \end{array} \text{ Ans.}$$

(67) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$$\begin{array}{r} 7 \\ 8\overline{)7.000} \\ \hline .875 \end{array}$$

Subtracting .807 from .875 the remainder is .068, as shown

$$\begin{array}{r} .875 \\ .807 \\ \hline .068 \end{array} \text{ Ans.}$$

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8}$) $\frac{3.000}{.375}$ and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .875 \\ .375 \\ \hline .500 \end{array} \text{ Ans.}$$

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 57).

Adding .15625 and .435, $\begin{array}{rr} & .15625 \\ & .435 \\ \hline \textit{sum} & .59125 \end{array}$ $\frac{21}{100} = .21$; subtracting, $\begin{array}{rr} & .21 \\ & .07 \\ \hline \textit{difference} & .14 \end{array}$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{rr} & .59125 \\ & .14 \\ \hline \textit{difference} & .45125 \end{array} \text{ Ans.}$$

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	0	0	0	3	3	
.0	1	7				
.0	1	7	0	3	3	*sum.*

tenths.	hundredths.	thousandths.	
.5	3		
.2	7	4	
.8	0	4	*sum.*

$$\begin{array}{rrl} & .804 & \textit{larger sum,} \\ & .017033 & \textit{smaller sum,} \\ \hline \textit{difference} & .786967 & \text{Ans.} \end{array}$$

(68) In addition of decimals the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

```
 .125
 .7
 .089
 .4005
 .9
 .000027
2.214527  Ans.
```

(69)

```
 927.416
   8.274
 372.6
  62.07938
1370.36938  Ans.
```

(70) tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.

```
.017
.2
.000047
.217047 = Ans.
```

(71) (*a*)

```
 .107
 .013
  321
 107
.001391  Ans.
```

There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

(*b*)

```
   203
   2.03
   609
  4060
  412.09
   .203
 123627
824180
83.65427  Ans.
```

There are two decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or two decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

$$\begin{array}{r} 31.85 \\ 2.7 \\ \hline 22295 \\ 6370 \\ \hline 85.995 \end{array}$$

The parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r} 3.160 \\ .316 \\ \hline 2.844 \end{array}$$

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

$$\begin{array}{r} 85.995 \\ 2.844 \\ \hline 343980 \\ 343980 \\ 687960 \\ 171990 \\ \hline 244.569780 \end{array} \text{ Ans.}$$

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

$$\begin{array}{r} 107.8 \\ +\quad 6.541 \\ \hline 114.341 \\ -\quad 31.96 \\ \hline 82.381 \\ \times\quad 1.742 \\ \hline 164762 \\ 329524 \\ 576667 \\ 82381 \\ \hline 143.507702 \end{array} \text{ Ans.}$$

(**72**) (*a*) $\left(\frac{7}{16} - .13\right) \times \overline{.625 + \frac{5}{8}} = ?$

First perform the operation indicated by the parenthesis.

$$\frac{7}{16}=16\,)\,7.0000\,(.4375$$

$$\begin{array}{r} 64 \\ \hline 60 \\ 48 \\ \hline 120 \\ 112 \\ \hline 80 \\ 80 \\ \hline \end{array}$$

We point off four decimal places since we annexed four ciphers.

$$\begin{array}{r} .4375 \\ .13 \\ \hline .3075 \end{array}$$

Subtracting, we obtain .3075

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$$\frac{5}{8}=8\,\underline{)\,5.000}$$
$$.625$$

Adding the terms included by the vinculum, we obtain

$$\begin{array}{r} .625 \\ .625 \\ \hline 1.250 \end{array}$$

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

$$\begin{array}{r} .3075 \\ 1.25 \\ \hline 15375 \\ 6150 \\ 3075 \\ \hline .384375 \end{array}$$ Ans.

(*b*) $\left(\frac{19}{32}\times .21\right)-\left(.02\times\frac{3}{16}\right)=?$

$.21=\frac{21}{100}$. $\frac{19}{32}\times\frac{21}{100}=\frac{399}{3200}$. $.02=\frac{2}{100}$. $\frac{2}{100}\times\frac{3}{16}=\frac{6}{1600}=\frac{3}{800}$.

$\frac{3}{800}=\frac{3\times 4}{800\times 4}=\frac{12}{3200}$. $\frac{399}{3200}-\frac{12}{3200}=\frac{399-12}{3200}=\frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$$\frac{387}{3200}\big)\,387.0000000\,(.1209375 \quad \text{Ans.}$$

```
3200)387.0000000(.1209375  Ans.
     3200
     ----
      6700
      6400
      ----
       30000
       28800
       -----
        12000
         9600
        -----
         24000
         22400
         -----
          16000
          16000
          -----
```

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $\left(\frac{13}{4} + .013 - 2.17\right) \times \overline{13\frac{1}{4} - 7\frac{5}{16}} = ?$

$\frac{13}{4} = \frac{13}{4})13.00$
$\quad\quad\quad 3.25$

Point off two decimal places, since two ciphers were annexed to the dividend.

$$\begin{array}{r} 3.25 \\ +\ .013 \\ \hline 3.263 \\ -\ 2.17 \\ \hline 1.093 \end{array}$$

$\frac{5}{16}$ reduced to a decimal is .3125, since

```
 5
--)5.0000(.3125
16 48
   --
    20
    16
    --
     40
     32
     --
      80
      80
      --
```

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16} = 7.3125$, and $13\frac{1}{4} = 13.25$, since $\frac{1}{4} = \frac{1}{4})\underline{1.00}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad .25$

$$\begin{array}{r} 13.25 \\ -\ \ 7.3125 \\ \hline 5.9375 \end{array} \qquad\qquad \begin{array}{r} 5.9375 \\ \times\ \ 1.093 \\ \hline 178125 \\ 534375 \\ 593750 \\ \hline 6.4896875 \end{array} \text{ Ans.}$$

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75$. Ans. Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{\cancel{8}_4} \times \frac{\cancel{2}}{1} = \frac{7}{4} = 1\frac{3}{4}$. Since $\frac{3}{4} = $

$$\begin{array}{r} 4)\,3.00\,(.75 \\ 28 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

$1\frac{3}{4} = 1.75$, the same answer as above.

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{\cancel{8}_4} \times \frac{\cancel{2}}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are $3 - 1$, or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1{,}000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1{,}000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to sixteenths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the de-

nominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\cancel{3}}{\underset{2}{\cancel{32}}} \times \frac{\cancel{16}}{\cancel{3}} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(74) $\dfrac{1.25 \times 20 \times 3}{\frac{87 + (11 \times 8)}{459 + 32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\dfrac{87 + (11 \times 8)}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The numerator is combined as though it were written $87 + (11 \times 8)$, and its result is

$$\begin{array}{r} 11 \\ 8 \text{ times} \\ \hline 88 \\ +87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1,473}{7} = 210\tfrac{3}{7}. \quad \text{Ans.}$$

(75) 1 plus $.001 = 1.001$. .01 plus $.000001 = .010001$. And $1.001 - .010001 =$

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \quad \text{Ans.}$$

ARITHMETIC.

(QUESTIONS 76–167.)

(76) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. $= .25 \times 8{,}428$ lb. $= 2{,}107$ lb. Ans.

(77) Here \$100 is the base and $1\% = .01$ is the rate. Then, $.01 \times \$100 = \$1$. Ans.

(78) $\frac{1}{2}\%$ means one-half of one per cent. Since 1% is .01, $\frac{1}{2}\%$ is .005, for, $2\,\underline{).010}$ / $.005$. And $.005 \times \$35{,}000 = \$175$. Ans.

(79) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **193,**

rate = percentage ÷ base;
rate $= 2 \div 50 = .04$ or 4%. Ans.

(80) By Art. **193,** rate = percentage ÷ base.*

As percentage = 10 and base = 10, we have rate $= 10 \div 10 = 1 = 100\%$. Hence, 10 is 100% of 10. Ans.

(81) (*a*) Rate = percentage ÷ by base. Art. **193.**

As percentage = \$176.54 and base = \$2,522, we have

rate $= 176.54 \div 2{,}522 = .07 = 7\%$. Ans.

$$\begin{array}{r} 2522\,\underline{)\,176.54} \\ .07 \end{array}$$

---

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

(*b*) Base = percentage ÷ rate. Art. **192.**
As percentage = 16.96 and rate = 8% = .08, we have

$$\text{base} = 16.96 \div .08 = 212. \quad \text{Ans.}$$

$$.08\overline{)16.96}$$
$$212$$

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}$%. Ans.

```
213.5 ) 3.2025 ( .015 = 1½%
        2135
        -----
        10675
        10675
        -----
```

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825, hence percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **193.**

$$\text{Therefore, rate} = 5.175 \div 207 = .025 = .02\frac{1}{2} = 2\frac{1}{2}\%. \quad \text{Ans.}$$

```
207 ) 5.175 ( .025
      414
      ----
      1035
      1035
      ----
```

(**82**) In this problem \$5,500 is the amount, since it equals what he paid for the farm + what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **197,**

base = amount ÷ (1 + rate); hence,
base = \$5,500 ÷ (1 + .15) = \$4,782.61. Ans.

$$\begin{array}{l} 1.15)5500.0000(4782.61 \\ \underline{460} \\ 900 \\ \underline{805} \\ 950 \\ \underline{920} \\ 300 \\ \underline{230} \\ 700 \\ \underline{690} \\ 100 \\ \underline{115} \end{array}$$

The example can also be solved as follows: 100% = cost; if he gained 15%, then $100+15=115\% = \$5,500$, the selling price.

If 115% = \$5,500, $1\% = \frac{1}{115}$ of \$5,500 = \$47.8261, and 100%, or the cost, = 100 × \$47.8261 = \$4,782.61. Ans.

(83)

$$\begin{array}{lll} 24\ \% \text{ of } \$950 = .24 & \times\ 950 = & \$228 \\ 12\frac{1}{2}\% \text{ of } \$950 = .125 & \times\ 950 = & 118.75 \\ \underline{17\ \% \text{ of } \$950 = .17} & \underline{\times\ 950 =} & \underline{161.50} \\ 53\frac{1}{2}\% \text{ of } \$950 & = & \$508.25 \end{array}$$

The total amount of his yearly expenses, then, is \$508.25, hence his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence, $100\% - 53\frac{1}{2}\% = 46\frac{1}{2}\%$ = per cent. saved. And \$950 × .465 = \$441.75 = his yearly savings. Ans.

(84) The percentage is 961.38, and the rate is $.37\frac{1}{2}$. By Art. **192**,

Base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

... method of solv- ... following:

... of a number is ... times the ... and the ... $37\frac{1}{2}$, ... 2,565.68 Ans.

$$
\begin{array}{r}
.375) 962.38000 (2566.6 \\
750 \\
\hline
2123 \\
1875 \\
\hline
2488 \\
2250 \\
\hline
2380 \\
2250 \\
\hline
2550 \\
2250 \\
\hline
3000 \\
3000 \\
\hline
\end{array}
$$

... 125 is 30% of some number; hence ... 30% = the rate, and the required ... Applying rule, Art. **192**,

... = \$1,125 ÷ .30 = \$3,750.

... of the property, one of the fourths is $\frac{1}{3}$ ... and $\frac{4}{4}$ of the entire property, is 4 × \$1,250 ...

... \$4,810 is the difference and 35% the rate. By Art. **195**

... (1 − rate)

\$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

	1.00
	.35
	.65

... can also be explained as follows: 100% = the sum ... diminished by 35%. 100% − 35% ... which is \$4,810

65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 ×
= \$7,400. Ans.

7) In this example the sales on Monday amounted to 55, which was $12\frac{1}{2}$% of the sales for the entire week; we have given the percentage, \$197.55, and the rate, , and the required number (or the amount of sales for reek) equals the base. By Art. **192,**

Base = percentage ÷ rate = \$197.55 ÷ .125;

or,

```
.125)197.5500(1580.4  Ans.
     125
     ---
      725
      625
      ----
      1005
      1000
      ----
         500
         500
         ---
```

ıerefore, base = \$1,580.40, which also equals the sales he week.

8) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road. vish to find the *entire* length.

.5 miles is the percentage, $12\frac{1}{2}$% is the rate, and the en- length will be the base. By Art. **192,**

Base = percentage ÷ rate = 16.5 ÷ $.12\frac{1}{2}$.

```
.125)16.500(132 miles.  Ans.
     125
     ---
      400
      375
      ---
       250
       250
       ---
```

(89) Here we have given the difference, or $35, and the rate, or 60%, to find the base. We use the rule in Art. **198**,

Base = difference ÷ (1 − rate)

= $35 ÷ (1 − .60) = $35 ÷ .40 = $87.50. Ans.

```
.40)35.000(87.5
    320
    ---
     300
     280
     ---
      200
      200
      ---
```

Or, 100% = whole debt; 100% − 60% = 40% = $35.

If 40% = $35, then $1\% = \frac{1}{40}$ of $\$35 = \frac{35}{40}$, and $100\% = \frac{35}{40} \times 100 = \$87.50$. Ans.

(90) 28 rd. 4 yd. 2 ft. 10 in. to inches.

```
  28
 × 5½
 ----
  154
 +  4
 ----
  158 yards
 ×  3
 ----
  474
 +  2
 ----
  476 feet
 × 12
 ----
 5712
 + 10
 ----
 5722 inches.  Ans.
```

Since there are 5½ yards in one rod, in 28 rods there are 28 × 5½ or 154 yards; 154 yards plus 4 yards = 158 yards. There are 3 feet in one yard; therefore, in 158 yards there are 3 × 158 or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in one foot, and in 476 feet there are 12 × 476 or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(91)

```
12 ) 5722 inches.
 3 ) 476 + 10 inches.
5½ ) 158 + 2 feet.
      28 + 4 yards.
```

Ans. = 28 rd. 4 yd. 2 ft. 10 in.

EXPLANATION.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 ft. and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in one rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches there are 28 rd. 4 yd. 2 ft. 10 in.

(92)

$$\begin{array}{rl} 5 & \text{weeks } 3.5 \text{ days.} \\ \times\ \underline{7} & \\ 35 & \text{days in 5 weeks.} \\ +\ \underline{3.5} & \\ 38.5 & \text{days.} \end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{rl} 38.5 & \text{days} \\ \times\ \underline{24} & \text{hours in one day.} \\ 1540 & \\ \underline{770} & \\ 924.0 & \text{hours in 38.5 days.} \\ \times\ \underline{60} & \text{minutes in one hour.} \\ 55440 & \text{minutes in 38.5 days.} \\ \times\ \underline{60} & \text{seconds in one minute.} \\ 3326400 & \text{seconds in 38.5 days. Ans.} \end{array}$$

(93) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining.

Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

[illegible]

Ans. [illegible]

94. [illegible]

Ans. [illegible]

[illegible] remaining [illegible] remaining [illegible]

95. [illegible]

Ans. [illegible]

[illegible] in one cubic foot; [illegible] as many cubic feet as [illegible] and [illegible] as a remainder. [illegible] contained times in [illegible] remaining. Then, in [illegible]

96. [illegible] in columns, [illegible] the same column.

[illegible]

	3	$2\frac{1}{2}$	[illegible]	[illegible]
or	3	2	2	1 Ans.

Explanation.—We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; as 12 in. make one foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) $+ 2 + 1 + 2 = 6$ ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) $+ 4 + 2 = 8$ yd. Since $5\frac{1}{2}$ yd. = 1 rod, 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column, and reserve 1 rd. for the next column; 1 (reserved) $+ 2 = 3$ rd.

Ans. =	3 rd.	$2\frac{1}{2}$ yd.	0 ft.	7 in.	
or,	3 rd.	2 yd.	1 ft.	13 in.	
or,	3 rd.	2 yd.	2 ft.	1 in.	Ans.

(97) We write the compound numbers so that the units of the same denomination shall stand in the same column. Beginning to add with the lowest denomination, we find that the sum of the gills is $1 + 2 + 3 = 6$. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) $+ 5 + 1 + 1 = 8$. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) $+ 8 + 3 = 15$. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) $+ 4 + 6 + 3 = 16$ gal. Since we can not reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

(102) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$ or $\frac{11}{2}$ is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div \frac{11}{2} = 100 \times \frac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

$$\begin{array}{r|l} 11\,)\;200 & (\,18\frac{2}{11}\text{ rd.} \\ 11 & \\ \hline 90 & \\ 88 & \\ \hline 2 & \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rods there are $\frac{\cancel{2}}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{2}}$, or one yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

(103)

A.	sq. rd.	sq. yd.	
114	80	25	
75	70	30	
39	9	$25\frac{1}{4}$	Ans.

EXPLANATION.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We can not subtract 30 from 25, so we take one square rod ($= 30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards leaves $25\frac{1}{4}$ square yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from 114 acres, which leaves 39 acres, which we place under the column of acres.

(104) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal $0 + 1$, or 1 pint. We can not reduce 1 pint to any higher denomination, so we place it under the pint column. The number of quarts is $3 + 2$, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quart column, and reserve the 1 gallon to add to the column of

gal.	qt.	pt.
10	2	1
26	3	0
37 gal.	1 qt.	1 pt.

(108) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $= 879$ ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3) 8 7 9 ft. 9 in.
5.5) 2 9 3 yd.
5 3 rd. $1\frac{1}{2}$ yd.

Then, answer $= 53$ rd. $1\frac{1}{2}$ yd. 0 ft. 9 in.; or 53 rd. 1 yd. 2 ft. 3 in.

(109)

	qt.	pt.	gi.	
	3	1	3	
			4.7	
	1 8.2 qt.	0	.1	
or,	1 8 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. $= 14.1$ gi. As 4 gi. $= 1$ pt., there are as many pints in 14.1 gi. as 4 is contained times in $14.1 = 3.5$ pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. $= 4.7$ pt.; $4.7 + 3.5$ pt. $= 8.2$ pt. As 2 pt. $= 1$ qt., there are as many quarts in 8.2 pt. as 2 is contained times in $8.2 = 4.1$ qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. $= 14.1$; $14.1 + 4.1 = 18.2$ qt. The answer now is 18.2 qt. 0 pt. .1

gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = .2 × 8 = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then, we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

$$\begin{array}{rl} & 3 \text{ qt.} \\ \times & 2 \text{ pt.} \\ \hline & 6 \text{ pt.} \\ + & 1 \text{ pt.} \\ \hline & 7 \text{ pt.} \\ \times & 4 \text{ gi.} \\ \hline & 28 \text{ gi.} \\ + & 3 \text{ gi.} \\ \hline & 31 \text{ gi.} \end{array}$$

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

$$\begin{array}{r|l} 4 \,)\, 145.7 & \text{gi.} \\ 2 \,)\, 36 & \text{pt.} + 1.7 \text{ gi.} \\ 18 & \text{qt.} + 0 \text{ pt.} \end{array}$$

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(110) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

$$\begin{array}{rrl} & 3 & \text{lb. 10 oz. 13 pwt. 12 gr.} \\ \times & 12 & \\ \hline & 36 & \text{oz.} \\ + & 10 & \\ \hline & 46 & \text{oz.} \\ \times & 20 & \\ \hline & 920 & \text{pwt.} \\ + & 13 & \\ \hline & 933 & \text{pwt.} \\ \times & 24 & \\ \hline & 22392 & \text{gr.} \\ + & 12 & \\ \hline & 22404 & \text{gr.} \end{array}$$

22.404 gr. × 1.5 = 33,606 gr.

$$\begin{array}{r|l} 24 \,)\, 33606 & \text{gr.} \\ 20 \,)\, 1400 & \text{pwt.} + 6 \text{ gr} \\ 12 \,)\, 70 & \text{oz.} + 0 \text{ pwt.} \\ 5 & \text{lb.} + 10 \text{ oz.} \end{array}$$

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pwt. as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We can not reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

	lb.	oz.	pwt.	gr.	
	3	10	13	12	
				1.5	
	4.5	15	19.5	18	
or,	4	21	19	30	
or,	5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. $\times$ 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr., but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. $\times$ 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the lb. column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(111) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain $9 \times$ (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. reserved equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu. for the product of the bushels. 9 times 2 bu. plus the 8 bu. reserved equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

	bu.	pk.	qt.
	2	3	6
			9
	18	27	54
or,	26	1	6

(112) (7 T. 15 cwt. 10.5 lb.) $\times 1.7 = ?$ When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then, multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

$$
\begin{array}{rl}
 & 7\text{ T. } 15\text{ cwt. } 10.5\text{ lb.} \\
\times & \underline{20} \\
 & 140\ \text{cwt.} \\
 & \underline{15} \\
 & 155\ \text{cwt.} \\
\times & \underline{100} \\
 & 15500\ \text{lb.} \\
 & \underline{10.5} \\
 & 15510.5\ \text{lb.}
\end{array}
$$

15,510.5 lb. $\times 1.7 =$ 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations

$$
\begin{array}{r l}
100\,) & 26367.85 \text{ lb.} \\ \hline
20\,) & 263 \text{ cwt.} + 67.85 \text{ lb.} \\ \hline
 & 13 \text{ T.} + 3 \text{ cwt.}
\end{array}
$$

There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 tons and 3 cwt. remaining. Since we cannot reduce 13 tons any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

(113) 7) 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft.
51 A. 31 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(114) 12) 282 bu. 3 pk. 1 qt. 1 pt.
23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt. Ans.

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write

2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(115) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mi. contains 5,280 ft.; hence, 23 mi. contain $5,280 \times 23 = 121,440$ ft.

121,440 ft. $\div$ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = $2 \times 4,048$, or 8,096 rails. Ans.

(116) In this case where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either and then divide as in simple numbers.

```
  1 bu. 1 pk. 7 qt.          3 5 6 bu. 3 pk. 5 qt.
  × 4                              × 4
  ----                       ---------
    4 pk.                     1 4 2 4 pk.
  + 1                              + 3
  ----                       ---------
    5 pk.                     1 4 2 7 pk.
  × 8                              × 8
  ----                       ---------
   4 0 qt.                  1 1 4 1 6 qt.
  + 7                              + 5
  ----                       ---------
   4 7 qt.                  1 1 4 2 1 qt.
47 ) 1 1 4 2 1 ( 2 4 3
     9 4
     ---                 11,421 qt. ÷ 47 qt. = 243 boxes
     2 0 2                                        Ans.
     1 8 8
     -----
       1 4 1
       1 4 1
       -----
```

(117) We must first reduce 16 square miles to acres.

In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16×640 A. = 10,240 A.

```
62 ) 1 0 2 4 0 A.
     ----------------------------------------------------
     1 6 5 A. 25 sq.rd. 24 sq.yd. 3 sq.ft. 80+ sq. in.  Ans.
```

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1,600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 times $30\frac{1}{4}$ sq. yd. $= 1,512\frac{1}{2}$ sq. yd. 62 is contained in $1,512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4,968$ sq. in. 62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining.

We write 80 sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(118) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11,664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 108 \\ \hline 11664 \end{array}$$

(119)

```
      181.25
      181.25
      ------
       90625
      36250
     18125
    145000
    18125
 -----------
  32851.5625
      181.25
 -----------
  1642578125
  657031250
 328515625
2628125000
328515625
--------------
5954345.703125
```

The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5{,}954{,}345.703125$. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the final product.

(120)

```
          27.61
          27.61
          -----
           2761
          16566
         19327
         5522
       --------
       762.3121
          27.61
       --------
        7623121
      45738726
     53361847
    15246242
   ------------
   21047.437081
          27.61
   ------------
    21047437081
  126284622486
 147332059567
42094874162
---------------
581119.73780641
```

The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is $27.61 \times 27.61 \times 27.61 \times 27.61 = 581{,}119.73780641$. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are $6 + 2 = 8$ decimal places in the final product.

(121) (*a*) $106^2 = 106 \times 106 = 11{,}236$. **Ans.**

$$\begin{array}{r} 106 \\ \underline{106} \\ 636 \\ \underline{1060} \\ 11236 \end{array}$$

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. **Ans.**

$\frac{1}{8} = 8\overline{)1.000}$
$\phantom{\frac{1}{8} = 8)}.125$

$$\begin{array}{r} 182.125 \\ \underline{182.125} \\ 910625 \\ 364250 \\ 182125 \\ 364250 \\ 1457000 \\ \underline{182125} \\ 33169.515625 \end{array}$$

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3 + 3 = 6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

$$\begin{array}{r} .005 \\ \underline{.005} \\ .000025 \end{array} \text{ Ans.}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

$$\begin{array}{r} .0063 \\ \underline{.0063} \\ 189 \\ \underline{378} \\ .00003969 \end{array} \text{ Ans.}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

$$\begin{array}{r} 10.06 \\ \underline{10.06} \\ 6036 \\ \underline{100600} \\ 101.2036 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2 + 2 = 4$ decimal places in the product.

(122) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

$$
\begin{array}{r}
753 \\
\underline{753} \\
2259 \\
3765 \\
\underline{5271} \\
567009 \\
\underline{753} \\
1701027 \\
2835045 \\
\underline{3969063} \\
426957777
\end{array}
$$

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

$$
\begin{array}{r}
987.4 \\
\underline{987.4} \\
39496 \\
69118 \\
78992 \\
\underline{88866} \\
974958.76 \\
\underline{987.4} \\
389983504 \\
682471132 \\
779967008 \\
\underline{877462884} \\
962674279.624
\end{array}
$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and one in the multiplier, there are $2 + 1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

$$
\begin{array}{r}
.005 \\
\underline{.005} \\
.000025 \\
\underline{.005} \\
000000125
\end{array}
$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the six decimal places.

Since there are six decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case **we prefix six** ciphers to form the nine decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$ Ans.

```
        .4044
        .4044
        -----
        16176
       16176
      161760
   ----------
    .16353936
        .4044
   ----------
     65415744
    65415744
   654157440
-------------
.066135317184
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(123) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$ Ans.

(124) $3^4 = 3 \times 3 \times 3 \times 3 = 81.$ Ans.

(125) (*a*) $67.85^2 = 67.85 \times 67.85 = 4{,}603.6225.$ Ans.

```
    67.85
    67.85
    -----
    33925
   54280
  47495
 40710
---------
4603.6225  Ans.
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the product.

(*b*) $967{,}845^2 = 967{,}845 \times 967{,}845 = 936{,}723{,}944{,}025.$ Ans.

```
      967845
      967845
      ------
     4839225
    3871380
   7742760
  6774915
 5807070
8710605
------------
936723944025
```

(*c*) **A fraction may be raised to any power by raising both numerator and denominator to the required term.**

$$\text{Thus, } \left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3}{8 \times 8} = \frac{9}{64}. \quad \text{Ans.}$$

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}$. Ans.

(126) (*a*) $5^{10} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 9{,}765{,}625$. Ans.

(*b*) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049$. Ans.

$$\begin{array}{r}
5 \\
5 \\
\hline
25 \\
5 \\
\hline
125 \\
5 \\
\hline
625 \\
5 \\
\hline
3125 \\
5 \\
\hline
15625 \\
5 \\
\hline
78125 \\
5 \\
\hline
390625 \\
5 \\
\hline
1953125 \\
5 \\
\hline
9765625
\end{array}
\qquad\qquad
\begin{array}{r}
9 \\
9 \\
\hline
81 \\
9 \\
\hline
729 \\
9 \\
\hline
6561 \\
9 \\
\hline
59049
\end{array}$$

(127) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736$. Ans.
Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2+1=3$ decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3+1=4$ decimal places in the final product.

$$\begin{array}{r}
1.2\\
1.2\\
\hline
2\,4\\
1\,2\\
\hline
1.4\,4\\
1.2\\
\hline
2\,8\,8\\
1\,4\,4\\
\hline
1.7\,2\,8\\
1.2\\
\hline
3\,4\,5\,6\\
1\,7\,2\,8\\
\hline
2.0\,7\,3\,6
\end{array}$$

(*b*) $11^6 = 11 \times 11 \times 11 \times 11 \times 11 \times 11 = 1{,}771{,}561$. Ans

$$\begin{array}{r}
11\\
11\\
\hline
121\\
11\\
\hline
1331\\
11\\
\hline
14641\\
11\\
\hline
161051\\
11\\
\hline
1771561
\end{array}$$

(*c*) $1^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$. Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

$$\begin{array}{r} .01 \\ .01 \\ \hline .0001 \\ .01 \\ \hline .000001 \\ .01 \\ \hline .00000001 \end{array}$$

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product. It is necessary to prefix 1 cipher to the product.

$$\begin{array}{r} .1 \\ .1 \\ \hline .01 \\ .1 \\ \hline .001 \\ .1 \\ \hline .0001 \\ .1 \\ \hline .00001 \end{array}$$

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4 + 1$ or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(128) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

$$\begin{array}{r}
.0133 \\
.0133 \\ \hline
399 \\
399 \\
133 \\ \hline
.00017689 \\
.0133 \\ \hline
53067 \\
53067 \\
17689 \\ \hline
.000002352637
\end{array}$$

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ 27,273,890.942264331. Ans.

$$\begin{array}{r}
301.011 \\
301.011 \\ \hline
301011 \\
301011 \\
3010110 \\
9030330 \\ \hline
90607.622121 \\
301.011 \\ \hline
90607622121 \\
90607622121 \\
906076221210 \\
2718228663630 \\ \hline
27273890.942264331
\end{array}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6 + 3 = 9$ decimal places in the final product.

(*c*) $\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}.$ Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734+.$$

Ans.

$$3\frac{3}{4} = \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \end{array} \qquad \begin{array}{l} 64)3375.000(52.734+ \\ 320 \\ \hline 175 \\ 128 \\ \hline 470 \\ 448 \\ \hline 220 \\ 192 \\ \hline .280 \\ 256 \\ \hline 24 \end{array}$$

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(129) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(130) (*a*)

```
        1
        1
       20
        8
       28
        8
      360
        6
      366
        6
     3720
        7
     3727
        7
     3734
```

$\sqrt{3'48'67'84.40'10} = 1867.29+$ Ans.

```
1
248
224
 2467
 2196
  27184
  26089
3734)1095.000(.293 or .29+
     7468
     34820
     33606
      12140
```

EXPLANATION.—Applying the short method described in Art. **272,** we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1095, and the last trial divisor (with the cipher omitted) is 3734. Dividing 1095 by 3734, as shown, the quotient is .293 +, or .29 + using two figures. Annexing to the root, gives 1,867.29 +. Ans.

```
(b) (a) 3      √9'00'00'99.40'09'00 = 3000.0165 + Ans.
        3  (b) 9
        ──     ─────────────
    (d) 60 (c)   0000994009
         0            600001
        ───          ──────────
        600          39400800
          0          36000156
        ────         ──────────
        6000          3400644
           0
        ─────
        60000
            0
        ──────
        600000
             1
        ──────
        600001
             1
        ───────
        6000020
              6
        ───────
        6000026
```

Explanation.—Beginning at the decimal point we point off the whole number into periods of *two* figures each, proceeding from ***right*** to ***left;*** also, point off the decimal into periods of *two* figures each, proceeding from *left* to *right.* The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher

in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017 –.

In this problem there are *seven* periods—four in the whole number and three in the decimal—hence, there will be *seven* figures in the root, *four* figures constituting the whole number, and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017 -$

(*c*)

3	$\sqrt{.00'12'25} = .035.$ Ans.
3	00
60	12
5	9
65	325
	325

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem solved in Art. **264.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(131) (*a*)

1	$\sqrt{1'07'95.21} = 103.9$ Ans.
1	1
20	0795
0	609
200	18621
3	18621
203	
3	
2060	
9	
2069	

(*b*)

2	$\sqrt{7'30'08.04} = 270.2$ Ans.
2	4
40	330
7	329
47	10804
7	10804
5400	
2	
5402	

(*c*)

```
                   √90.00'00'00 = 9.487 −
 9                  81                Ans.
 9                  ---
----                 900
 180                 736
   4                 -----
----                 16400
 184                 15104
   4                 -----
----        1896 ) 1296.00 (.68 + or .7 −
 1880                11376
    8                -----
-----                 15840
 1888                 15168
    8                 -----
-----
 1896
```

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.09} = .3$. Ans.

(132) (*a*)

```
  6        36        ∛.327'680'000 = .6894 +
  6        72         216                    Ans.
---       -------     ------
 12        10800      111680
  6         1504       98432
---       -------     --------
180        12304      13248000
  8         1568      12650769
---       -------     --------
188       1387200   1424163 ) 597231.00 (.41+ or .4+
  8         18441             5696652
---       -------             -------
196       1405641              2756580
  8         18522              1424163
----      -------              -------
2040      1424163
   9
----
2049
   9
----
2058
   9
----
2067
```

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **284.**

EXPLANATION.—(1) When extracting the *cube* root we divide the power into periods of three figures each. Always begin at the decimal point, and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680; annex it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root, and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and add the second figure of the root, and we obtain 188. This, multiplied by the second figure of the root, 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor* 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the *first new*

correction. This, multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *new second correction.* Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex *one* cipher to the *new second correction,* and to this add the third figure of the root, 9, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor, 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

(*b*)

4	16	$\sqrt[3]{74'088} = 42$ Ans.
4	32	64
8	4800	10088
4	244	10088
120	5044	
2		
122		

(*c*)

4	16	$\sqrt[3]{92'416} = 45.212 -$ Ans.
4	32	64
8	4800	28416
4	625	27125
120		
5	5425	1291000
125	650	1220408
5	607500	612912)70592.000(.115
130	2704	612912
5	610204	930080
1350	2708	612912
2		
1352	612912	3171680
2		3064560
1354		107120

(*d*)

7	49	$\sqrt[3]{.373'248} = .72$ Ans.
7	98	343
14	14700	30248
7	424	30248
210	15124	
2		
212		

(133)

1	1	$\sqrt[3]{2.000'000'000} = 1.259921+$ Ans.
1	2	1
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4755243)4383021.000(9217 or .922−
2	1850	42797187
360	4687500	10330230
5	33831	9510486
365	4721331	8197440
5	33912	4755243
370	4755243	34421970
5		
3750		
9		
3759		
9		
3768		

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9217, thus making the last figures of the answer 922, according to Art. **272.** This is not correct in this case; the true answer to eight decimal places being 1.25992104 +; hence, the first three figures

found by division should be used in this case. The reason for the apparent failure of the method in this case to give the seventh figure of the root correctly is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **272,** if it is desired to obtain absolutely correct results.

(134) (*a*)

1	1	$\sqrt[3]{1'758.416'743} = 12.07$ Ans.
1	2	1
2	300	758
1	64	728
30	364	30416743
2	68	30416743
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*b*)

1	1	$\sqrt[3]{1'191'016} = 106$ Ans.
1	2	1
2	30000	191016
1	1836	191016
300	31836	
6		
306		

(*c*) $\sqrt[3]{\frac{4}{32}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}.$ Ans.

(*d*) $\sqrt[3]{\frac{27}{512}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}.$ Ans.

(135)

1	1	$\sqrt[3]{3.000'000'000} = 1.442250-$ Ans.
1	2	1
2	300	2000
1	136	1744
30	436	256000
4	152	241984
34	58800	14016000
4	1696	12458888
38	60496	6238092) 1557112.000 (.2496 or .250 −
4	1712	12476184
420	6220800	30949360
4	8644	24952368
424	6229444	59969920
4	8648	56142828
428	6238092	3827092
4		
4320		
2		
4322		
2		
4324		

(136) (*a*)

1	$\sqrt{1'23.21} = 11.1$ Ans.
1	1
20	23
1	21
21	221
1	221
220	
1	
221	

(*b*)

1	$\sqrt{1'14.92'10} = 10.72+$ Ans.
1	1
200	1492
7	1449
207	4310
7	4284
2140	26
2	
2142	

(*c*)

7	$\sqrt{50'26'81} = 709$ Ans.
7	49
140	12681
0	12681
1400	
9	
1409	

(*d*)

2	$\sqrt{.00'04'12'09} = .0203$ Ans.
2	00
400	04
3	4
403	1209
	1209

(137) (*a*)

1	1	$\sqrt[3]{.006'500'000} = .18663 -$ **Ans**
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 65144.00 (.627 or .63 −
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000} = .2759-$ Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9—
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2	4	$\sqrt[3]{8'036'054'027} = 2,003$ Ans.
2	8	8
4	12000000	036054027
2	18009	36054027
6000	12018009	
3		
6003		

(*d*)

1	1	$\sqrt[3]{.000'004'096} = .016$ Ans.
1	2	000
2	300	004
1	216	1
30	516	3096
6		3096
36		

(*e*)

2	4	$\sqrt[3]{17.000'000} = 2.5713-$ Ans.
2	8	8
4	1200	9000
2	325	7625
60	1525	1375000
5	350	1349593
65	187500	198147) 25407.00 (.128 or .13−
5	5299	198147
70	192799	559230
5	5348	396294
750	198147	162936
7		
757		
7		
764		

(138) (*a*) In this example the index is 4, and equals 2×2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6561} = \sqrt{\sqrt{6561}} = \sqrt{81} = 9$. Ans.

8	$\sqrt{65'61} = 81$	$\sqrt{81} = 9$ **Ans.**
8	64	
160	161	
1	161	
161		

(*b*) In this example the index is 6, and 6 equals 3×2 or 2×3. The root indicated is the sixth root; hence, extract both the square and cube root, it making no particular difference as to which root is extracted first. Thus,

$$\sqrt[6]{\ } = \sqrt[3]{\ } \text{ of the } \sqrt{\ }, \text{ or } \sqrt{\ } \text{ of the } \sqrt[3]{\ }.$$

Hence, $\sqrt[6]{117,649} = \sqrt[3]{\sqrt{117,649}} = \sqrt[3]{343} = 7$. Ans.

3	$\sqrt{11'76'49} = 343$	$\sqrt[3]{343} = 7$ Ans.
3	9	
60	276	
4	256	
64	2049	
1	2049	
680		
3		
683		

(*c*) $\sqrt[6]{.000064} = \sqrt[3]{\sqrt{.000064}} = .2$. **Ans.**

$\sqrt{.000064} = .008$. $\sqrt[3]{.008} = .2$. Hence, $\sqrt[6]{.000064} = .2$. Ans.

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since $8\overline{)3.000}$
$.375$

7	49	$\sqrt[3]{.375'000'000} = .72112+$ Ans.
7	98	343
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523) 194639.00 (.124 or .12+
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+$. Ans.

(139) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1225}}{\sqrt{5476}}$.

3	$\sqrt{12'25} = 35$
3	9
60	325
5	325
65	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}$. Ans.

7	$\sqrt{54'76} = 74$
7	49
140	576
4	576
144	

(*b*)

5	$\sqrt{.33'64} = .58$ Ans.
5	25
100	864
8	864
108	

(*c*)

3	$\sqrt{.10'00'00'00} = .31623-$ Ans.
3	9
60	100
1	61
61	3900
1	3756
620	632) 144.00 (.227 or .23 −
6	1264
626	1760
6	1264
632	496

(*d*) $25.0\frac{3}{4} = 25.075.$

5	$\sqrt{25.07'50'00'00'00} = 5.00749+$ Ans.
5	25
10000	075000
7	70049
10007	495100
7	400576
100140	9452400
4	9013401
100144	438999
4	
1001480	
9	
1001489	

(*e*) $.000\frac{4}{9} = .0004444444+.$

2	$\sqrt{.00'04'44'44'44} = .02108+$ Ans.
2	00
40	04
1	4
41	44
1	41
4200	34444
8	33664
4208	780

(140) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}$.

1	$\sqrt{2.00'00'00'00'00} = 1.41421356+$
1	1
20	100
4	96
24	400
4	281
280	11900
1	11296
281	60400
1	56564
2820	28284) 3836.0000 (.13562 or .1356+
4	28284
2824	100760
4	84852
28280	159080
2	141420
28282	176600
2	169704
28284	6896

1	$\sqrt{1.41'42'13'56} = 1.1892+$ Ans.
1	1
20	41
1	21
21	2042
1	1824
220	21813
8	21321
228	49256
8	47564
2360	1692
9	
2369	
9	
23780	
2	
23782	

It is required in this problem to extract the fourth root of 2 to four decimal places; hence, we must extract the square root twice, since $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$. In the first operation we carry the root to 8 decimal places, in order to carry the root in the second operation to 4 decimal places.

(*b*) $\sqrt[6]{6} = \sqrt[3]{\sqrt{6}}.$

2	$\sqrt{6.00'00'00'00'00} = 2.4494897428+$
2	4
40	200
4	176
44	2400
4	1936
480	46400
4	44001
484	239900
4	195936
4880	4396400
9	3919104
4889	489896)477296.00000(.974280 or .97428+
9	4409064
48980	3638960
4	3429272
48984	2096880
4	1959584
489880	1372960
8	979792
489888	3931680
8	3919168
489896	12512

It is required in this problem to find the sixth root of 6; hence it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1	1	$\sqrt[3]{2.449'489'742'800} = 1.34801 -$
1	2	1 Ans.
2	300	1449
1	99	1197
30	399	252489
3	108	209104
33	50700	43385742
3	1576	43352192
36	52276	5451312) 33550.000 (.006 or .01 −
3	1592	32707872
390	5386800	842128
4	32224	
394	5419024	
4	32288	
398	5451312	
4		
4020		
8		
4028		
8		
4036		

(141) (*a*)

1	$\sqrt{3.14'16} = 1.7725 -$ Ans.
1	1
20	214
7	189
27	2516
7	2429
340	354) 87.00 (.245 + or 25 −
7	708
347	1620
7	1416
354	204

(*b*)

```
   8
   8
 ---
 160
   8
 ---
 168
   8
 ----
 1760
    6
 ----
 1766
    6
 ----
 1772
```

```
√.78'54'00 = .8862 +   Ans.
  64
  ----
  1454
  1344
  -----
   11000
   10596
1772 ) 404.0 (.22 or .2 +
       3544
       ----
        496
```

(142) (*a*)

```
1       1
1       2
-       ----
2       300
1       136
--      ----
30      436
 4      152
--      -----
34      58800
 4       2556
--      -----
38      61356
 4       2592
---     -------
420     6394800
  6       17536
---     -------
426     6412336
  6       17552
---     -------
432     6429888
  6
----
4380
   4
----
4384
   4
----
4388
```

```
∛3.141'600'000 = 1.4646 –
 1                      Ans.
 ----
 2141
 1744
 ------
  397600
  368136
  --------
   29464000
   25649344
6429888 ) 3814656.0 ( .59 or .6 –
          32149440
          --------
           5997120
```

(*b*)

8	64	$\sqrt[3]{.523'600'000}=.80599+$ or $.8060-$ Ans.
8	128	512
16	1920000	11600000
8	12025	9660125
2400	1932025	1944075)1939875.00(.99
5	12050	17496675
2405	1944075	1902075
5		
2410		

(143) $11.7 : 13 :: 20 : x.$ The product of the means equals the product of the extremes.

$11.7x = 13 \times 20$

$11.7x = 260$

$x = \frac{260}{11.7}$) 260.000 (22.22+ Ans.

```
234
 260
 234
  260
  234
   260
   234
    26
```

(144) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27x = 18 \times 3$

$27x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $12^2 : 100^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144x = 10{,}000 \times 4$

$144x = 40{,}000$

$$x = \frac{40{,}000}{144}$$

$$\begin{array}{r|l}
144 \,) & 40000.0 \,(\, 277.7 + \quad \text{Ans.} \\
 & \underline{288} \\
 & 1120 \\
 & \underline{1008} \\
 & \ \ 1120 \\
 & \ \ \underline{1008} \\
 & \ \ \ \ 1120 \\
 & \ \ \ \ \underline{1008} \\
 & \ \ \ \ \ \ 112
\end{array}$$

(145) (*a*) $\frac{4}{x} = \frac{7}{21}$, is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \text{Ans.}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x$.

$$15x = 45 \times 60$$
$$15x = 2{,}700$$
$$x = \frac{2{,}700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600$.

$$150x = 10 \times 600$$
$$150x = 6{,}000$$
$$x = \frac{6{,}000}{150} = 40. \quad \text{Ans.}$$

(146) $x : 5 :: 27 : 12.5.$

$$\begin{array}{r} 5 \\ \hline 12.5\,)\,135.0\,(\,10\frac{4}{5}\ \text{Ans.} \\ 125 \\ \hline 100 \end{array} \quad \frac{100}{125} = \frac{4}{5}$$

(147) $45 : 60 :: x : 24$

$60x = 45 \times 24$

$60x = 1{,}080$

$x = \frac{1{,}080}{60} = 18.$ Ans.

(148) $x : 35 :: 4 : 7.$

$7x = 35 \times 4$

$7x = 140$

$x = \frac{140}{7} = 20.$ Ans.

(149) $9 : x :: 6 : 24.$

$6x = 9 \times 24$

$6x = 216$

$x = \frac{216}{6} = 36.$ Ans

(150)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$10x = 297.$

$x = \frac{297}{10} = 29.7.$ Ans.

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$\begin{array}{r} 1 \\ 1 \\ \hline 2 \\ 1 \\ \hline 30 \\ 1 \\ \hline 31 \end{array} \qquad \begin{array}{r} 1 \\ 2 \\ \hline 300 \\ 31 \\ \hline 331 \end{array} \qquad \begin{array}{r} 1'331\,(\,11 \\ 1 \\ \hline 331 \\ 331 \\ \hline \end{array}$$

(151) $64 : 81 = 21^2 : x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} = \sqrt{21^2} : \sqrt{x^2}$ is the same as

$8 : 9 = 21 : x$

$8x = 189$

$x = 23.625.$ Ans.

(152) $7 + 8 : 7 = 30 : x$ is equivalent to

$15 : 7 = 30 : x.$

$15x = 7 \times 30$

$15x = 210$

$x = \frac{210}{15} = 14.$ Ans.

C. Vol. VI.—7.

(153) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, 29 : 31 = 2,480 : x. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since, when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29 : 31 = x : 2{,}480,$$
or, $31x = 71{,}920$;
whence, $x = 71{,}920 \div 31 = 2{,}320$ steps. Ans.

(154) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96 : 900 = 12 : x,$$
or, $96x = 10{,}800$;
whence, $x = 10{,}800 \div 96 = 112.5$ mi. Ans.

(155) This is also a direct proportion; hence,

$$27.63 : 29.4 = .76 : x,$$
or, $27.63x = 29.4 \times .76 = 22.344$;
whence, $x = 22.344 \div 27.63 = .808 +$ lb. Ans.

(156) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt. Hence,

$$23 : 46 = 5 : x,$$
or, $23x = 46 \times 5 = 230$;
whence, $x = 230 \div 23 = 10$ days. Ans.

(157) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2 : 12^2 = 24 : x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2 : 12^2 = x : 24.$$

Dividing both terms of the first couplet (see Art. **310**) by 6

$1^2 : 2^2 = x : 24$; or $1 : 4 = x : 24$;
whence, $4x = 24$, or $x = 6$ degrees. Ans.

(158) Taking the dimensions as the causes,

$$\begin{array}{c|c|l} \cancel{12} & 15 & \\ \cancel{4} & 5 & \\ 2 & \cancel{2} = \cancel{12} & x, \text{ whence, } 2x = 75, \text{ or, } x = \$37.50. \\ \cancel{8} & \cancel{6} & \qquad\qquad \text{Ans.} \end{array}$$

(159) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min.

Hence, $120 : 868 = 100 : x$,

or, $120x = 86,800$;

whence, $x = 723\frac{1}{3}$ gal. Ans.

(160) Taking the dimensions as the causes,

$$\begin{array}{c|cc|l} \cancel{14} & \cancel{2} & & \\ \cancel{28} & \cancel{20} & & \\ 2 & & = \cancel{798} & x, \text{ whence, } 2x = 17 \times 57 = 969, \\ \cancel{12} & 17 & 57 & \qquad \text{or, } x = 484\frac{1}{2} \text{ bbl. Ans} \\ \cancel{10} & \cancel{6} & & \end{array}$$

(161) 8 hr. 40 min. = 520 min. Hence,

$$444 : 1,060 = 520 : x,$$

$$\text{or, } x = \frac{1,060 \times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{137,800}{111} = 1,241.44 + \text{ min.} = 20 \text{ hr. } 41.44 + \text{ min. Ans.}$$

(162) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2} : 60 = 6,160 : x,$$

$$\text{or, } x = \frac{60 \times 6,160}{5.5} = 67,200 \text{ ft. Ans.}$$

(163) Writing the statement as a direct proportion, $8 : 10 = 5 : x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8 : 10 = x : 5,$$

$$\text{or, } x = \frac{\overset{4}{\cancel{8}} \times \cancel{5}}{\underset{2}{\cancel{10}}} = 4 \text{ men. Ans.}$$

(164) Taking the times as the causes,

$$\begin{array}{r|rr|l} \cancel{20} & \cancel{25} & & 14 \\ & \cancel{5} & & \cancel{70} \\ & & = \cancel{540} & \cancel{630}; \\ \cancel{10} & x & \cancel{27} & \\ 2 & & 3 & \end{array}$$

whence, $3x = 2 \times 14 = 28$, or $x = 9\frac{1}{3}$ hr. Ans.

(165) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **349,** 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{r|l|} 14^2 & 30^2 \\ 500 & 660 = 112 \\ 48 & 42 \end{array}\ x, \text{ or } \begin{array}{r|l|l} & 9 & \\ \cancel{14} & \cancel{900} & \\ \cancel{196} & 22 & \\ \cancel{5} & \cancel{110} & \\ \cancel{500} & \cancel{660} = \cancel{112} & x; \\ \cancel{6} & 3 \quad \cancel{8} & \\ \cancel{48} & \cancel{42} & \end{array}$$

whence, $x = 9 \times 22 \times 3 = 594$ horsepower. Ans.

(166) First find the volume of the cylinder in cubic inches, as in the example, Art. **345.** The volume, multiplied by the weight of one cubic inch (.261 lb.), will evidently be the weight of the cylinder. Hence,

$$\begin{array}{r|l|} 10^2 & 12^2 \\ 20 & 60 \end{array} = 1{,}570.8 \ \Big|\ x, \text{ or } \begin{array}{r|l|l} 100 & 144 & \\ & 3 \ = 1{,}570.8 & x; \\ \cancel{20} & \cancel{60} & \end{array}$$

whence, $x = \dfrac{144 \times 3 \times 1{,}570.8}{100} = 6{,}785.856$ cu. in. Therefore, weight of cylinder $= 6{,}785.856 \times .261 = 1{,}771.11$ lb. Ans.

(167) Referring to the example in Art. **348,**

$$\begin{array}{r|l|} 15 & 40 \\ & \\ 20^2 & 18^2 = 187 \\ 10 & 12 \end{array}\ x, \text{ or } \begin{array}{r|l|l} & \cancel{4} & \\ 5 & \cancel{40} & \\ \cancel{15} & & \\ 100 & & \\ \cancel{400} & 324 = 187 & x; \\ \cancel{10} & 4 & \\ & \cancel{12} & \end{array}$$

whence, $x = \dfrac{324 \times 4 \times 187}{500} = 484.7$ lb. Ans.

ALGEBRA.

(QUESTIONS 168-217.)

(168) $-\dfrac{c-(a-b)}{c+(a+b)} = \dfrac{(a-b)-c}{c+(a+b)}$. Ans. (Art. **482.**)

(169) (*a*) Factoring each expression (Art. **457**), we have $9x^4 + 12x^2y^2 + 4y^4 = (3x^2 + 2y^2)(3x^2 + 2y^2) = (3x^2 + 2y^2)^2$.

(*b*) $49a^4 - 154a^2b^2 + 121b^4 = (7a^2 - 11b^2)(7a^2 - 11b^2) = (7a^2 - 11b^2)^2$. Ans.

(*c*) $64x^2y^2 + 64xy + 16 = 16(2xy + 1)^2$. Ans.

(170) (*a*) Arrange the dividend according to the decreasing powers of x and divide. Thus,

$$
\begin{array}{l}
3x - 1\,)\;9x^3 + 3x^2 + x - 1\;(\,3x^2 + 2x + 1 \quad \text{Ans.} \\
\underline{9x^3 - 3x^2} \\
\quad 6x^2 + x \\
\quad \underline{6x^2 - 2x} \qquad \text{(Art. } \mathbf{444.}\text{)} \\
\qquad 3x - 1 \\
\qquad \underline{3x - 1}
\end{array}
$$

(*b*)

$$
\begin{array}{l}
a - b\,)\;a^3 - 2ab^2 + b^3\;(\,a^2 + ab - b^2 \quad \text{Ans.} \\
\underline{a^3 - a^2b} \\
\quad a^2b - 2ab^2 \\
\quad \underline{a^2b - ab^2} \qquad \text{(Art. } \mathbf{444.}\text{)} \\
\qquad -ab^2 + b^3 \\
\qquad \underline{-ab^2 + b^3}
\end{array}
$$

(*c*) Arranging the terms of the dividend according to the decreasing powers of x, we have

$$
\begin{array}{l}
7x - 3\,)\;7x^3 - 24x^2 + 58x - 21\;(\,x^2 - 3x + 7 \quad \text{Ans.} \\
\underline{7x^3 - 3x^2} \\
\quad -21x^2 + 58x \\
\quad \underline{-21x^2 + 9x} \\
\qquad 49x - 21 \\
\qquad \underline{49x - 21}
\end{array}
$$

(171) See Arts. **352** and **353.**

(172) (*a*) In the expression $4x^3y - 12x^2y^2 + 8xy^3$, it is evident that each term contains the common factor $4xy$. Dividing the expression by $4xy$, we obtain $x^2 - 3x^2y + 2y^2$ for a quotient. The two factors, therefore, are $4xy$ and $x^2 - 3x^2y + 2y^2$. Hence, by Art. **452,**

$$4x^3y - 12x^2y^2 + 8xy^3 = 4xy(x^2 - 3x^2y + 2y^2). \quad \text{Ans.}$$

(*b*) The expression $(x^4 - y^4)$ when factored, equals $(x^2 + y^2)(x^2 - y^2)$. (Art. **463.**) But, according to Art. **463,** $x^2 - y^2$ may be further resolved into the factors $(x + y)(x - y)$.

Hence, $(x^4 - y^4) = (x^2 + y^2)(x + y)(x - y)$. Ans.

(*c*) $8x^3 - 27y^3$. See Art. **466.** The cube root of the first term is $2x$, and of the second term is $3y$, the sign of the second term being $-$. Hence, the first factor of $8x^3 - 27y^3$ is $2x - 3y$. The second factor we find to be $4x^2 + 6xy + 9y^2$, by division. Hence, the factors are $2x - 3y$ and $4x^2 + 6xy + 9y^2$.

(173) Arranging the terms according to the decreasing powers of m.

$$\begin{array}{lllllll}
3m^3 & +10m^2n & +10mn^2 & +3n^3 & & & \\
3m^4n & -5m^3n^2 & +5m^2n^3 & -mn^4 & & & \\
\hline
9m^7n & +30m^6n^2 & +30m^5n^3 & +9m^4n^4 & & & \\
 & -15m^6n^2 & -50m^5n^3 & -50m^4n^4 & -15m^3n^5 & & \\
 & & +15m^5n^3 & +50m^4n^4 & +50m^3n^5 & +15m^2n^6 & \\
 & & & -3m^4n^4 & -10m^3n^5 & -10m^2n^6 & -3mn^7 \\
\hline
9m^7n & +15m^6n^2 & -5m^5n^3 & +6m^4n^4 & +25m^3n^5 & +5m^2n^6 & -3mn^7
\end{array}$$

Ans.

(174) $(2a^2bc^3)^4 = 16a^8b^4c^{12}$. Ans.

$(-3a^2b^2c)^5 = -243a^{10}b^{10}c^5$. Ans.

$(-7m^3nx^2y^4)^2 = 49m^6n^2x^4y^8$. Ans.

(175) (*a*) $4a^2 - b^2$ factored $= (2a + b)(2a - b)$. Ans.

(*b*) $16x^{10} - 1$ factored $= (4x^5 + 1)(4x^5 - 1)$. **(Art. 463.)** Ans.

(*c*) $16x^6 - 8x^4y^2 + x^2y^4$, when factored $=$

$(4x^3 - xy^2)(4x^3 - xy^2)$. (Art. **457,** Rule.)

But, $(4x^3 - xy^2) = x(2x + y)(2x - y)$. (Arts. **452** and **463.**)

Hence, $16x^6 - 8x^4y^2 + x^2y^4 = x^2(2x + y)(2x + y)(2x - y)(2x - y)$. Ans.

(176) $4a^6 - 12a^5x + 5a^4x^2 + 6a^3x^3 + a^2x^4 (2a^3 - 3a^2x - ax^2$ Ans.

$$\begin{array}{l|l}
 & 4a^6 \\
4a^3 - 3a^2x & -12a^5x + 5a^4x^2 \\
 & -12a^5x + 9a^4x^2 \\
\hline
4a^3 - 6a^2x - ax^2 & -4a^4x^2 + 6a^3x^3 + a^2x^4 \\
 & -4a^4x^2 + 6a^3x^3 + a^2x^4 \\
\hline
\end{array}$$

(177) (*a*) $6a^4b^4 + a^3b^2 - 7a^2b^3 + 2abc + 3$.

(*b*) $3 + 2abc + a^3b^2 - 7a^2b^3 + 6a^4b^4$.

(*c*) $1 + ax + a^2 + 2a^3$. Written like this, the a in the second term is understood as having 1 for an exponent; hence, if we represent the first term by a^0, in value it will be equal to 1, since $a^0 = 1$. (Art. **439.**) Therefore, 1 should be written as the first term when arranged according to the increasing powers of a.

(178) $\sqrt[4]{16a^{12}b^4c^8} = \pm 2a^3bc^2$. Ans. (Art. **521.**)

$\sqrt[5]{-32a^{15}} = -2a^3$. Ans.

$\sqrt[3]{-1,728a^6d^{12}x^3y^9} = -12a^2d^4xy^3$. Ans.

(179) (*a*) $(a - 2x + 4y) - (3z + 2b - c)$. Ans. (Art. **408.**)

(*b*) $-3b - 4c + d - (2f - 3e)$ becomes $-[3b + 4c - d + (2f - 3e)]$ when placed in brackets preceded by a minus sign. Ans. (Art. **408.**)

(*c*) The subtraction of one expression or quantity from another, when none of the terms are alike, can be represented only by combining the subtrahend with the minuend by means of the sign $-$.

In this case, where we are to subtract $2b-(3c+2d)-a$ from x, the result will be indicated by $x-[2b-(3c+2d)-a.]$ Ans. (Art. **408.**)

(180) (*a*)

$$\begin{array}{l}
2x^3+2x^2+2x-2 \\
x-1 \\
\hline
2x^4+2x^3+2x^2-2x \\
\quad\;\; -2x^3-2x^2-2x+2 \\
\hline
2x^4 \qquad\qquad\quad -4x+2 \quad \text{Ans.}
\end{array}$$

(*b*)

$$\begin{array}{l}
x^2-4ax+c \\
2x+a \\
\hline
2x^3-8ax^2+2cx \\
\qquad\quad ax^2 \qquad -4a^2x+ac \\
\hline
2x^3-7ax^2+2cx-4a^2x+ac \quad \text{Ans.}
\end{array}$$

(*c*)

$$\begin{array}{l}
-a^3+3a^2b-2b^3 \\
5a^2+9ab \\
\hline
-5a^5+15a^4b-10a^2b^3 \\
\qquad -9a^4b \qquad\qquad +27a^3b^2-18ab^4 \\
\hline
-5a^5+6a^4b-10a^2b^3+27a^3b^2-18ab^4
\end{array}$$

Arranging the terms according to the decreasing powers of a, we have $-5a^5+6a^4b+27a^3b^2-10a^2b^3-18ab^4$. Ans.

(181) (*a*)

$$\begin{array}{r}
4xyz \\
-3xyz \\
-5xyz \\
6xyz \\
-9xyz \\
3xyz \\
\hline
-4xyz
\end{array} \quad \text{Ans.}$$

The sum of the coefficients of the positive terms we find to be $+13$, since $(+3)+(+6)+(+4)=(+13)$.

When no sign is given before a quantity the $+$ sign must always be understood. The sum of the coefficients of the negative terms we find to be -17 since $(-9)+(-5)+(-3)=(-17)$. Subtracting the *lesser* sum from the *greater*, and prefixing the sign of the greater sum $(-)$ (Art. **390,** rule II), we have $(+13)+(-17)=-4$. Since the terms are all alike, we have only to annex the common symbols xyz to -4, thereby obtaining $-4xyz$ for the result or sum.

(*b*)

$$\begin{array}{rrr}
3a^2 + & 2ab + & 4b^2 \\
5a^2 - & 8ab + & b^2 \\
-\ a^2 + & 5ab - & b^2 \\
18a^2 - & 20ab - & 19b^2 \\
14a^2 - & 3ab + & 20b^2 \\
\hline
39a^2 - & 24ab + & 5b^2
\end{array} \quad \text{Ans.}$$

When adding polynomials, always place like terms under each other. (Art. **393.**)

The coefficient of a^2 in the result will be 39, since $(+14) + (+18) + (-1) + (+5) + (+3) = 39$. When the coefficient of a term is not written, 1 is always understood to be its coefficient. (Art. **359.**) The coefficient of ab will be -24, since $(-3) + (-20) + (+5) + (-8) + (+2) = -24$. The coefficient of b^2 will be $(+20) + (-19) + (-1) + (+1) + (+4) = +5$. Hence, the result or sum is $39a^2 - 24ab + 5b^2$.

(*c*)

$$\begin{array}{l}
\quad 4mn + 3ab - 4c \\
+\,2mn - 4ab \qquad + 3x + 3m^2 - 4p \\
\hline
6mn - \ ab - 4c + 3x + 3m^2 - 4p
\end{array} \quad \text{Ans.}$$

(182) The reciprocal of 3.1416 is $\frac{1}{3.1416} = .3183 +$. Ans.

Reciprocal of $.7854 = \frac{1}{.7854} = 1.273 +$. Ans.

Of $\frac{1}{64.32} = \dfrac{1}{\frac{1}{64.32}} = 1 \times \frac{64.32}{1} = 64.32$. Ans.
(See Art. **481.**)

(183) (*a*) $\frac{x}{x-y} + \frac{x-y}{y-x}$. If the denominator of the second fraction were written $x - y$, instead of $y - x$, then $x - y$ would be the common denominator.

By Art. **482,** the signs of the denominator and the sign before the fraction $\frac{x-y}{y-x}$ may be changed, giving $-\frac{x-y}{x-y}$. We now have

$$\frac{x}{x-y} - \frac{x-y}{x-y} = \frac{x-x+y}{x-y} = \frac{y}{x-y}. \quad \text{Ans.}$$

(*b*) $\frac{x^2}{x^2-1} + \frac{x}{x+1} - \frac{x}{1-x}$. If we write the denominator of the third fraction $x - 1$ instead of $1 - x$, $x^2 - 1$ will then be the common denominator.

By Art. 482, the signs of the denominator and the sign before the fraction may be changed, thereby giving $\frac{x}{x-1}$. We now have

$$\frac{x^2}{x^2-1}+\frac{x}{x-1}+\frac{x}{x+1}=\frac{x^2+x(x+1)+x(x-1)}{x^2-1}=$$

$$\frac{x^2+x^2+x+x^2-x}{x^2-1}=\frac{3x^2}{x^2-1}. \text{ Ans.}$$

(c) $\frac{3a-4b}{7}-\frac{2a-b+c}{3}+\frac{13a-4c}{12}$, when reduced to a common denominator

$$=\frac{12(3a-4b)-28(2a-b+c)+7(13a-4c)}{84}.$$

Expanding the terms and removing the parentheses, we have

$$\frac{36a-48b-56a+28b-28c+91a-28c}{84}.$$

Combining like terms in the numerator, we have as the result,

$$\frac{71a-20b-56c}{84}. \text{ Ans.}$$

(184) (a) $45x^7y^{10}-90x^5y^7-360x^4y^8=45x^4y^7(x^3y^3-2x-8y)$. Ans. (Art. **452.**)

(b) $a^2b^2+2abcd+c^2d^2=(ab+cd)^2$. Ans. (Art. **457.**)

(c) $(a+b)^2-(c-d)^2=(a+b+c-d)(a+b-c+d)$. Ans. (Art. **463.**)

(185) (a) If a man builds 20 rods of stone wall, and we consider this work as positive, or +, the work which he does in tearing it down may be considered as negative, or −. If he tore down 10 rods, we could say that he *built* − 10 rods.

(b) See Arts. **388** and **398.**

(186) (a) $\frac{2ax+x^2}{a^3-x^3}\div\frac{x}{a-x}=\frac{x(2a+x)}{a^3-x^3}\times\frac{a-x}{x}$.

(Art. **502.**)

Canceling common factors, the result equals $\frac{2a+x}{a^2+ax+x^2}$. Ans.

$$\begin{array}{l} a-x\,)\,a^3-x^3\,(\,a^2+ax+x^2 \\ \quad\;\; \underline{a^3-a^2x} \\ \qquad\quad a^2x-x^3 \\ \qquad\quad \underline{a^2x-ax^2} \\ \qquad\qquad\quad ax^2-x^3 \\ \qquad\qquad\quad \underline{ax^2-x^3} \end{array}$$

(*b*) Inverting the divisor and factoring, we have

$$\frac{3n(2m^2n-1)}{(2m^2n-1)\,(2m^2n-1)} \times \frac{(2m^2n+1)\,(2m^2n-1)}{3n}.$$

Canceling common factors, we have $2m^2n+1$. Ans.

(*c*) $9+\frac{5y^2}{x^2-y^2} \div \left(3+\frac{5y}{x-y}\right)$ simplified $= \frac{9x^2-4y^2}{x^2-y^2} \div \frac{3x+2y}{x-y}$.

Inverting the divisor, we have $\frac{9x^2-4y^2}{x^2-y^2} \times \frac{x-y}{3x+2y}$.

Canceling common factors, the result equals $\frac{3x-2y}{x+y}$. Ans.

(187) According to Art. **456,** the trinomials $1-2x^2+x^4$ and $4x^2+4x+1$ are perfect squares. (See Art. **458.**) The remaining trinomials are not perfect squares, since they do not comply with the foregoing principles.

(188) (*a*) By Art. **481,** the reciprocal of $\frac{24}{49} = 1 \div \frac{24}{49} = 1 \times \frac{49}{24} = \frac{49}{24}$. Ans.

(*b*) Since, by Art. **481,** a number may be found from its reciprocal by dividing 1 by the reciprocal, the number $= 1 \div 700 = .0014\frac{2}{7}$. Ans.

(189) Applying the method of Art. **474,**

$$\begin{array}{r|llll}
x+y & 12xy(x^2-y^2), & 2x^2(x^2+2xy+y^2), & 3y^2(x-y)^2, & 6(x^2+xy) \\ \hline
x-y & 12xy(x-y), & 2x^2(x+y), & 3y^2(x-y)^2, & 6x \\ \hline
3xy & 12xy & 2x^2(x+y), & 3y^2(x-y), & 6x \\ \hline
2 & 4, & 2x(x+y), & y(x-y), & 2 \\ \hline
 & 2, & x(x+y), & y(x-y), & 1
\end{array}$$

Whence, L. C. M. $= (x+y)\ (x-y)\ 3xy \times 2 \times 2 \times x(x+y) \times y(x-y) = 12x^2y^2(x+y)^2(x-y)^2$.

(190) (*a*)
$$\begin{array}{l}
2+4a-5a^2-6a^3 \\
7a^3 \\ \hline
14a^3+28a^4-35a^5-42a^6
\end{array}$$
Ans. (Art. **423.**)

(*b*)
$$\begin{array}{l}
4x^2-4y^2+6z^2 \\
3x^2y \\ \hline
12x^4y-12x^2y^3+18x^2yz^2
\end{array}$$
Ans.

(*c*)
$$\begin{array}{l}
3b+5c-2d \\
6a \\ \hline
18ab+30ac-12ad
\end{array}$$
Ans.

(191) (*a*) See Arts. **359** and **361.**
(*b*) See Arts. **419** and **440.**
(*c*) See Art. **416.**

(192) (*a*) On removing the vinculum, we have

$$2a - [3b + \{4c - 4a - (2a+2b)\} + \{3a - b - c\}].$$

(Art. **405.**)

Removing the parenthesis,

$$2a - [3b + \{4c - 4a - 2a - 2b\} + \{3a - b - c\}].$$

Removing the braces,

$$2a - [3b + 4c - 4a - 2a - 2b + 3a - b - c].$$

(Art. **406.**)

Removing the brackets,

$$2a - 3b - 4c + 4a + 2a + 2b - 3a + b + c.$$

Combining like terms, the result is $5a - 3c$. **Ans.**

(*b*) Removing the parenthesis, we have

$$7a - [3a - \{2a - 5a + 4a\}].$$

Removing the brace,

$$7a - [3a - 2a + 5a - 4a].$$

Removing the brackets,

$$7a - 3a + 2a - 5a + 4a.$$

Combining terms, the result is $5a$. Ans.

(*c*) Removing the parentheses, we have

$$a - [2b + \{3c - 3a - a - b\} + \{2a - b - c\}].$$

Removing the braces,

$$a - [2b + 3c - 3a - a - b + 2a - b - c].$$

Removing the brackets,

$$a - 2b - 3c + 3a + a + b - 2a + b + c.$$

Combining like terms, the result is $3a - 2c$. Ans.

(193) (*a*) $(x^3 + 8) = (x + 2)(x^2 - 2x + 4)$. Ans.

(*b*) $x^3 - 27y^3 = (x - 3y)(x^2 + 3xy + 9y^2)$. Ans.

(*c*) $xm - nm + xy - ny = m(x - n) + y(x - n)$,

or $(x - n)(m + y)$. Ans.

(Arts. **466** and **468.**)

(194) Arrange the terms according to the decreasing powers of x. (Art. **523.**)

$4x^4 + 8ax^3 + 4a^2x^2 + 16b^2x^2 + 16ab^2x + 16b^4 (2x^2 + 2ax + 4b^2$. Ans.
$(2x^2)^2 = 4x^4$.

$$\begin{array}{l|l|l}
4x^2 + 2ax & 8ax^3 + 4a^2x^2 & \\
 & 8ax^3 + 4a^2x^2 & \\
\hline
4x^2 + 4ax + 4b^2 & & 16b^2x^2 + 16ab^2x + 16b^4 \\
 & & 16b^2x^2 + 16ab^2x + 16b^4 \\
\hline
\end{array}$$

(195) $\dfrac{c(a + b) + cd}{(a + b)c} = \dfrac{ac + bc + cd}{ac + bc}$. Canceling c, which is common to each term, we have $\dfrac{a + b + d}{a + b} = 1 + \dfrac{d}{a + b}$. Ans.

(201) The subtraction of one expression from another, if none of the terms are similar, may be represented only by connecting the subtrahend with the minuend by means of the sign $-$. Thus, it is required to subtract $5a^3b - 7a^2b^2 + 5ab^3$ from $a^4 - b^4$, the result will be represented by $a^4 - b^4 - (5a^3b - 7a^2b^2 + 5ab^3)$, which, on removing the parentheses (Art. **405**), becomes $a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3$. From this result, subtract $3a^4 - 4a^3b + 6a^2b^2 + 5ab^3 - 3b^4$.

$$\begin{array}{rrrrrl} a^4 & -\ b^4 & -\ 5a^3b & +\ 7a^2b^2 & -\ 5ab^3 & \textit{minuend.} \\ -\ 3a^4 & +\ 3b^4 & +\ 4a^3b & -\ 6a^2b^2 & -\ 5ab^3 & \textit{subtrahend}, \text{ with signs} \\ \hline -\ 2a^4 & +\ 2b^4 & -\ a^3b & +\ a^2b^2 & -\ 10ab^3 & \text{changed. (Art. } \mathbf{401}.) \end{array}$$

Or, $-2a^4 - a^3b + a^2b^2 - 10ab^3 + 2b^4$. Answer arranged according to the decreasing powers of a.

(202) (a)

$$\begin{array}{r} 3a - 2b + 3c \\ 2a - 8b - \ c \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 3a - 2b + 3c \\ -2a + 8b + \ c \\ \hline a + 6b + 4c \end{array}$$

when the signs of the subtrahend are changed. Now, adding each term (with its sign changed) in the subtrahend to its corresponding term in the minuend, we have $(-2a) + (3a) = a$; $(+8b) + (-2b) = +6b$; $(+c) + (3c) = +4c$. Hence, $a + 6b + 4c$ equals the difference. Ans.

(b)

$$\begin{array}{l} 2x^3 - 3x^2y + 2xy^3 \\ x^3 \qquad\qquad\qquad + y^3 - xy^2 \text{ becomes} \\ \hline \end{array}$$

$$\begin{array}{r} 2x^3 - 3x^2y + 2xy^3 \qquad\qquad \\ -\ x^3 \qquad\qquad\qquad - y^3 + xy^2 \\ \hline x^3 - 3x^2y + 2xy^3 - y^3 + xy^2 \end{array}$$

when the signs of the subtrahend are changed. Adding each term in the subtrahend (with its sign changed) to its corresponding term in the minuend, we have $x^3 - 3x^2y + 2xy^3 - y^3 + xy^2$, which, arranged according to the decreasing powers of x, equals $x^3 - 3x^2y + xy^2 + 2xy^3 - y^3$. Ans.

(c)

$$\begin{array}{r} 14a + 4b - 6c - 3d \\ 11a - 2b + 4c - 4d \\ \hline \end{array}$$

On changing the sign of each term in the subtrahend, the problem becomes

$$\begin{array}{r} 14a + 4b - 6c - 3d \\ -11a + 2b - 4c + 4d \\ \hline 3a + 6b - 10c + d \end{array}$$

Adding each term of the subtrahend (with the sign changed) to its corresponding term in the minuend, the difference, or result, is $3a + 6b - 10c + d$. Ans.

(203) The numerical values of the following, when $a = 16$, $b = 10$, and $x = 5$, are:

(*a*) $(ab^2x + 2abx)\,4a = (16 \times 10^2 \times 5 + 2 \times 16 \times 10 \times 5) \times 4 \times 16$. It must be remembered that when no sign is expressed between symbols or quantities, the sign of multiplication is understood.

$(16 \times 100 \times 5 + 2 \times 16 \times 10 \times 5) \times 64 = (8{,}000 + 1{,}600) \times 64 = 9{,}600 \times 64 = 614{,}400$. Ans.

(*b*) $2\sqrt{4a} - \frac{2bx}{a-b} + \frac{b-x}{x} = 2\sqrt{64} - \frac{2 \times 10 \times 5}{16 - 10} + \frac{10-5}{5} = 16 - \frac{100}{6} + 1 = \frac{96 - 100 + 6}{6} = \frac{2}{6} = \frac{1}{3}$. Ans.

(*c*) $(b - \sqrt{a})\,(x^3 - b^2)\,(a^2 - b^2) = (10 - \sqrt{16})\,(5^3 - 10^2)\,(16^2 - 10^2) = (10 - 4)\,(125 - 100)\,(256 - 100) = 6 \times 25 \times 156 = 23{,}400$. Ans.

(204) (*a*) Dividing both numerator and denominator by $15mxy^2$, $\frac{15mxy^2}{75mx^2y^3} = \frac{1}{5xy}$. Ans.

(*b*) $\frac{x^2 - 1}{4x(x+1)} = \frac{(x+1)(x-1)}{4x(x+1)}$ when the numerator is factored.

Canceling $(x + 1)$ from both the numerator and denominator (Art. **484**), the result is $\frac{x-1}{4x}$. Ans.

(*c*) $\frac{(a^3 + b^3)(a^2 + ab + b^2)}{(a^3 - b^3)(a^2 - ab + b^2)}$ when factored becomes

$$\frac{(a + b)\,(a^2 - ab + b^2)\,(a^2 + ab + b^2)}{(a - b)\,(a^2 + ab + b^2)\,(a^2 - ab + b^2)}. \quad \text{(Art. 466.)}$$

Canceling the factors common to both the numerator and denominator, we have

$$\frac{(a+b)(a^2-ab+b^2)(a^2+ab+b^2)}{(a-b)(a^2+ab+b^2)(a^2-ab+b^2)}=\frac{a+b}{a-b}. \text{ Ans.}$$

$$a+b\,)\,a^3+b^3\,(\,a^2-ab+b^2 \qquad a-b\,)\,a^3-b^3\,(\,a^2+ab+b^2$$

$$\begin{array}{rr} a^3+a^2b & \\ \hline -a^2b+b^3 & \\ -a^2b-ab^2 & \\ \hline ab^2+b^3 & \\ ab^2+b^3 & \\ \hline \end{array} \qquad \begin{array}{rr} a^3-a^2b & \\ \hline a^2b-b^3 & \\ a^2b-ab^2 & \\ \hline ab^2-b^3 & \\ ab^2-b^3 & \\ \hline \end{array}$$

(205) (*a*) $$\frac{\dfrac{1}{1-x}-\dfrac{1}{1+x}}{\dfrac{1}{1-x}+\dfrac{1}{1+x}}=\frac{\dfrac{1+x-1+x}{1-x^2}}{\dfrac{1+x+1-x}{1-x^2}}=\frac{2x}{1-x^2}\div$$

$$\frac{2}{1-x^2}=\frac{2x}{1-x^2}\times\frac{1-x^2}{2}=x.$$ Ans. (See Art. **509.**)

(*b*) $$\frac{\dfrac{a^2}{b^3}+\dfrac{1}{a}}{\dfrac{a}{b^2}-\dfrac{a-b}{ab}}=\frac{\dfrac{a^3+b^3}{ab^3}}{\dfrac{a^2b-b^2(a-b)}{ab^3}}=\frac{\dfrac{a^3+b^3}{ab^3}}{\dfrac{a^2b-ab^2+b^3}{ab^3}}=$$

$$\frac{a^3+b^3}{ab^3}\div\frac{a^2b-ab^2+b^3}{ab^3}=\frac{a^3+b^3}{ab^3}\times\frac{ab^3}{b(a^2-ab+b^2)}=\frac{a+b}{b}.$$ Ans.

(*c*) $$\frac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}=\frac{1}{x+\dfrac{3-x}{4}}=\frac{4}{3x+3}.$$ Ans. (Art. **509.**)

(206) $\frac{3+2x}{2-x}-\frac{2-3x}{2+x}+\frac{16x-x^2}{x^2-4}$. If the denominator of the third fraction were written $4-x^2$, instead of x^2-4, the common denominator would then be $4-x^2$.

By Art. **482**, $\frac{16x-x^2}{x^2-4}$ becomes $-\frac{16x-x^2}{-x^2+4}=-\frac{16x-x^2}{4-x^2}$.

Hence, $\frac{3+2x}{2-x}-\frac{2-3x}{2+x}-\frac{16x-x^2}{4-x^2}$, when reduced to a common denominator, becomes

$$\frac{(3+2x)(2+x)-(2-3x)(2-x)-(16x-x^2)}{4-x^2}=$$
$$\frac{(6+7x+2x^2)-(4-8x+3x^2)-(16x-x^2)}{4-x^2}.$$

Removing the parentheses (Art. **405**), we have

$$\frac{6+7x+2x^2-4+8x-3x^2-16x+x^2}{4-x^2}.$$

Combining like terms in the numerator, we have

$$\frac{2-x}{4-x^2}.$$

Factoring the denominator by Art. **463**, we have

$$\frac{2-x}{(2+x)(2-x)}.$$

Canceling the common factor $(2-x)$, the result equals

$$\frac{1}{2+x}, \text{ or } \frac{1}{x+2}. \quad \text{Ans.} \qquad \text{(Art. } \mathbf{373.}\text{)}$$

(207) (*a*)

$$1+2x-\frac{4x-4}{5x}=\frac{5x+10x^2-4x+4}{5x}=\frac{10x^2+x+4}{5x}. \quad \text{Ans.}$$

(Art. **504.**)

(*b*) $\dfrac{3x^2+2x+1}{x+4}=3x-10+\dfrac{41}{x+4}.$ Ans. (Art. **505.**)

$$\begin{array}{r|l}
x+4 \,)\; 3x^2+2x+1 & (\, 3x-10+\dfrac{41}{x+4} \\
3x^2+12x & \\
\hline
-10x+1 & \\
-10x-40 & \\
\hline
41 &
\end{array}$$

(*c*) Reducing, the problem becomes

$$\frac{x^2+4x-5}{x^2}\times\frac{x-7}{x^2-8x+7}.$$

Factoring, we have

$$\frac{(x+5)(x-1)}{x^2}\times\frac{x-7}{(x-1)(x-7)}.$$

Canceling common factors, the result equals $\dfrac{x+5}{x^2}$. Ans.

(208) (*a*) Writing the work as follows, and canceling common factors in both numerator and denominator (Arts. **496** and **497**), we have

$$\frac{9m^2n^2}{8p^3q^2} \times \frac{5p^3q}{2xy} \times \frac{24x^2y^2}{90mn} =$$

$$\frac{9 \times 5 \times 24 \times m^2n^2p^3qx^2y^2}{8 \times 2 \times 90 \times m \times n \times p^3 \times q^2 \times x \times y} = \frac{3mnxy}{4pq^2}. \quad \text{Ans.}$$

(*b*) Factoring the numerators and denominators of the fraction (Art. **498**), and writing the factors of the numerators over the factors of the denominators, we have

$$\frac{(a-x)(a^2+ax+x^2)(a+x)(a+x)}{(a+x)(a^2-ax+x^2)(a-x)(a-x)} =$$

$$\frac{(a+x)(a^2+ax+x^2)}{(a-x)(a^2-ax+x^2)}. \quad \text{Ans.}$$

(*c*) This problem may be written as follows, according to Art. **480**:

$$\frac{3ax+4}{1} \times \frac{a^2}{a(3ax+4)(3ax+4)}.$$

Canceling a and $(3ax+4)$, we have $\dfrac{a}{3ax+4}$. Ans.

(209) (*a*) $-7my\,)\,35m^2y + 28m^2y^2 - 14my^3$
$\qquad -5m^2 - 4my + 2y^2$ Ans. (Art. **442**.)

(*b*) $a^4\,)\,4a^4 - 3a^5b - a^6b^2$
$\qquad 4 - 3ab - a^2b^2$ Ans.

(*c*) $4x^2\,)\,4x^3 - 8x^5 + 12x^7 - 16x^9$
$\qquad x - 2x^3 + 3x^5 - 4x^7$ Ans.

(210) (*a*) $16a^2b^3$; $a^4 + 4ab$; $4a^2 - 16a^3b + 5a^6 + 7ax$.

(*b*) Since the terms are not alike, we can only indicate the sum, connecting the terms by their proper signs. (Art. **389**.)

(*c*) Multiplication: $4ac^2d$ means $4 \times a \times c^2 \times d$. (Art. **358**.)

(211) $$\frac{a^2+c^2+ac}{a^2+b^2-c^2-2ab} \times \frac{a^2+c^2-b^2-2ac}{a^4c-ac^4}.$$

Arranging the terms, we have

$$\frac{a^2+ac+c^2}{a^2-2ab+b^2-c^2} \times \frac{a^2-2ac+c^2-b^2}{a^4c-ac^4},$$

which, being placed in parentheses, become

$$\frac{a^2+ac+c^2}{(a^2-2ab+b^2)-c^2} \times \frac{(a^2-2ac+c^2)-b^2}{a^4c-ac^4}.$$

By Art. **456**, we know that $a^2-2ab+b^2$, also $a^2-2ac+c^2$, are perfect squares, and may be written $(a-b)^2$ and $(a-c)^2$.

Factoring a^4c-ac^4 by Case I, Art. **452**, we have

$$\frac{a^2+ac+c^2}{(a-b)^2-c^2} \times \frac{(a-c)^2-b^2}{ac(a^3-c^3)} =$$

$$\frac{a^2+ac+c^2}{(a-b-c)(a-b+c)} \times \frac{(a-c-b)(a-c+b)}{ac(a-c)(a^2+ac+c^2)}.$$

(Arts. **463** and **466.**)

Canceling common factors and multiplying, we have

$$\frac{a-c+b}{(a-b+c)\,ac\,(a-c)}, \text{ or } \frac{a+b-c}{ac(a-b+c)(a-c)}. \quad \text{Ans.}$$

(212) The square root of the fraction a plus b plus c divided by n, plus the square root of a, plus the fraction b plus c divided by n, plus the square root of a plus b, plus the fraction c divided by n, plus the quantity a plus b, into c, plus a plus bc.

(213) (a) $\frac{4x+5}{3} - \frac{3x-7}{5x} + \frac{9}{12x^2}$.

We will first reduce the fractions to a common denominator. The L. C. M. of the denominator is $60x^2$, since this is the smallest quantity that each denominator will divide without a remainder. Dividing $60x^2$ by 3, the first denominator, the quotient is $20x^2$; dividing $60x^2$ by $5x$, the second denominator, the quotient is $12x$; dividing $60x^2$ by $12x^2$, the third denominator, the quotient is 5. Multiplying the corresponding numerators by these respective quotients, we obtain $20x^2(4x+5)$ for the first new numerator; $12x(3x-7)$ for the second new numerator, and $5 \times 9 = 45$ for the third new numerator. Placing these new numerators over the common denominator and expanding the terms, we have

$$\frac{20x^2(4x+5)-12x(3x-7)+45}{60x^2}=\frac{80x^3+100x^2-36x^2+84x+45}{60x^2}$$

Collecting like terms, the result is

$$\frac{80x^3+64x^2+84x+45}{60x^2}. \quad \text{Ans.}$$

(b) In $\frac{1}{2a(a+x)}+\frac{1}{2a(a-x)}$, the L. C. M. of the denominators is $2a(a^2-x^2)$, since this is the smallest quantity that each denominator will divide without a remainder. Dividing $2a(a^2-x^2)$ by $2a(a+x)$, the first denominator, we will have $a-x$; dividing $2a(a^2-x^2)$ by $2a(a-x)$, the second denominator, we have $a+x$. Multiplying the corresponding numerators by these respective quotients, we have $(a-x)$ for the first new numerator, and $(a+x)$ for the second new numerator. Arranging the work as follows:

$$\begin{array}{rl} 1\times(a-x)=a-x & = \text{1st numerator.} \\ 1\times(a+x)=a+x & = \text{2d numerator.} \\ \hline \text{or } 2a & = \text{the sum of the numerators.} \end{array}$$

Placing the $2a$ over the common denominator $2a(a^2-x^2)$, we find the value of the fraction to be

$$\frac{2a}{2a(a^2-x^2)}=\frac{1}{a^2-x^2}. \quad \text{Ans.}$$

(c) $\frac{x}{y}+\frac{y}{x+y}+\frac{x^2}{x^2+xy}=\frac{x}{y}+\frac{y}{x+y}+\frac{x}{x+y}.$

The common denominator $=y\,(x+y)$. Reducing the fractions to a common denominator, we have

$$\frac{x\,(x+y)+y^2+xy}{y(x+y)}=\frac{x^2+2xy+y^2}{y(x+y)}=\frac{x+y}{y}. \quad \text{Ans.}$$

(214) (a) Apply the method of Art. **474**:

$$\begin{array}{r|lll} 6ax & 18ax^2, & 72ay^2, & 12xy \\ \hline 2y & 3x, & 12y^2, & 2y \\ \hline 3 & 3x, & 6y, & 1 \\ \hline & x, & 2y, & 1 \end{array}$$

Whence, $6ax\times 2y\times 3\times x\times 2y=72ax^2y^2$. Ans.

$$\begin{array}{r|lll} (b)\ 2(1+x) & 4(1+x), & 4(1-x), & 2(1-x^2) \\ \hline 2(1-x) & 2, & 2(1-x), & 1-x \\ \hline & 1, & 1, & 1 \end{array}$$

Hence, L. C. M. $= 2(1+x) \times 2(1-x) = 4(1-x^2)$. Ans.

$$\begin{array}{r|lll} (c)\ a-b & (a-b)\,(b-c), & (b-c)\,(c-a), & (c-a)\,(a-b) \\ \hline b-c & (b-c), & (b-c)\,(c-a), & (c-a) \\ \hline c-a & 1, & c-a, & c-a \\ \hline & 1, & 1, & 1 \end{array}$$

Hence, L. C. M. $= (a-b)\,(b-c)\,(c-a)$. Ans.

(215) $3x^6 - 3 + a - ax^6 = (3-a)x^6 - 3 + a =$ $(3-a)\,(x^6-1)$. Regarding $x^6 - 1$ as $(x^3)^2 - 1$, we have, by Art. **462,** $x^6 - 1 = (x^3)^2 - 1 = (x^3-1)\,(x^3+1)$. $x^3 - 1 =$ $(x-1)\,(x^2+x+1)$; $x^3 + 1 = (x+1)\,(x^2-x+1)$. Art. **466.**

Hence, the factors are

$(x^2+x+1)\,(x^2-x+1)\,(x+1)\,(x-1)\,(3-a)$. Ans.

(216) Arranging the terms according to the decreasing powers of x, and extracting the square root, we have

$$\begin{array}{l|l} & x^4 + x^3y + 4\tfrac{1}{4}x^2y^2 + 2xy^3 + 4y^4 \ (x^2 + \tfrac{1}{2}xy + 2y^2. \text{ Ans.} \\ & x^4 \\ \hline 2x^2 + \tfrac{1}{2}xy & x^3y + 4\tfrac{1}{4}x^2y^2 \\ & x^3y + \ \tfrac{1}{4}x^2y^2 \\ \hline 2x^2 + xy + 2y^2 & 4x^2y^2 + 2xy^3 + 4y^4 \\ & 4x^2y^2 + 2xy^3 + 4y^4 \\ \hline \end{array}$$

(217) The arithmetic ratio of $x^4 - 1$ to $x + 1$ is $x^4 - 1 -$ $(x+1) = x^4 - x - 2$. Art. **381.**

The geometric ratio of $x^4 - 1$ to $x + 1$ is $\dfrac{x^4-1}{x+1} = x^3 - x^2$ $+ x - 1 = (x^2+1)\,(x-1)$. Ans.

ALGEBRA.

(QUESTIONS 218–257.)

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(218) (*a*) According to Art. **528**, $x^{\frac{3}{4}}$ expressed radically is $\sqrt[4]{x^3}$;

$$3x^{\frac{1}{2}}y^{-\frac{3}{2}} \text{ expressed radically is } 3\sqrt{xy^{-3}};$$

$$3x^{\frac{1}{6}}y^{-\frac{5}{6}}z^{\frac{1}{3}} = 3\sqrt[6]{xy^{-5}z^2}, \text{ since } z^{\frac{1}{3}} = z^{\frac{2}{6}}. \quad \text{Ans.}$$

$$(b) \quad a^{-1}b^{\frac{1}{2}} + \frac{c^{-2}}{a+b} + (m-n)^{-1} - \frac{a^2b^{-2}c}{c^{-3}} =$$

$$\frac{b^{\frac{1}{2}}}{a} + \frac{1}{c^2(a+b)} + \frac{1}{m-n} - \frac{a^2c^4}{b^2}. \quad \text{Ans.}$$

$$(c) \quad \sqrt[7]{x^6} = x^{\frac{6}{7}}. \text{ Ans.} \quad \sqrt[3]{x^{-4}} = x^{-\frac{4}{3}}. \text{ Ans.}$$

$$(\sqrt[4]{b^5x^2})^3 = (b^{\frac{5}{4}}x^{\frac{1}{2}})^3 = b^{\frac{15}{4}}x^{\frac{3}{2}}. \text{ Ans.}$$

(219) $3\sqrt{21} = \sqrt{189}$. Ans. (Art. **542.**)

$a^2b\sqrt{b^3c} = \sqrt{a^4b^5c}$. Ans. $2x\sqrt[5]{x} = \sqrt[5]{32x^6}$. Ans.

(220) Let x = the length of the post.

Then, $\frac{x}{5}$ = the amount in the earth.

$\frac{3x}{7}$ = the amount in the water.

$$\frac{x}{5} + \frac{3x}{7} + 13 = x.$$

$$7x + 15x + 455 = 35x.$$

$$-13x = -455.$$

$$x = 35 \text{ feet.} \quad \text{Ans.}$$

(221) $t = \dfrac{W_1s_1t_1 + W_2s_2t_2}{W_1s_1 + W_2s_2}$.

In order to transform this formula so that t_2 may stand alone in the first member, we must first clear of fractions. Clearing of fractions, we have

$$t\,W_1\,s_1 + t\,W_2\,s_2 = W_1\,s_1\,t_1 + W_2\,s_2\,t_2.$$

Transposing, we have

$$-W_2\,s_2\,t_2 = W_1\,s_1\,t_1 - t\,W_1\,s_1 - t\,W_2\,s_2.$$

Factoring (Arts. **452** and **408**), we have

$$-W_2\,s_2\,t_2 = W_1\,s_1\,t_1 - (W_1\,s_1 + W_2\,s_2)\,t;$$

whence, $t_2 = \dfrac{(W_1s_1 + W_2s_2)\,t - W_1s_1t_1}{W_2s_2}$. Ans.

(222) Let $x =$ number of miles he traveled per hour.

Then, $\dfrac{48}{x} =$ time it took him.

$\dfrac{48}{x+4} =$ time it would take him if he traveled 4 miles more per hour.

In the latter case the time would have been 6 hours *less;* whence, the equation

$$\frac{48}{x+4} = \frac{48}{x} - 6.$$

Clearing of fractions,

$$48x = 48x + 192 - 6x^2 - 24x.$$

Combining like terms and transposing,

$$6x^2 + 24x = 192.$$

Dividing by 6, $x^2 + 4x = 32$.

Completing the square, $x^2 + 4x + 4 = 36$.

Extracting square root, $x + 2 = \pm 6$;

whence, $x = -2 + 6 = 4$, or the number of miles he traveled per hour. Ans.

(223) (*a*) $S = \sqrt[3]{\dfrac{CPD^2}{f\left(2 + \dfrac{D^2}{d^2}\right)}} = \sqrt[3]{\dfrac{CPD^2}{2f + \dfrac{fD^2}{d^2}}}$.

Cubing both members to remove the radical,

$$S^3 = \frac{CPD^2}{2f + \dfrac{fD^2}{d^2}}.$$

Simplifying the result, $S^3 = \dfrac{CPD^2d^2}{2fd^2 + fD^2}$.

Clearing of fractions,

$$2S^3fd^2 + S^3fD^2 = CPD^2d^2.$$

Transposing, $CPD^2d^2 = 2S^3fd^2 + S^3fD^2$;

whence, $P = \frac{2S^3fd^2 + S^3fD^2}{CD^2d^2} = \frac{(2d^2 + D^2)fS^3}{CD^2d^2}$. Ans.

(*b*) Substituting the values of the letters in the given formula, we have

$$P = \frac{(2 \times 18^2 + 30^2) \times 864 \times 6^3}{10 \times 30^2 \times 18^2} = \frac{(648 + 900) \times 864 \times 216}{9{,}000 \times 324} =$$

$$\frac{288{,}893{,}952}{2{,}916{,}000} = 99.1, \text{ nearly. Ans.}$$

(224) (*a*) $3x + 6 - 2x = 7x$. Transposing 6 to the second member, and $7x$ to the first member (Art. **561**),

$$3x - 2x - 7x = -6.$$

Combining like terms, $-6x = -6$;

whence, $x = 1$. Ans.

(*b*) $5x - (3x - 7) = 4x - (6x - 35)$.

Removing the parentheses (Art. **405**),

$$5x - 3x + 7 = 4x - 6x + 35.$$

Transposing 7 to the second member, and $4x$ and $-6x$ to the first member, $5x - 3x - 4x + 6x = 35 - 7$. (Art. **561**.)

Combining like terms, $4x = 28$;

whence, $x = 28 \div 4 = 7$. Ans.

(*c*) $(x + 5)^2 - (4 - x)^2 = 21x$.

Performing the operations indicated, the equation becomes

$$x^2 + 10x + 25 - 16 + 8x - x^2 = 21x.$$

Transposing, $x^2 - x^2 + 10x + 8x - 21x = 16 - 25$.

Combining like terms, $-3x = -9$.

Dividing by -3, $x = 3$. Ans.

(225) (*a*) Simplifying by Art. **538**,

$$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}.$$

$$2\sqrt{48} = 2\sqrt{16} \times \sqrt{3} = 8\sqrt{3}.$$

$$3\sqrt{108} = 3\sqrt{36} \times \sqrt{3} = 18\sqrt{3}.$$

Sum $= 29\sqrt{3}$. Ans. (Art. **544**.)

[illegible]

Sum = [illegible] Ans. (Art. 541.)

[illegible] (Art. 540.)

[illegible]

Sum = [illegible] Ans.

(226) Let x = the capacity.

Then $x - 42$ = amount held at first;

$$7(x - 42) = x;$$

$$7x - 294 = x;$$

$$6x = 294;$$

$x = 49$ gallons. Ans.

(227) (*a*) $2\sqrt{3x-4} - x = 4$.

Transposing, Art. **579**, so that the radical stands alone in the first member, $2\sqrt{3x+4} = x + 4$.

Squaring both members, since the index of the radical is understood to be 2, $4(3x + 4) = (x + 4)^2$,

or $12x + 16 = x^2 + 8x + 16$.

Transposing and uniting terms,

$$-x^2 - 8x + 12x = 16 - 16.$$

$$-x^2 + 4x = 0.$$

Dividing by $-x$, $x - 4 = 0$;

whence, $x = 4$. Ans.

(*b*) $\sqrt{3x-2} = 2(x-4)$.

Squaring, $3x - 2 = 4(x-4)^2$,

or $3x - 2 = 4x^2 - 32x + 64$.

Transposing, $-4x^2+32x+3x=64+2$.

Combining terms, $-4x^2+35x=66$.

Dividing by -4, $x^2-\frac{35x}{4}=-\frac{66}{4}$.

Completing the square,

$$x^2-\frac{35x}{4}+\left(\frac{35}{8}\right)^2=-\frac{66}{4}+\frac{1,225}{64}.$$

$$x^2-\frac{35x}{4}+\left(\frac{35}{8}\right)^2=-\frac{1,056}{64}+\frac{1,225}{64}=\frac{169}{64}.$$

Extracting the square root,

$$x-\frac{35}{8}=\pm\frac{13}{8}.$$

Transposing, $x=\frac{35}{8}\pm\frac{13}{8}=6$, or $2\frac{3}{4}$. Ans.

(*c*) $\sqrt{x+16}=2+\sqrt{x}$ becomes $x+16=4+4\sqrt{x}+x$, when squared. Canceling x (Art. **562**), and transposing,

$$-4\sqrt{x}=4-16.$$

$$-4\sqrt{x}=-12.$$

$$\sqrt{x}=3;$$

whence, $x=3^2=9$. Ans.

(228) (*a*) $\sqrt{3x-5}=\frac{\sqrt{7x^2+36x}}{x}$.

Clearing of fractions,

$$x\sqrt{3x-5}=\sqrt{7x^2+36x}.$$

Removing radicals by squaring,

$$x^2(3x-5)=7x^2+36x.$$

$$3x^3-5x^2=7x^2+36x.$$

Dividing by x, $3x^2-5x=7x+36$.

Transposing and uniting,

$$3x^2-12x=36.$$

Dividing by 3, $x^2-4x=12$.

Completing the square,

$$x^2 - 4x + 4 = 16.$$

Extracting the square root,

$$x - 2 = \pm 4;$$

whence, $x = 6$, or -2. Ans.

$$x^2 - (b - a)c = ax - bx + cx.$$

Transposing, $x^2 - ax + bx - cx = (b - a)c$.

Factoring, $x^2 - (a - b + c)x = bc - ac$.

Regarding $a - b + c$ as the coefficient of x, and completing the square,

$$x^2 - (a - b + c)x + \left(\frac{a - b + c}{2}\right)^2 = bc - ac + \left(\frac{a - b + c}{2}\right)^2.$$

$$x^2 - (a - b + c)x + \left(\frac{a - b + c}{2}\right)^2 =$$

$$\frac{a^2 - 2ab + b^2 - 2ac + 2bc + c^2}{4}.$$

$$x - \frac{a - b + c}{2} = \pm \frac{a - b - c}{2}.$$

$$x = \frac{2a - 2b}{2}, \text{ or } \frac{2c}{2}.$$

$$x = a - b, \text{ or } c. \quad \text{Ans.}$$

[illegible] $= 0$, becomes

[illegible] $= 0$, when expanded.

[illegible] terms,

$$-x^2 - 2x = -2$$

[illegible]

$$x^2 - 2x = 2$$

Completing the square, $x^2 - 2x + 1 = 3$.

Extracting the square root, $x - 1 = \pm\sqrt{3}$;

whence, $x = 1 \pm \sqrt{3}$. Ans.

[illegible]

Squaring and clearing of fractions,

[illegible] $= a^2 - b^2$.

Squaring both members,

[illegible] $= a^4 - 2a^2b^2 + b^4$.

pleting the square,

$$x^2 - 4abx + 4a^2b^2 = a^4 + 2a^2b^2 + b^4.$$
$$x - 2ab = \pm (a^2 + b^2).$$
$$x = (a^2 + 2ab + b^2),$$
$$\text{or } - (a^2 - 2ab + b^2).$$
$$x = (a + b)^2, \text{ or } - (a - b)^2. \text{ Ans.}$$

$$-\frac{1}{\sqrt{x+1}} + \frac{1}{\sqrt{x-1}} = \frac{1}{\sqrt{x^2-1}} \text{ becomes}$$

$-\sqrt{x-1} + \sqrt{x+1} = 1$ when cleared of fractions.

uring,

$$1 - 2\sqrt{x^2-1} + x + 1 = 1.$$
$$-2\sqrt{x^2-1} = 1 - 2x.$$

uring again, $4x^2 - 4 = 1 - 4x + 4x^2$.

celing $4x^2$ and transposing,

$$4x = 5.$$
$$x = \frac{5}{4} = 1\frac{1}{4} \quad \text{Ans.}$$

0) $5x - 2y = 51.$ (1)

$19x - 3y = 180.$ (2)

will first find the value of x by transposing $-2y$ to the member of equation (1), whence $5x = 51 + 2y$, and

$$x = \frac{51 + 2y}{5}. \qquad (3)$$

gives the value of x in terms of y. Substituting the of x for the x in (2), (Art. **609.**)

$$\frac{19(51 + 2y)}{5} - 3y = 180.$$

anding, $\frac{969 + 38y}{5} - 3y = 180.$

ring of fractions, $969 + 38y - 15y = 900.$

nsposing and uniting, $23y = -69.$

$$y = -3. \quad \textbf{Ans.}$$

stituting this value in equation (3), we have

$$x = \frac{51 - 6}{5} = 9. \quad \text{Ans.}$$

[illegible]

Transposing and [illegible]

$$4ab + a^2 - 2ab - b^2 \over 4 = \frac{a^2 - 2ab - b^2}{4}$$

acting square root,

$$x+\frac{a-b}{2}=\pm\frac{a+b}{2}.$$

$$x=-\frac{a-b}{2}+\frac{a+b}{2}=b,$$

or $$x=-\frac{a-b}{2}-\frac{a+b}{2}=-a.$$

:efore, $x=b$, or $-a$. Ans.

2) Let x = rate of current.
y = rate of rowing.

n stream, the rowers are aided by the current, so = 12.

e it takes them twice as long to row a given distance :am as it does down stream, they will go only $\frac{1}{2}$ as far ›ur, or $\frac{1}{2}$ of 12 = 6 miles per hour up stream.

$$x+y=12. \quad (1)$$
$$-x+y=\ \ 6. \quad (2)$$

tracting, $2x = 6$, and $x=3$ miles per hour. Ans.

33) (*a*) $\frac{10x+3}{3}-\frac{6x-7}{2}=10(x-1)$.

ucing the last member to a simpler form, the equation es

$$\frac{10x+3}{3}-\frac{6x-7}{2}=10x-10.$$

ring of fractions by multiplying each term of both ers by 6, the L. C. M. of the denominators, and changing e sign of each term of the numerator of the second ›n, since it is preceded by the minus sign (Art. **567**), re

$$20x+6-18x+21=60x-60.$$

asposing terms, $20x-18x-60x=-60-21-6.$
ıbining like terms, $-58x=-87.$
nging signs, $58x=87;$

whence, $x=\frac{87}{58}=1\frac{1}{2}$. Ans.

Vol. VI.—9.

[illegible]

$$x = -2. \quad \text{Ans.}$$

[illegible]

[illegible] Art. 182.

$$x = \frac{[illegible]}{[illegible]}. \quad \text{Ans.}$$

[illegible]

Clearing of fractions,

$$(\sqrt{x} - 3)(\sqrt{x} - 1) = (\sqrt{x} - 7)(\sqrt{x} - 4) =$$

$$x - 2\sqrt{x} - 3 = x - 3\sqrt{x} - 28.$$

Transposing and canceling x (Art. **562**),

$$-2\sqrt{x} - 3\sqrt{x} = 3 - 28.$$

$$-5\sqrt{x} = -25.$$

$$\sqrt{x} = 5.$$

$$x = 25. \quad \textbf{Ans.}$$

(238) (a) $\sqrt{\frac{3}{2}}$ by Art. **540** $=\sqrt{\frac{3}{2}\times\frac{2}{2}}=\sqrt{\frac{6}{4}}=\frac{1}{2}\sqrt{6}.$ Ans.

(b) $\frac{3}{11}\sqrt{\frac{4}{7}}=\frac{3}{11}\sqrt{\frac{4}{7}\times\frac{7}{7}}=\frac{3}{11}\times\frac{2}{7}\sqrt{7}=\frac{6}{77}\sqrt{7}.$ Ans.

(c) $z\sqrt[3]{\frac{2x}{z}}=z\sqrt[3]{\frac{2x}{z}\times\frac{z^2}{z^2}}=\frac{z}{z}\sqrt[3]{2xz^2}=\sqrt[3]{2xz^2}.$ Ans.

(239) (a) $$\frac{9x+20}{36}=\frac{4(x-3)}{5x-4}+\frac{x}{4}=?$$

When the denominators contain both simple and compound expressions, it is best to remove the simple expressions rst, and then remove each compound expression in order. 'hen, after each multiplication, the result should be reduced ɔ the simplest form.

Multiplying both sides by 36,

$$9x+20=\frac{144(x-3)}{5x-4}+9x,$$

or $$\frac{144x-432}{5x-4}=20.$$

Clearing of fractions,

$$144x-432=100x-80.$$

Transposing and combining,

$$44x=352;$$

whence, $x=8.$ Ans.

(b) $ax-\frac{3a-bx}{2}=\frac{1}{2}$ becomes, when cleared of fractions,

$$2ax-3a+bx=1.$$

Transposing and uniting terms,

$$2ax+bx=3a+1.$$

Factoring, $(2a+b)x=3a+1;$

whence, $x=\frac{3a+1}{2a+b}.$ Ans.

(c) $am - b - \frac{ax}{b} - \frac{x}{m} = 0$, when cleared of fractions =

$$abm^2 - b^2m - amx + bx = 0.$$

Transposing, $bx - amx = b^2m - abm^2$.

Factoring, $(b - am)x = bm(b - am)$;

whence, $x = \frac{bm(b - am)}{(b - am)} = bm$. Ans.

(240) $x + y = 13$. **(1)**

$xy = 36$. **(2)**

Squaring (1) we have

$x^2 + 2xy + y^2 = 169$. **(3)**

Multiplying (2) by 4, $4xy = 144$. **(4)**

Subtracting (4) from (3),

$x^2 - 2xy + y^2 = 25$. **(5)**

Extracting the square root of (5),

$x - y = \pm 5$. **(6)**

Adding (6) and (1), $2x = 18$ or 8,

$x = 9$ or 4. **Ans.**

Substituting the value of x in (1),

$9 + y = 13$,

or $4 + y = 13$;

whence, $\left.\begin{array}{l} y = 4, \\ y = 9. \end{array}\right\}$ **Ans.**

(241) $x^3 - y^3 = 98$. **(1)**

$x - y = 2$. **(2)**

From (2), $x = 2 + y$. **(3)**

Substituting the value of x in (1),

$8 + 12y + 6y^2 + y^3 - y^3 = 98$.

Combining and transposing,

$6y^2 + 12y = 90$.

$y^2 + 2y = 15$.

$y^2 + 2y + 1 = 15 + 1 = 16$.

$y + 1 = \pm 4$.

$y = 3$, or -5. **Ans.**

Substituting the value of y in (3), $x = 5$, or -3. **Ans.**

(242) Let $x =$ the whole quantity.

Then, $\frac{2x}{3} + 10 =$ the quantity of niter.

$\frac{x}{6} - 4\frac{1}{2} =$ the quantity of sulphur.

$\frac{1}{7}\left(\frac{2x}{3} + 10\right) - 2 =$ the quantity of charcoal.

Hence, $x = \frac{2x}{3} + 10 + \frac{x}{6} - 4\frac{1}{2} + \frac{1}{7}\left(\frac{2x}{3} + 10\right) - 2.$

Clearing of fractions and expanding terms,

$$42x = 28x + 420 + 7x - 189 + 4x + 60 - 84.$$

Transposing,

$$42x - 28x - 7x - 4x = 420 - 189 + 60 - 84.$$

$$3x = 207.$$

$$x = 69 \text{ lb.} \quad \text{Ans.}$$

(243) Let $x =$ number of revolutions of hind wheel.

Then, $51 + x =$ number of revolutions of fore wheel.

Since, in making these revolutions both wheels traveled the same distance, we have

$$16x = 14(51 + x).$$

$$16x = 714 + 14x.$$

$$2x = 714.$$

$$x = 357.$$

Since the hind wheel made 357 revolutions, and since the distance traveled for each revolution is equal to the circumference of the wheel, or 16 feet, the whole distance traveled $= 357 \times 16$ ft. $= 5,712$ feet. Ans.

(244) (*a*) Transposing,

$$5x^2 - 2x^2 = 24 + 9.$$

Uniting terms, $3x^2 = 33.$

$$x^2 = 11.$$

Extracting the square root of both members,

$$x = \pm\sqrt{11}. \quad \text{Ans.}$$

(*b*) $$\frac{3}{4x^2} - \frac{1}{6x^2} = \frac{7}{3}.$$

Clearing of fractions, $9 - 2 = 28x^2$.

Transposing terms, $28x^2 = 7$.

$$x^2 = \frac{1}{4}.$$

Extracting the square root of both members,

$$x = \pm\frac{1}{2}. \quad \text{Ans.}$$

(*c*) $$\frac{x^2}{5} - \frac{x^2 - 10}{15} = 7 - \frac{50 + x^2}{25}.$$

Clearing of fractions by multiplying each term of both members by 75, the L. C. M. of the denominators, and expanding,

$$15x^2 - 5x^2 + 50 = 525 - 150 - 3x^2.$$

Transposing and uniting terms,

$$13x^2 = 325.$$

Dividing by 13, $$x^2 = \frac{325}{13} = 25,$$

or $x = \pm 5$. Ans.

(245) $$4x + 3y = 48. \quad (1)$$
$$-3x + 5y = 22. \quad (2)$$

From (1), $$y = \frac{48 - 4x}{3}. \quad (3)$$

From (2), $$y = \frac{22 + 3x}{5}. \quad (4)$$

Placing (3) and (4) equal to each other,

$$\frac{48 - 4x}{3} = \frac{22 + 3x}{5}.$$

Clearing of fractions,

$$240 - 20x = 66 + 9x.$$

Transposing and uniting terms,

$$-29x = -174,$$

or $x = 6$. Ans.

Substituting this value in (4),

$$y = \frac{22 + 18}{5} = 8. \quad \text{Ans.}$$

(246) Let $x =$ speed of one.

$x + 10 =$ speed of other.

Then, $\dfrac{1,200}{x} =$ number of hours one train required.

$\dfrac{1,200}{x+10} =$ number of hours other train required.

$$\frac{1,200}{x} = \frac{1,200}{x+10} + 10.$$

$$1,200x + 12,000 = 1,200x + 10x^2 + 100x.$$

$$-10x^2 - 100x = -12,000.$$

$$10x^2 + 100x = 12,000.$$

$$x^2 + 10x = 1,200.$$

$$x^2 + 10x + 25 = 1,200 + 25 = 1,225.$$

$$x + 5 = \pm 35.$$

$$\left.\begin{aligned} x &= 30 \text{ miles per hour.} \\ x + 10 &= 40 \text{ miles per hour.} \end{aligned}\right\} \text{Ans.}$$

(247) $\left.\begin{aligned} 2x - \frac{y-3}{5} - 4 &= 0 \\ 3y + \frac{x-2}{3} - 9 &= 0 \end{aligned}\right\}$ cleared of fractions, becomes

$$10x - y + 3 - 20 = 0. \qquad (1)$$

$$9y + x - 2 - 27 = 0. \qquad (2)$$

Transposing and uniting, $10x - y = 17. \qquad (3)$

$$x + 9y = 29. \qquad (4)$$

Multiplying (4) by 10 and subtracting (3) from the result,

$$\begin{aligned} 10x + 90y &= 290 \\ 10x - y &= 17 \\ \hline 91y &= 273 \\ y &= 3. \quad \text{Ans.} \end{aligned}$$

Substituting value of y in (4),

$$x + 27 = 29.$$

$$x = 2. \quad \text{Ans.}$$

(248) (*a*) $\sqrt[3]{2} \times \sqrt[5]{3} = 2^{\frac{1}{3}} \times 3^{\frac{1}{5}}$. (Art. **547.**)

$2^{\frac{5}{15}} \times 3^{\frac{3}{15}} = \sqrt[15]{2^5} \times \sqrt[15]{3^3} = \sqrt[15]{32 \times 27} = \sqrt[15]{864}$. Ans.

(*b*) $\sqrt[4]{2ax} \times \sqrt[3]{ax^2} = (2ax)^{\frac{1}{4}} \times (ax^2)^{\frac{1}{3}} = \sqrt[12]{8a^3x^3} \times \sqrt[12]{a^4x^8} =$
$\sqrt[12]{8a^7x^{11}}$. Ans.

(*c*) $2\sqrt{xy} \times 3\sqrt[5]{x^3y} = 2 \times 3(xy)^{\frac{1}{2}} \times (x^3y)^{\frac{1}{5}} = 6\sqrt[10]{x^{11}y^7}$. Ans.

(249) Let $x =$ the part of the work which they all can do in 1 day when working together.

Then, since $\frac{1}{7\frac{1}{2}} = \frac{2}{15}$, $\frac{1}{5} + \frac{1}{6} + \frac{2}{15} = x$;

or, clearing of fractions and adding,

$$15 = 30x, \text{ and } x = \frac{1}{2}.$$

Since they can do $\frac{1}{2}$ the work in 1 day, they can do all of the work in 2 days. Ans.

(250) Let $x =$ value of first horse.
$y =$ value of second horse.

If the saddle be put on the first horse, its value will be $x + 10$. This value is double that of the second horse, or $2y$, whence the equation, $x + 10 = 2y$.

If the saddle be put on the second horse, its value is $y + 10$. This value is \$13 less than the first, or $x - 13$, whence the equation, $y + 10 = x - 13$.

$$x + 10 = 2y. \quad (1)$$
$$y + 10 = x - 13. \quad (2)$$

Transposing, $x - 2y = -10$. (3)
$$-x + y = -23. \quad (4)$$

Adding (3) and (4), $-y = -33$.

$y = \$33$, or value of second horse. Ans.

Substituting in (1), $x + 10 = 66$;

or $x = \$56$, or value of first horse. Ans.

(251) Let $x =$ A's money.
$y =$ B's money.

If A should give B \$5, A would have $x - 5$, and B, $y + 5$. B would then have \$6 more than A, whence the equation,

$$y + 5 - (x - 5) = 6. \qquad (1)$$

But if A received \$5 from B, A would have $x + 5$, and B, $y - 5$, and 3 times his money, or 3 $(x + 5)$, would be \$20 more than 4 times B's, or 4 $(y - 5)$, whence the equation,

$$3(x + 5) - 4(y - 5) = 20. \qquad (2)$$

Expanding equations (1) and (2),

$$y + 5 - x + 5 = 6. \qquad (3)$$

$$3x + 15 - 4y + 20 = 20. \qquad (4)$$

Transposing and combining,

$$y - x = -4. \qquad (5)$$

$$-4y + 3x = -15. \qquad (6)$$

Multiplying (5) by 4, and adding to (6),

$$4y - 4x = -16.$$

$$\underline{-4y + 3x = -15.}$$

$$-x = -31.$$

$$x = 31.$$

Substituting value of x in (5),

$$y - 31 = -4.$$

$$y = 27.$$

Hence, $\left.\begin{array}{l} x = \$31, \text{ A's money.} \\ y = \$27, \text{ B's money.} \end{array}\right\}$ Ans.

(252) (*a*) $x^2 - 6x = 16.$

Completing the square (Art. **597**),

$$x^2 - 6x + 9 = 16 + 9.$$

Extracting the square root, $x - 3 = \pm 5.$

Transposing, $x = 8$, or -2. Ans.

(*b*) $x^2 - 7x = 8.$

$$x^2 - 7x + \left(\frac{7}{2}\right)^2 = 8 + \left(\frac{7}{2}\right)^2 = \frac{81}{4}.$$

$$x - \frac{7}{2} = \pm \frac{9}{2};$$

whence, $x = 8$, or -1. Ans.

(c) $$9x^2 - 12x = 21.$$

Dividing by 9, $$x^2 - \frac{12x}{9} = \frac{21}{9},$$

or $$x^2 - \frac{4x}{3} = \frac{7}{3}.$$

Completing the square,

$$x^2 - \frac{4x}{3} + \left(\frac{2}{3}\right)^2 = \frac{7}{3} + \frac{4}{9} = \frac{25}{9}.$$

Extracting square root, $$x - \frac{2}{3} = \pm\frac{5}{3}.$$

Transposing, $$x = \frac{2}{3} + \frac{5}{3} = \frac{7}{3},$$

or $$x = \frac{2}{3} - \frac{5}{3} = -\frac{3}{3} = -1.$$

Therefore, $$x = \frac{7}{3}, \text{ or } -1. \quad \text{Ans.}$$

(253) $(c^{-\frac{2}{3}})^{-\frac{3}{4}} = c^{\frac{1}{2}}$. Ans. (Art. **526**, III.)

$$(m\sqrt{n^3})^{-\frac{2}{3}} = m^{-\frac{2}{3}}(n^{\frac{3}{2}})^{-\frac{2}{3}} = m^{-\frac{2}{3}}n^{-1} = \frac{1}{m^{\frac{2}{3}}n}. \quad \text{Ans.}$$

$(cd^{-2})^{\frac{1}{a}} = c^{\frac{1}{a}}d^{-\frac{2}{a}}$, or $\sqrt[a]{cd^{-2}}$, or $\sqrt[a]{\frac{c}{d^2}}$. Ans. (Art. **530.**)

(254)

Let x = number of quarts of 90-cent wine in the mixture.

y = number of quarts of 50-cent wine in the mixture

Then, $x + y = 60$, (1)

and $90x + 50y = 4,500 = 75 \times 60$. (2)

Multiplying (1) by 50,

$50x + 50y = 3,000$. (3)

Subtracting (3) from (2),

$$40x = 1,500;$$

whence, $x = 37\frac{1}{2}$ qt. Ans.

Multiplying (1) by 90, $90x + 90y = 5,400$ (4)

Subtracting (2), $90x + 50y = 4,500$ (2)

$$40y = 900;$$

whence, $y = 22\frac{1}{2}$ qt. Ans.

(255) Let $x =$ the numerator of the fraction.
$y =$ the denominator of the fraction.

Then, $\frac{x}{y} =$ the fraction.

From the conditions, $$\frac{2x}{y+7} = \frac{2}{3}, \qquad (1)$$

and $$\frac{x+2}{2y} = \frac{3}{5}. \qquad (2)$$

Clearing (1) and (2) of fractions, and transposing,

$$6x = 2y + 14, \quad (3)$$

and $$5x = 6y - 10. \quad (4)$$

Solving for x, $$x = \frac{2y+14}{6} = \frac{y+7}{3}. \quad (5)$$

$$x = \frac{6y-10}{5}. \quad (6)$$

Equating (5) and (6), $$\frac{y+7}{3} = \frac{6y-10}{5}.$$

Clearing of fractions, $5y + 35 = 18y - 30$
whence, $13y = 65$,
or, $y = 5$.

Substituting this value of y in (3),

$$6x = 10 + 14 = 24;$$

whence, $x = 4$.

Therefore, the fraction is $\frac{4}{5}$. Ans.

(256) Let $x =$ digit in tens place.
$y =$ digit in units place.

Then $10x + y =$ the number.

From the conditions of the example,

$$10x + y = 4(x+y) = 4x + 4y;$$

whence, $3y = 6x$,
or $y = 2x$.

From the conditions of the example,

$$10x + y + 18 = 10y + x;$$

whence, $9y - 9x = 18$.

Substituting the value of y, found above,

$$18x - 9x = 18;$$

$$\text{whence,}\quad x = 2.$$

$$y = 2x = 4.$$

Hence, the number $= 10x + y = 20 + 4 = 24$. Ans.

(257) Let x = greater number.
y = less number.

Then,
$$x + 4 = 3\tfrac{1}{4}y, \qquad (1)$$

$$\text{and}\quad y + 8 = \frac{x}{2}. \qquad (2)$$

Clearing of fractions, $4x + 16 = 13y$,

$$\text{and}\quad 2y + 16 = x;$$

$$\text{whence,}\quad 13y - 4x = 16. \qquad (3)$$

$$2y - x = -16. \qquad (4)$$

Multiplying (4) by 4, and subtracting from (3)

$$5y = 80,$$

$$\text{or}\quad y = 16. \text{ Ans.}$$

Substituting in (4), $32 - x = -16$;

$$\text{whence,}\quad x = 48. \text{ Ans.}$$

LOGARITHMS.

(QUESTIONS 258–272.)

(258) First raise $\frac{200}{100}$ to the .29078 power. Since $\frac{200}{100} = 2$, $\left(\frac{200}{100}\right)^{.29078} = 2^{.29078}$, and $\log 2^{.29078} = .29078 \times \log 2 = .29078 \times .30103 = .08753$. Number corresponding $= 1.2233$. Then,

$$1 - \left(\frac{200}{100}\right)^{.29078} = 1 - 1.2233 = -.2233.$$

We now find the product required by adding the logarithms of 351.36, 100, 24, and .2233, paying no attention to the negative sign of .2233 until the product is found. (Art. **647.**)

$$\begin{aligned} \text{Log } 351.36 &= 2.54575 \\ \log\ 100 &= 2 \\ \log\ 24 &= 1.38021 \\ \log\ .2233 &= \bar{1}.34889 \\ \hline sum &= 5.27485 = \end{aligned}$$

$$\log 351.36 \times 100 \times 24 \left(1 - \left(\frac{200}{100}\right)^{.29078}\right)$$

Number corresponding $= 188,300$.

The number is negative, since multiplying positive and negative signs gives negative; and the sign of .2233 is minus. Hence,

$$x = -188,300. \quad \text{Ans.}$$

(259) (*a*) Log 2,376 = 3.37585. Ans. (See Arts. **625** and **627.**)

(*b*) Log .6413 = $\bar{1}.80706$. Ans.

(*c*) Log .0002507 = $\bar{4}.39915$. Ans.

(260) (*a*) Apply rule, Art. **652.**

$$\begin{aligned} \text{Log } 755.4 &= 2.87818 \\ \log .00324 &= \overline{3}.51055 \\ \hline \textit{difference} &= 5.36763 = \text{logarithm of quotient.} \end{aligned}$$

The mantissa is not found in the table. The next less mantissa is 36754. The difference between this and the next greater mantissa is 773 − 754 = 19, and the P. P. is 763 − 754 = 9. Looking in the P. P. section for the column headed 19, we find opposite 9.5, 5, the fifth figure of the number. The fourth figure is 1, and the first three figures 233; hence, the figures of the number are 23315. Since the characteristic is 5, 755.4 ÷ .00324 = 233,150. Ans.

(*b*) Apply rule, Art. **652.**

$$\begin{aligned} \text{Log } .05555 &= \overline{2}.74468 \\ \log .0008601 &= \overline{4}.93455 \\ \hline \textit{difference} &= 1.81013 = \text{logarithm of quotient.} \end{aligned}$$

The number whose logarithm is 1.81013 equals 64.584.

Hence, .05555 ÷ .0008601 = 64.584. Ans.

(*c*) Apply rule, Art. **652.**

$$\begin{aligned} \text{Log } 4.62 &= .66464 \\ \log .6448 &= \overline{1}.80943 \\ \hline \textit{difference} &= .85521 = \text{logarithm of quotient.} \end{aligned}$$

Number whose logarithm = .85521 = 7.1648.

Hence, 4.62 ÷ .6448 = 7.1648. Ans.

(261) $$x^{.74} = \frac{238 \times 1{,}000}{.0042^{.6602}}.$$

$$\begin{aligned} \text{Log } 238 &= 2.37658 \\ \log 1{,}000 &= 3. \\ \hline \textit{sum} &= 5.37658 = \log (238 \times 1{,}000) \end{aligned}$$

$$\begin{array}{r} \text{Log } .0042 = \overline{3}.62325 \\ .6602 \\ \hline 124650 \\ 373950 \\ 373950 \\ \hline .411469650 \end{array} \text{ or } .41147.$$

$$\begin{array}{r} .6602 \\ -3 \\ \hline -1.9806 \end{array} = \text{characteristic.}$$

Adding,
$$\begin{array}{r} .41147 \\ -1.9806 \\ \hline \bar{2}.43087 \end{array} \text{ (See Art. **659.**)}$$

Then, $\log\left(\frac{238 \times 1,000}{.0042^{.6602}}\right) = 5.37658 - \bar{2}.43087 = 6.94571 =$ $74 \log x$; whence, $\log x = \frac{6.94571}{.74} = 9.38609$. Number whose logarithm $= 9.38609$ is $2,432,700,000 = x$. Ans.

(262) Log $.00743 = \bar{3}.87099$.
log $.006 = \bar{3}.77815$.

$\sqrt[5]{.00743} = \log .00743 \div 5$ (Art. **662**), and $\sqrt[.6]{.006} = \log .006 \div .6$. Since these numbers are wholly decimal, we apply Art. **663.**

$$\begin{array}{r} 5\,)\,\bar{3}.87099 \\ \hline \bar{1}.57419 \end{array} = \log \sqrt[5]{.00743}.$$

The characteristic $\bar{3}$ will not contain 5. We then add $\bar{2}$ to it, making $\bar{5}$. 5 is contained in $\bar{5}$, $\bar{1}$ times. Hence, the characteristic is $\bar{1}$. Adding the same number, 2, to the mantissa, we have 2.87099. $2.87099 \div 5 = .57419$. Hence, $\log \sqrt[5]{.00743} = \bar{1}.57419$.

$$\begin{array}{r} .6\,)\,\bar{3}.77815 \\ \hline \bar{5}. \\ 1.29691 \\ \hline \end{array} \quad \begin{array}{l} .6 \text{ is contained in } \bar{3}, -5 \text{ times.} \\ .6 \text{ is contained in } .77815, 1.29691 \text{ times.} \end{array}$$

sum $= \bar{4}.29691 = \log \sqrt[.6]{.006}$.

$$\begin{array}{rl} \text{Log } \sqrt[5]{.00743} & = \bar{1}.57419 \\ \log \sqrt[.6]{.006} & = \bar{4}.29691 \\ \hline \end{array}$$

difference $= 3.27728 = \log$ of quotient.

Number corresponding $= 1,893.6$.

Hence, $\sqrt[5]{.00743} \div \sqrt[.6]{.006} = 1,893.6$. Ans.

[illegible]

244. [illegible]

[illegible] Ans.

246. [illegible]

[illegible]

[illegible] the logarithm of the required power. ([illegible] 687.)

[illegible] = 7.22472 is 16,777,000.

Hence, 32[illegible] 16,777,000. Ans.

(*b*) $.76^{3.62}$.

Log $.76 = \bar{1}.88081$.

(See Arts. **658** and **659.**)

$$\begin{array}{r} \bar{1}+.88081 \\ 3.62 \\ \hline 176162 \\ 528486 \\ 264243 \\ \hline 3.1885322 \\ -\ 3.62 \\ \hline \bar{1}.56853 = \log .37028. \end{array}$$

Hence, $.76^{3.62} = .37028$. Ans.

(*c*) $.84^{.38}$.

Log $.84 = \bar{1}.92428$.

$$\begin{array}{r} \bar{1}+.92428 \\ .38 \\ \hline 739424 \\ 277284 \\ \hline .3512264 \\ -\ .38 \\ \hline \bar{1}.97123 = \log .93590. \end{array}$$

Hence, $.84^{.38} = .93590$. Ans.

(267) $\text{Log} \sqrt[6]{\frac{1}{249}} - \log \sqrt[5]{\frac{23}{71}} =$ logarithm of answer.

$\text{Log} \sqrt[6]{\frac{1}{249}} = \frac{1}{6}(\log 1 - \log 249) = \frac{1}{6}(0 - 2.39620) = -.39937$ $=$ (adding $+1$ and -1) $\bar{1}.60063$.

$\text{Log} \sqrt[5]{\frac{23}{71}} = \frac{1}{5}(\log 23 - \log 71) = \frac{1}{5}(1.36173 - 1.85126) =$ $\frac{1}{5}(-.48953) = -.097906 =$ (adding $+1$ and -1) $\bar{1}.902094$, or $\bar{1}.90209$ when using 5-place logarithms.

Hence, $\bar{1}.60063 - \bar{1}.90209 = \bar{1}.69854 = \log .49950$. Therefore,

$$\sqrt[6]{\frac{1}{249}} \div \sqrt[5]{\frac{23}{71}} = .49950. \quad \text{Ans.}$$

(268) The mantissa is not found in the table. The next less mantissa is .81291; the difference between this and the

ext greater mantissa is 298 − 291 = 7, and the P. P. is 91293 − .81291 = 2. Looking in the P. P. section for the olumn headed 7, we find opposite 2.1, 3, the fifth figure of he number; the fourth figure is 0, and the first three igures, 650. Hence, the number whose logarithm is .81293 s 6.5003. Ans.

2.52460 = logarithm of 334.65. Ans. (See Art. **640.**)

$\overline{1}.27631$ = logarithm of .18893. Ans. We choose 3 for the fifth figure because, in the proportional parts column headed 23, 6.9 is nearer 8 than 9.2.

(**269**) The most expeditious way of solving this example is the following:

$$p\, v^{1.41} = p_1 v_1^{\;1.41}, \text{ or } v_1 = \sqrt[1.41]{\frac{p\, v^{1.41}}{p_1}} = v \sqrt[1.41]{\frac{p}{p_1}}.$$

Substituting values given, $v_1 = 1.495 \sqrt[1.41]{\dfrac{134.7}{16.421}}$.

$$\text{Log } v_1 = \log\ 1.495 + \frac{\log\ 134.7 - \log\ 16.421}{1.41} = .17464 +$$

$$\frac{2.12937 - 1.21540}{1.41} = .17464 + .64821 = .82285 = \log\ 6.6504;$$

whence, $v_1 = 6.6504$. Ans.

(**270**) $\text{Log} \sqrt[5]{\dfrac{7.1895 \times 4{,}764.2^2 \times 0.00326^5}{.000489 \times 457^3 \times .576^2}} = \dfrac{1}{5}[\log 7.1895$ $+ 2 \log 4{,}764.2 + 5 \log .00326 - (\log .000489 + 3 \log 457 + 2$ $\log .576)] = \dfrac{\overline{5}.77878 - 4.18991}{5} = \overline{2}.31777 = \log .020786.$ Ans

$$\begin{aligned} \text{Log } 7.1895 &= .85670 \\ 2 \log 4{,}764.2 = 2 \times 3.67799 &= 7.35598 \\ 5 \log\ .00326 = 5 \times \overline{3}.51322 &= \overline{13}.56610 \\ \hline sum &= \overline{5}.77878 \end{aligned}$$

$$\begin{aligned} \text{Log } .000489 &= \overline{4}.68931 \\ 3 \log\ 457 = 3 \times 2.65992 &= 7.97976 \\ 2 \log .576 = 2 \times \overline{1}.76042 &= \overline{1}.52084 \\ \hline sum &= 4.18991. \end{aligned}$$

(271) Substituting the values given,

$$p = \frac{\frac{8,000}{\cancel{960,000}} \times \left(\frac{3}{16}\right)^{2.18}}{\cancel{120} \times 2.25} = \frac{8,000\left(\frac{3}{16}\right)^{2.18}}{2.25}.$$

$\text{Log}\, p = \log 8,000 + 2.18 \log \frac{3}{16} - \log 2.25 = 3.90309 + 2.18 (\log 3 - \log 16) - .35218 = 3.55091 + 2.18 \times (.47712 - 1.20412) = 1.96605 = \log 92.480.$ Ans.

(272) Solving for t, $t = \sqrt[2.18]{\frac{p l d}{960,000}}.$

Substituting values given,

$$t = \sqrt[2.18]{\frac{\overset{44}{\cancel{160} \times \cancel{132} \times \cancel{2}}}{\underset{\underset{\underset{1,000}{\cancel{3,000}}}{\cancel{6,000}}}{\cancel{960,000}}}} = \sqrt[2.18]{.044}.$$

$$\text{Log}\, t = \frac{\log .044}{2.18} = \frac{\bar{2}.64345}{2.18} = \frac{-2.18 + .82345}{2.18} = \bar{1}.37773 = \log .23863. \text{ Ans.}$$

GEOMETRY AND TRIGONOMETRY.

(QUESTIONS 273–354.)

(273) When one straight line meets another straight line at a point between the ends, the sum of the two adjacent angles equals two right angles. Therefore, since one of the angles equals $\frac{4}{5}$ of a right angle, then, the other angle equals $\frac{10}{5}$, or two right angles, minus $\frac{4}{5}$. We have, then, $\frac{10}{5} - \frac{4}{5} = \frac{6}{5}$, or $1\frac{1}{5}$ right angles.

(274) The size of one angle is $\frac{1}{6}$ of two right angles, or $\frac{1}{3}$ of a right angle.

(275) The pitch being 4, the number of teeth in the wheel equals 4×12, or 48. The angle formed by drawing lines from the center to the middle points of two adjacent teeth equals $\frac{1}{48}$ of 4 right angles, or $\frac{1}{12}$ of a right angle.

(276) It is an isosceles triangle, since the sides opposite the equal angles are equal.

(277) An equilateral heptagon has seven equal sides; therefore, the length of the perimeter equals 7×3, or 21 inches.

(278) A regular decagon has 10 equal sides; therefore, the length of one side equals $\frac{40}{10}$, or 4 inches.

(279) The sum of all the interior angles of any polygon equals two right angles, multiplied by the number of sides

[illegible]

286. [illegible] angles of an equiangular [illegible] is [illegible] right angles. [illegible] since the [illegible] angles of the equiangular [illegible] equals [illegible]

287. [illegible] acute angles of a right-angled [illegible] therefore, if one of them [illegible] the other equals [illegible] $-\frac{5}{8}$, or $\frac{3}{8}$ of a [illegible]

(288) (See Art. **734.**)

(289) In Fig. 2, $AB = 4$ inches, and $AO = 6$ inches. We first find the length of DO. $DO = \sqrt{\overline{OA}^2 - \overline{DA}^2}$; but $\overline{OA}^2 = 6^2$, or 36, and $\overline{DA}^2 = \left(\frac{4}{2}\right)^2$ or 4; therefore, $DO = \sqrt{36 - 4}$, or 5.657.

$DC = CO - DO$, or $DC = 6 - 5.657$, or .343 inch. In the right-angled triangle ADC, we have AC, which is the chord of one-half the arc ACB, equals $\sqrt{2^2 + .343^2}$, or 2.03 inches.

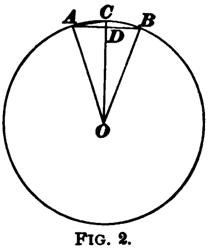

FIG. 2.

(290) The method of solving this is similar to the last problem.

$$DO = \sqrt{9 - 4}, \text{ or } 2.236. \quad DC = 3 - 2.236 = .764.$$

$$AC = \sqrt{2^2 + .764^2}, \text{ or } 2.14 \text{ inches.}$$

(291) Let HK of Fig. 3 be the section; then, $BI = 2$ inches, and $HK = 6$ inches, to find AB. HI ($= 3$ inches) being a mean proportional between the segments AI and IB, we have

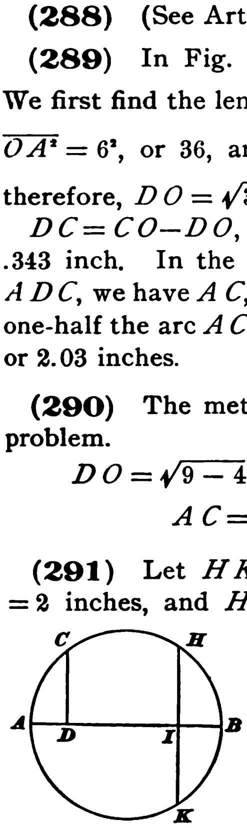

FIG. 3.

$$BI : HI :: HI : IA,$$

$$\text{or } 2 : 3 :: 3 : IA.$$

$$\text{Therefore, } IA = 4\frac{1}{2}.$$

$AB = AI + IB$; therefore, $AB = 4\frac{1}{2} + 2$, or $6\frac{1}{2}$ inches.

(292) Given $OC = 5\frac{3}{4}$ inches, and $OA = \frac{17}{2}$, or $8\frac{1}{2}$ inches, to find AB (see Fig. 4). CA, which is one-half the chord AB, equals

$$\sqrt{\overline{OA}^2 - \overline{OC}^2};$$

$$\text{therefore, } CA = \sqrt{(8\tfrac{1}{2})^2 - (5\tfrac{3}{4})^2},$$

or 6.26 inches. Now, $AB = 2 \times CA$; therefore, $AB = 2 \times 6.26$, or 12.52 inches.

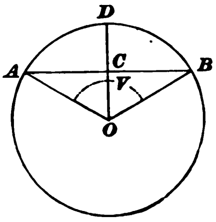

FIG. 4.

(293) The arc intercepted equals $\frac{3}{4}$ of 4, or 3 quadrants. As the inscribed angle is measured by one-half the intercepted arc, we have $\frac{3}{2} = 1\frac{1}{2}$ quadrants is the size of the angle.

(294) Four right angles $\div \frac{2}{7} = 4 \times \frac{7}{2}$, or 14 equal arcs.

(295) Since 24 inches equals the perimeter, we have $\frac{24}{8}$, or 3 inches, as the length of each side or chord.

Then [illegible] $= 7.84$ inches diameter.

(296) Given $AC = \frac{AB}{2} = \frac{10.5}{2}$, or 5.25 inches. AO and $AP = 13$ inches. (See Fig. 5.)

The required distance between the arcs DD' is equal to $OA + AP - OP$. In the right-angled triangle ACO, we have

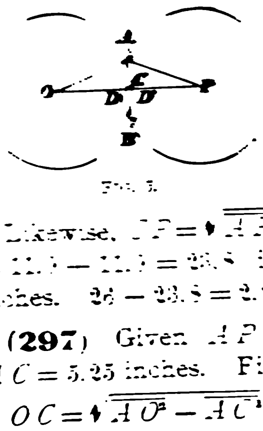

Fig. 5.

$$OC = \sqrt{\overline{AO}^2 - \overline{AC}^2},$$

or $OC = \sqrt{169 - 27.5625} = 11.9$ inches.

Likewise, $CP = \sqrt{\overline{AP}^2 - \overline{AC}^2} = 11.9$. $OP = OC + CP = 11.9 + 11.9 = 23.8$ inches. $OA + AP = 13 + 13 = 26$ inches. $26 - 23.8 = 2.2$ inches. Ans.

(297) Given $AP = 13$ inches, $OA = 8$ inches, and $AC = 5.25$ inches. Fig. 6.

$OC = \sqrt{\overline{AO}^2 - \overline{AC}^2} = \sqrt{8^2 - 5.25^2} =$ 6.03 inches.

$CP = \sqrt{\overline{AP}^2 - \overline{AC}^2} = 11.9$ inches.

$OP = OC + CP = 6.03 + 11.9 =$ 17.93 inches.

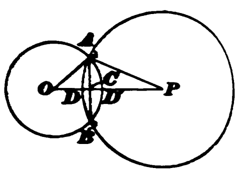

FIG. 6.

$DD' = OA + AP - OP = 8 + 13 - 17.93 = 3.07$ inches. Ans.

(298) $AB = 14$ inches, and $AE = 3\frac{1}{4}$ inches, Fig. 7. $CE = ED$ is a mean proportional between the segments

AE and EB. Then,

$$AE : CE :: CE : EB,$$

or $3\frac{1}{4} : CE :: CE : 10\frac{3}{4}$,

or $\overline{CE}^2 = 3\frac{1}{4} \times 10\frac{3}{4} = 34.9375.$

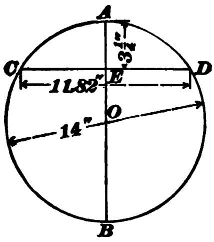

FIG. 7.

Extracting the square root, we have

$$CE = 5.91.$$

$2 \times CE = CD = 2 \times 5.91$, or 11.82 inches. Ans.

(299) In 19° 19′ 19″ there are 69,559 seconds, and in 360°, or a circle, there are 1,296,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{1,296,000}$, or .053672 part of a circle. Ans.

(300) In an angle measuring 19° 19′ 19″ there are 69,559 seconds, and in a quadrant, which is $\frac{1}{4}$ of 360°, or 90°, there are 324,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{324,000}$, or .214688 part of a quadrant. Ans.

(301) Given, $OB = OA = \frac{23}{2}$, or $11\frac{1}{2}$ inches, and angle $AOB = \frac{1}{10}$ of 360°, or 36°. (See Fig. 8.) In the right-angled triangle COB, we have

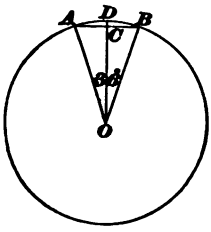

FIG. 8.

$\sin COB = \frac{CB}{OB}$, or $CB = OB \times \sin COB$.

Substituting the values of OB and $\sin COB$, we have

$$CB = 11\frac{1}{2} \times \sin 18°,$$

or $CB = 11\frac{1}{2} \times .30902 = 3.55.$

Since $AB = 2\,CB$, $AB = 2 \times 3.55 = 7.1$ inches.

The perimeter then equals $10 \times 7.1 = 71$ inches, nearly. Ans.

(302)

$$\begin{array}{rrrl} 90^\circ = 89^\circ & 59' & 60'' & \\ 35^\circ & 24' & 25.8'' & \\ \hline 54^\circ & 35' & 34.2'' & \text{Ans.} \end{array}$$

(303) The side $B\,C = \sqrt{\overline{A\,B}^2 - \overline{A\,C}^2}$, or $B\,C = \sqrt{17.69^2 - 9.75^2} = \sqrt{217.8736} = 14$ ft. 9 in. To find the angle $B\,A\,C$, we have $\cos B\,A\,C = \frac{A\,C}{A\,B}$, or $\cos B\,A\,C = \frac{9.75}{17.69} = .55116$.

.55116 equals the cos of 56° 33′ 12.5″.

Angle $A\,B\,C = 90°$ — angle $B\,A\,C$, or 90° — 56° 33′ 12.5″ = 33° 26′ 47.5″.

(304)

$$\begin{array}{rrr} 159^\circ & 27' & 34.6'' \\ 25^\circ & 16' & 8.7'' \\ 3^\circ & 48' & 53\ \ '' \\ \hline 188^\circ & 32' & 36.3'' \end{array}$$

(305)

Sin 17° 28′ = .30015.
Sin 17° 27′ = .29987.

.30015 — .29987 = .00028, the difference for 1′.

$.00028 \times \frac{37}{60} = .00017$, difference for 37″.

.29987 + .00017 = .30004 = sin 17° 27′ 37″.

Cos 17° 27′ = .95398.
Cos 17° 28′ = .95389.

.95398 — .95389 = .00009, difference for 1′.

$.00009 \times \frac{37}{60} = .00006$, difference for 37″.

.95398 — .00006 = .95392 = cos 17° 27′ 37″.

Tan 17° 28′ = .31466.
Tan 17° 27′ = .31434.

.31466 — .31434 = .00032, difference for 1′.

$.00032 \times \frac{37}{60} = .00020$, difference for 37″.

.31434 + .0002 = .31454 = tan 17° 27′ 37″.

$$\left.\begin{array}{l} \text{Sin } 17^\circ\ 27'\ 37'' = .30004 \\ \text{Cos } 17^\circ\ 27'\ 37'' = .95392 \\ \text{Tan } 17^\circ\ 27'\ 37'' = .31454 \end{array}\right\} \text{Ans.}$$

306) From the vertex B, draw BD perpendicular to , forming the right-angled triangles ADB and BDC. he right-angled triangle ADB, AB is known, and also angle A. Hence, $BD = 26.583 \times \sin 36° 20' 43'' =$ $83 \times .59265 = 15.754$ feet. $AD = 26.583 \times \cos 36° 20' 43'' =$ $83 \times .80546 = 21.411$. $AC - AD = 40 - 21.411 =$ 89 feet $= DC$. In the right-angled triangle BDC, the sides BD and DC are known; hence, $\tan C = \frac{BD}{DC} =$

$\frac{54}{89} = .84749$, and angle $C = 40° 16' 52''$. Ans.

$= \frac{BD}{\sin C} = \frac{15.754}{\sin 40° 16' 52''} = \frac{15.754}{.64654} = 24.37$, or 24 ft. 4.4 in. Ans.

ngle $B = 180° - (36° 20' 43'' + 40° 16' 52'') = 180° - 76°$ 35$'' = 103° 22' 25''$. Ans.

307) This problem is solved exactly like problem 305.

Sin of $63° 4' 51.8'' = .89165$.
Cos of $63° 4' 51.8'' = .45273$.
Tan of $63° 4' 51.8'' = 1.96949$.

308) $.27038 = \sin 15° 41' 12.9''$.
$.27038 = \cos 74° 18' 47.1''$.
$2.27038 = \tan 66° 13' 43.2''$.

309) The angle formed by drawing radii to the emities of one of the sides equals $\frac{360°}{11}$, or $32° 43' 38.2''$.

. The length of one side of the undecagon equals $\frac{3 \text{ in.}}{1}$, or 4.6364 inches. The radius of the circle equals

$\frac{\frac{1}{2} \text{ of } 4.6364}{\text{f } \frac{1}{2}(32° 43' 38.2'')} = \frac{2.3182}{.28173} = 8.23$ inches. Ans.

310) If one of the angles is twice the given one, then it t be $2 \times (47° 13' 29'')$, or $94° 26' 58''$. Since there are right angles, or 180°, in the three angles of a triangle, third angle must be $180 - (47° 13' 29'' + 94° 26' 58'')$, or 19$' 33''$.

... angles is one-half as large as the ... must be $\frac{1}{2}$ of 75° 48' 17", or 37° 54' 8.5'. ... equals 180° — (75° 48' 17" + 37° 54' 8.5'), or

... B, draw BD perpendicular to ... right-angled triangles ADB and BDC. ... triangle ADB, AB is known, and also ... $= \sin A \times AB = \sin 54° 54'$

... = 13.434 feet.

... $= \frac{13.434}{13.542} = .99202$, and, hence,

... Ans.

... 54' + 82° 45' 30") = 180 – 137°

... Ans.

... $\times \cos 54° 54' 54" = 16\frac{5}{12} \times$

... $\times \cos 82° 45' 30" = 13\frac{13}{24} \times$

... − 1.70692 = 11.143 = 11 ft.

... certain angle equals 14° 47' 10",

... 47' 10", or 44° 21' 30". $2\frac{1}{2}$

... equals one of the other two ... equals 180° — (110° 53' 45" + 44°

... feet and AC = 792 feet, to find ... angles A and B.

... $-$ 437² = $\sqrt{818,233}$ = 904 ft $6\frac{3}{4}$ in. Ans.

Tan $A = \frac{437}{792} = .55177$; therefore, $A = 28° \ 53' \ 19''$. Ans.
Angle $B = 90° - 28° \ 53' \ 19'' = 61° \ 6' \ 41''$. Ans.

(315) In Fig. 9, angle $A \, O \, B = \frac{1}{8}$ of 360°, or 45°. Angle $m \, O \, B = \frac{1}{2}$ of 45°, or $22\frac{1°}{2}$. Side $A \, B = \frac{1}{8}$ of 56 feet, or 7 feet. Now, in the triangle $m \, O \, B$, we have the angle $m \, O \, B = 22\frac{1°}{2}$, and $m \, B = \frac{7}{2}$, or $3\frac{1}{2}$ feet, given, to find $O \, B$ and the angle $m \, B \, O$.

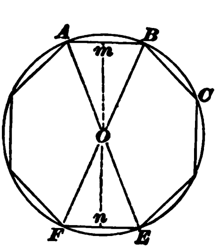

FIG. 9.

$$\text{Sin } m \, O \, B = \frac{m \, B}{O \, B}, \text{ or } O \, B = \frac{m \, B}{\sin m \, O \, B}.$$

Substituting their values, $O \, B = \frac{3.5}{\sin 22\frac{1}{2}°} = \frac{3.5}{.38268} = 9.146$ feet.

$B \, F$, the diameter of the circle, equals $2 \times B \, O$; therefore, $BF = 2 \times 9.146 = 18.292$ feet $= 18$ feet $3\frac{1}{2}$ inches.

$$\begin{aligned} &\text{Angle } B \ O \ m = 22° \ 30'. \\ &B \ O \ m + O \ B \ m = 90°. \\ \text{Therefore,} \quad &O \ B \ m = 90° - B \ O \ m = 90° - 22° \ 30'. \\ &\qquad\qquad = 67° \ 30'. \\ &A \ B \ C = 2 \ O \ B \ m = 2 \ (67° \ 30') = 135°. \end{aligned}$$

Ans.

By Art. **703**, the sum of the interior angles of an octagon is $2 \ (8 - 2) = 12$ right angles. Since the octagon is regular, the interior angles are equal, and since there are eight of them, each one is $\frac{12}{8} = 1\frac{1}{2}$ right angles. $1\frac{1}{2} \times 90° = 135°$.

(316) Lay off with a protractor the angle $A \ O \ C$ equal to $67° \ 8' \ 49''$, Fig. 10. Tangent to the circle at A, draw the line $A \ T$. Through the point C, draw the line $O \ C$, and continue it until it intersects the line $A \ T$ at T. From C

[illegible] perpendicular, respectively, [illegible] the radii [illegible] and [illegible]. [illegible] is the sine, [illegible] the cosine, and [illegible] the tangent.

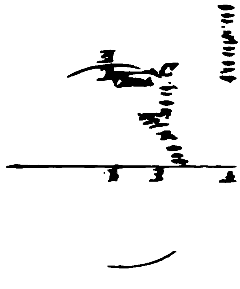

FIG. 10.

317. Suppose that in Fig. 10, the line [illegible] has been drawn equal to 3 times the radius [illegible]. From [illegible] draw [illegible]; then the tangent of [illegible] $= \frac{[illegible]}{[illegible]} = 3$. Where [illegible] cuts the circle at [illegible], draw [illegible] and [illegible] perpendicular, respectively, to [illegible] and $O\,A$. [illegible] is the cosine and [illegible] the sine. The angle corresponding to tan 3 is found in the table to equal 71° 33′ 54″; therefore, sin [illegible] = [illegible] and [illegible] 71° 33′ 54″ = .31623.

318. The angle whose cos is .39278 = 68° 52′ 20″.

Sin of 68° 52′ 20″ = .91963.

Tan of 68° 52′ 20″ = 2.34132.

For a circle with a diameter $4\frac{3}{4}$ times as large, the values of the above cos, sin, and tan will be

$$\left.\begin{array}{l} 4\frac{3}{4} \times .39278 = 1.86570 \text{ cos.} \\ 4\frac{3}{4} \times .91963 = 4.36824 \text{ sin.} \\ 4\frac{3}{4} \times 2.34132 = 11.12127 \text{ tan.} \end{array}\right\} \text{Ans.}$$

(319) See Fig. 11. Angle B = 180° − (29° 21′ + 76° 44′ 18″) = 180° − 106° 5′ 18″ = 73° 54′ 42″.

From C, draw $C\,D$ perpendicular to $A\,B$.

$A\,D = A\,C \cos A = 31.833 \times \cos 29° 21' = 31.833 \times .87164 = 27.747$ ft. $C\,D = A\,C \sin A = 31.833 \times \sin 29° 21' = 31.833 \times .49014 = 15.603$.

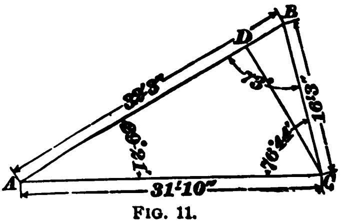

FIG. 11.

$$BC = \frac{CD}{\sin B} = \frac{15.603}{\sin 73° 54' 42''} = 16.24 \text{ feet} = 16 \text{ ft. } 3 \text{ in.}$$

$$BD = \frac{DC}{\tan B} = \frac{15.603}{\tan 73° 54' 42''} = 4.5 \text{ feet.}$$

$$AB = AD + DB = 27.747 + 4.5 = 32.247 = 32 \text{ ft. } 3 \text{ in.}$$

$$\text{Ans.} \begin{cases} AB = 32 \text{ ft. } 3 \text{ in.} \\ BC = 16 \text{ ft. } 3 \text{ in.} \\ B = 73° 54' 42''. \end{cases}$$

(320) In Fig. 8, problem 301, AB is the side of a regular decagon; then, the angle $COB = \frac{1}{20}$ of 360°, or 18°. To find the side CB, we have $CB = OB \times \sin 18°$, or $CB = 9.75 \times .30902 = 3.013$ inches. Since $AB = 2 \times CB$, $AB = 2 \times 3.013$, or 6.026 inches, which multiplied by 10, the number of sides, equals 60.26 inches. Ans.

(321) Perimeter of circle equals $2 \times 9.75 \times 3.1416$, or 61.26 inches. $61.26 - 60.26 = 1$ inch, the difference in their perimeters. Ans.

In order to find the area of the decagon, we must first find the length of the perpendicular CO (see Fig. 8 in answer to question 301); $CO = OB \times \cos 18°$, or $CO = 9.75 \times .95106 = 9.273$. Area of triangle $AOB = \frac{1}{2} \times 9.273 \times 6.026$, or 27.939, which multiplied by 10, the number of triangles in the decagon, equals 279.39 square inches. Area of the circle $= 3.1416 \times 9.75 \times 9.75$, or 298.65 square inches.

$298.65 - 279.39 = 19.26$ square inches difference. Ans.

(322) The diameter of the circle equals $\sqrt{\frac{89.42}{.7854}} = \sqrt{113.8528}$, or 10.67 inches. Ans.

The circumference equals 10.67×3.1416, or 33.52 inches. Ans.

In a regular hexagon inscribed in a circle, each side is equal to the radius of the circle; therefore, $\frac{10.67}{2} = 5.335$ inches is the length of a side. Ans.

(323) Angle $m\,O\,B = \frac{1}{16}$ of $360°$, or $22\frac{1}{2}°$. $m\,O = \frac{1}{2}$ of $m\,n = \frac{1}{2}$ of 2, or 1 inch. (See Fig. 12).

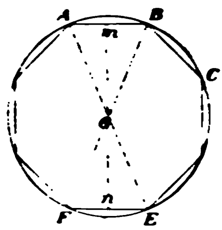

FIG. 12.

Side $m\,B = O\,m \times \tan 22\frac{1}{2}°$, or $m\,B = 1 \times .41421 = .41421$.

$A\,B = 2\,m\,B$; therefore, $A\,B = .82842$ inch.

Area of $A\,O\,B = \frac{1}{2} \times .82842 \times 1 = .41421$ square inch, which, multiplied by 8, the number of equal triangles, equals 3.31368 square inches.

Wt. of bar equals $3.31368 \times 10 \times 12 \times .282$, or 112 pounds 2 ounces. Ans.

(324) $16 \times 16 \times 16 \times \frac{1}{6} \times 3.1416 = 2{,}144.66$ cu. in. equals the volume of a sphere 16 inches in diameter.

$12 \times 12 \times 12 \times \frac{1}{6} \times 3.1416 = 904.78$ cu. in. equals the volume of a sphere 12 inches in diameter.

The difference of the two volumes equals the volume of the spherical shell, and this multiplied by the weight per cubic inch equals the weight of the shell. Hence, we have $(2{,}144.66 - 904.78) \times .261 = 323.61$ lb. Ans.

(325) The circumference of the circle equals $\frac{5\frac{13}{32} \times 360}{27}$, or 72.0833 inches. The diameter, therefore, equals $\frac{72.0833}{3.1416}$, or 22.95 inches.

(326) The number of square inches in a figure 7 inches square equals 7×7 or 49 square inches. $49 - 7 = 42$ square inches difference in the two figures.

$\sqrt{7} = 2.64$ inches is the length of side of square containing 7 square inches. The length of one side of the other square equals 7 inches.

(327) (*a*) $17\frac{1}{64}$ inches $= 17.016$ inches.

Area of circle $= 17.016^2 \times .7854 = 227.41$ sq. in. Ans.
Circumference $= 17.016 \times 3.1416 = 53.457$ inches.

$$16° \; 7' \; 21'' = 16.1225°.$$

(*b*) Length of the arc $= \dfrac{16.1225 \times 53.457}{360} = 2.394$ inches. Ans.

(328) Area $= 12 \times 8 \times .7854 = 75.4$ sq. in. Ans.
Perimeter $= (12 \times 1.82) + (8 \times 1.315) = 32.36$ in. Ans.

(329) Area of base $= \frac{1}{4} \times 3.1416 \times 7 \times 7 = 38.484$ sq. in.

The slant height of the cone equals $\sqrt{11^2 + 3\frac{1}{2}^2}$, or 11.54 in.

Circumference of base $= 7 \times 3.1416 = 21.9912$.

Convex area of cone $= 21.9912 \times \dfrac{11.54}{2} = 126.927$.

Total area $= 126.927 + 38.484 = 165.41$ square inches. Ans.

(330) Volume of sphere equals $10 \times 10 \times 10 \times \frac{1}{6} \times 3.1416 = 523.6$ cu. in.

Area of base of cone $= \frac{1}{4} \times 3.1416 \times 10 \times 10 = 78.54$ sq. in.

$\dfrac{3 \times 523.6}{78.54} = 20$ inches, the altitude of the cone. Ans.

(331) Volume of sphere $= \frac{1}{6} \times 3.1416 \times 12 \times 12 \times 12 = 904.7808$ cu. in.

Area of base of cylinder $= \frac{1}{4} \times 3.1416 \times 12 \times 12 = 113.0976$ sq. in.

Height of cylinder $= \dfrac{904.7808}{113.0976} = 8$ inches. Ans.

(332) (*a*) Area of the triangle equals $\frac{1}{2} A\,C \times B\,D$, or $\frac{1}{2} \times 9\frac{1}{2} \times 12 = 57$ square inches. Ans.

(341) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet.

Length of stroke plus the clearance $= 1.14 \times 2$ ft. (24 in. $= 2$ ft.) $= 2.28$ ft.

$1.9689 \times 2.28 = 4.489$ cubic feet, or the volume of steam in the small cylinder. Ans.

(*b*) Area of piston $= 31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance $= 1.08 \times 2 = 2.16$ ft.

$5.2414 \times 2.16 = 11.321$ cubic feet, or the volume of steam in the large cylinder. Ans.

$$(c)\quad \text{Ratio} = \frac{11.321}{4.489}, \text{ or } 2.522:1. \quad \text{Ans.}$$

(342) (*a*) Area of cross-section of pipe $= 8^2 \times .7854 = 50.2656$ sq. in.

$$\text{Volume of pipe} = \frac{50.2656 \times 7}{144} = 2.443 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) Ratio of volume of pipe to volume of small cylinder $= \dfrac{2.443}{4.489}$, or $0.544:1$. Ans.

(343) (*a*) In Fig. 15, given $OB = \frac{16}{2}$ or 8 inches, and $OA = \frac{13}{2}$ or $6\frac{1}{2}$ inches, to find the volume, area and weight:

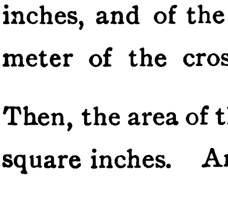

FIG. 15.

Radius of center circle equals $\frac{8 + 6.5}{2}$ or $7\frac{1}{4}$ inches. Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring = $1\frac{1}{2}$ inches.

Area of cross-section of ring $= \left(1\frac{1}{2}\right)^2 \times .7854 = 1.76715$ sq. in. Ans.

Volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

(344) The problem may be solved like the one in Art. **790.** A quicker method of solution is by means of the principle given in Art. **826.**

(345) The convex area $= 4 \times 5\frac{1}{4} \times 18 = 378$ sq. in. Ans.

Area of the bases $= 5\frac{1}{4} \times 5\frac{1}{4} \times 2 = 55.125$ sq. in.

Total area $= 378 + 55.125 = 433.125$ sq. in. Ans.

Volume $= \left(5\frac{1}{4}\right)^2 \times 18 = 496.125$ cu. in. Ans.

(346) In Fig. 16, $OC = \frac{AC}{\tan 30°}$. $\left(\frac{1}{6}\text{ of } 360° = 60°\text{, and since } AOC = \frac{1}{2}\text{ of } AOB,\ AOC = 30°\right)$.

$OC = \frac{6}{.57735} = 10.392.$

Area of $AOB = \frac{12 \times 10.392}{2} = 62.352$ square feet.

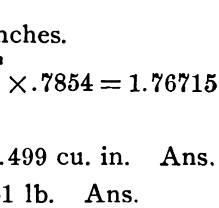

FIG. 16.

Since there are 6 equal triangles in a hexagon, then the area of the base $= 6 \times 62.352$, or 374.112 square feet.

Perimeter $= 6 \times 12$, or 72 feet.

Convex area $= \frac{72 \times 37}{2} = 1,332$ sq. ft. Ans.

Total area $= 1,332 + 374.112 = 1,706.112$ sq. ft. Ans.

(347) Area of the base = 374.112 square feet, and altitude = 37 feet. Since the volume equals the area of the base multiplied by $\frac{1}{3}$ of the altitude, we have

$$\text{Volume} = 374.112 \times \frac{37}{3} = 4,614 \text{ cubic feet.}$$

(348) Area of room $= 15 \times 18$ or 270 square feet.

One yard of carpet 27 inches wide will cover $3 \times 2\frac{1}{4}$ (27 inches $= 2\frac{1}{4}$ ft.) $= 6\frac{3}{4}$ sq. ft. To cover 270 sq. ft., it will take $\frac{270}{6\frac{3}{4}}$, or 40 yards. Ans.

(349) Area of ceiling $= 16 \times 20 =$ 320 square feet.
Area of end walls $= 2(16 \times 11) =$ 352 square feet.
Area of side walls $= 2(20 \times 11) =$ 440 square feet.
Total area $=$ 1,112 square feet.

From the above number of square feet, the following deductions are to be made:

Windows $= 4(7 \times 4) = 112$ square feet.
Doors $= 3(9 \times 4) = 108$ square feet.

Baseboard less the width of the three doors

equals $(72' - 12') \times \frac{6}{12} = 30$ square feet.

Total No. of feet to be deducted $= 250$ square feet.

Number of square feet to be plastered, then, equals 1,112 $-$ 250, or 862 square feet, or $95\frac{7}{9}$ square yards. Ans.

(350) Given $A\,B = 6\frac{7}{8}$ inches, and $O\,B = O\,A = \frac{10}{2}$ or 5 inches, Fig. 17, to find the area of the sector.

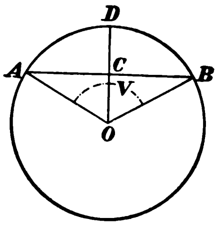

FIG 17.

Area of circle $= 10^2 \times .7854 = 78.54$ square inches.

$\text{Sin } A\,O\,C = \frac{A\,C}{O\,A} = \frac{6\frac{7}{8} \div 2}{5} = .6875$;

therefore, $A\,O\,C = 43° \; 26'$.

$A\,O\,B = 2 \times A\,O\,C = 2 \times 43° \; 26' = 86° \; 52' = 86.8666°$.

$\frac{86.8666}{360} \times 78.54 = 18.95$ square inches.

Ans.

(351) Area of parallelogram equals

$7 \times 10\frac{3}{4}$ (129 inches $= 10\frac{3}{4}$ ft.) $= 75\frac{1}{4}$ sq. ft. Ans.

(352) (*a*) See Art. **778.**

Area of the trapezoid $= \dfrac{15\frac{7}{12} + 21\frac{11}{12}}{2} \times 7\frac{2}{3} = 143.75$ sq. ft. Ans.

(*b*) In the equilateral triangle ABC, Fig. 18, the area, 143.75 square feet, is given to find a side. Since the triangle is equilateral all the angles are equal to $\frac{1}{3}$ of 180° or 60°. In the triangle $ABD = ADC$, we have $AD = AB \times \sin 60°$. The area of any triangle is equal to one-half the product of the base by the altitude, therefore, $\dfrac{BC \times AD}{2} = 143.75$. $BC = AB$ and $AD = AB \times \sin 60°$; then, the above becomes

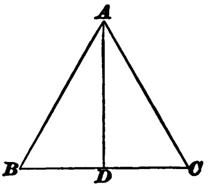

FIG. 18.

$$\frac{AB \times AB \sin 60°}{2} = 143.75,$$

$$\text{or} \quad \frac{AB^2 \times .86603}{2} = 143.75,$$

$$\text{or} \quad AB^2 = \frac{2 \times 143.75}{.86603}.$$

Therefore, $AB = \sqrt{\dfrac{287.50}{.86603}} = 18$ ft. 2.64 in. Ans.

(353) (*a*) Side of square having an equivalent area $= \sqrt{143.75} = 11.99$ feet. Ans.

(*b*) Diameter of circle having an equivalent area $= \sqrt{\dfrac{143.75}{.7854}} = \sqrt{183.0277} = 13\frac{1}{2}$ feet. Ans.

(*c*) Perimeter of square $= 4 \times 11.99 = 47.96$ ft.

Circumference or perimeter of circle $= 13\frac{1}{2} \times 3.1416 = 42.41$ ft.

Difference of perimeter $= 5.55$ ft. $=$ 5 feet 6.6 inches. Ans.

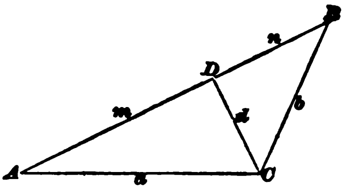

FIG. 19.

(354) In the triangle ABC, Fig. 19,

$AB = 24$ feet,
$BC = 11.25$ feet, and
$AC = 18$ feet.

$m+n : a+b :: a-b : m-n$,
or $24 : 29.25 :: 6.75 : m-n$.

$$m - n = \frac{29.25 \times 6.75}{24} = 8.226562.$$

Adding $m+n$ and $m-n$, we have

$$\begin{aligned} m + n &= 24 \\ m - n &= 8.226562 \\ \hline 2m &= 32.226562 \\ m &= 16.113281. \end{aligned}$$

Subtracting $m-n$ from $m+n$, we have

$$\begin{aligned} 2n &= 15.773438 \\ n &= 7.886719. \end{aligned}$$

In the triangle ADC, side $AC = 18$ feet, side $AD = 16.113281$; hence, according to Rule **3**, Art. **754**, $\cos A = \frac{16.113281}{18} = .89518$, or angle $A = 26° 28' 5''$. In the triangle BDC, side $BD = 7.886719$, and side $BC = 11.25$ ft. Hence, $\cos B = \frac{7.886719}{11.25} = .70104$, or angle $B = 45° 29' 23''$. Angle $C = 180° - (45° 29' 23'' + 26° 28' 5'') = 108° 2' 32''$.

$$\text{Ans.} \begin{cases} A = 26° 28' 5''. \\ B = 45° 29' 23''. \\ C = 108° 2' 32''. \end{cases}$$

ELEMENTARY MECHANICS.

(QUESTIONS 355–453.)

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(355) Use formulas **18** and **8**.

Time it would take the ball to fall to the ground $= t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 5.5}{32.16}} = .58484$ sec.

The space passed through by a body having a velocity of 500 ft. per sec. in .58484 of a second $= S = Vt = 500 \times .58484 = 292.42$ ft. Ans.

(356) Use formula **7**.

$$\frac{\frac{40}{12} \times 3.1416 \times 160}{60} = 55.85 \text{ ft. per sec. Ans.}$$

(357) $160 \div 60 \times 7 = \frac{8}{21}$ revolution in $\frac{1}{7}$ sec. $360^\circ \times \frac{8}{21} = 137\frac{1}{7}^\circ = 137^\circ\, 8'\, 34\frac{2}{7}''$. Ans.

(358) (*a*) See Fig. 20. $36'' = 3'$. $4 \div 3 = \frac{4}{3}$ = number of revolutions of pulley to one revolution of fly-wheel. $54 \times \frac{4}{3} = 72$ revolutions of pulley and drum per min. $100 \div \left(\frac{18}{12} \times 3.1416\right) = 21.22$ revolutions of drum to raise elevator 100 ft. $\frac{21.22}{72} \times 60 = 17.68$ sec. to travel 100 ft. Ans.

(*b*) $21.22 : x :: 30 : 60$, or $x = \dfrac{21.22 \times 60}{30} = 4$[illegible]

per min. of drum. The diameter of the pulley divided by

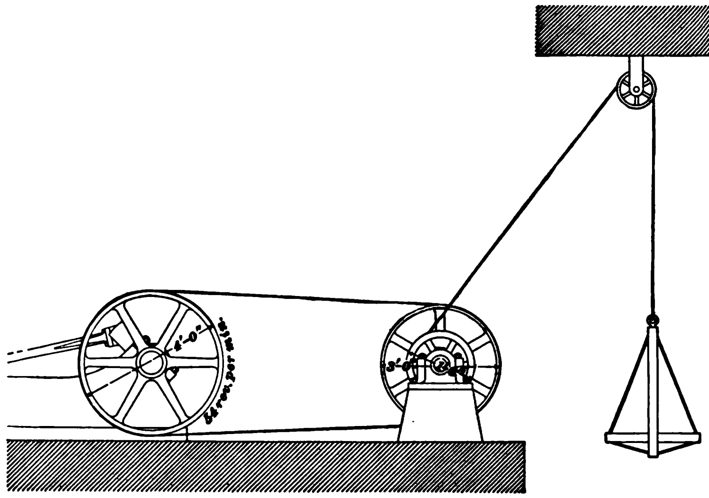

FIG. 20.

the diameter of the fly-wheel $= \frac{3}{4}$, which multiplied by $42.44 = 31.83$ revolutions per min. of fly-wheel. Ans.

(359) See Arts. **857** and **859.**

(360) See Art. **861.**

(361) See Arts. **843** and **871.**

(362) See Art. **871.**

(363) See Arts. **842, 886, 887,** etc.

The relative weight of a body is found by comparing it with a given standard by means of the balance. The absolute weight is found by noting the pull which the body will exert on a spring balance.

The absolute weight increases and decreases according to the laws of weight given in Art. **890**; the relative weight is always the same.

(364) See Art. **861.**

(365) See Art **857.**

(366) See Art. **857.**

(367) If the mountain is at the same height above, and the valley at the same depth below sea-level respectively, it will weigh more at the bottom of the valley.

(368) $\frac{31,680}{5,280} = 6$ miles. Using formula **12,** $d^2 : R^2 ::$ $W : w$, we have $w = \frac{R^2 W}{d^2} = \frac{3,960^2 \times 20,000}{3,966^2} = 19,939.53 +$ lb. $= 19,939$ lb. $8\frac{1}{2}$ oz. Ans.

(369) Using formula **11,** $R : d :: W : w$, we have $w = \frac{d W}{R} = \frac{3,958 \times 20,000}{3,960} = 19,989.89$ lb. $= 19,989$ lb. $14\frac{1}{4}$ oz. Ans.

(370) See Art. **870.**

(371) See Art. **894.**

(372) The velocity which a body may have at the instant the time begins to be reckoned.

(373) Because the man after jumping tends to continue in motion with the same velocity as the train, and the sudden stoppage by the earth causes a shock, the severity of which varies with the velocity of the train.

(374) See Arts. **870** and **871.**

(375) See Art. **872.**

(376) That force which will produce the same final effect upon a body as all the other forces acting together is called the resultant.

(377) (*a*) If a 5-in. line = 20 lb., a 1-in. line = 4 lb. $1 \div 4 = \frac{1}{4}$ in. = 1 lb. Ans. (*b*) $6\frac{1}{4} \div 4 = 1.5625$ in. = $6\frac{1}{4}$ lb. Ans.

(378) Those forces by which a given force may be

replaced, and which will produce the same effect upon a body.

(379) Southeast, in the direction of the diagonal of a square. See Fig. 21.

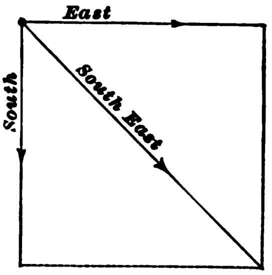

FIG. 21.

(380) $4' \, 6' = 54'$. $54 \times 2 \times \frac{3}{4} \times .261 = 21.141$ lb. = weight of lever. Center of gravity of lever is in the middle, at a, Fig. 22, 27′ from each end. Consider that the lever has no weight. The center of gravity of the two weights is at b, at a distance from c equal to $\frac{47 \times 54}{47 + 71} = 21.508' = bc$. Formula **20**, Art. **911**.

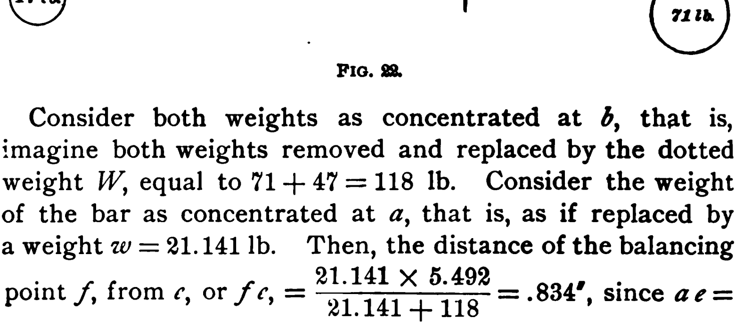

FIG. 22.

Consider both weights as concentrated at b, that is, imagine both weights removed and replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. Consider the weight of the bar as concentrated at a, that is, as if replaced by a weight $w = 21.141$ lb. Then, the distance of the balancing point f, from c, or fc, $= \frac{21.141 \times 5.492}{21.141 + 118} = .834'$, since $ae = 27 - 21.508 = 5.492''$. Finally, $fe + eh = fh = .834 + 21.508 = 22.342''$ = the short arm. Ans. $54 - 22.342 = 31.658'$ = long arm. Ans.

(381) See Fig. 23.

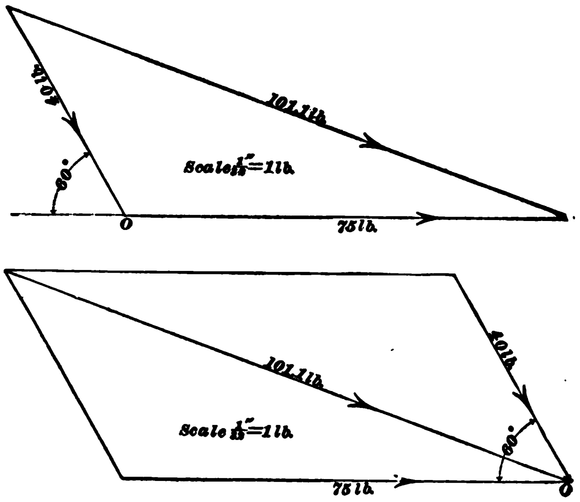

FIG. 23.

(382) See Fig. 24.

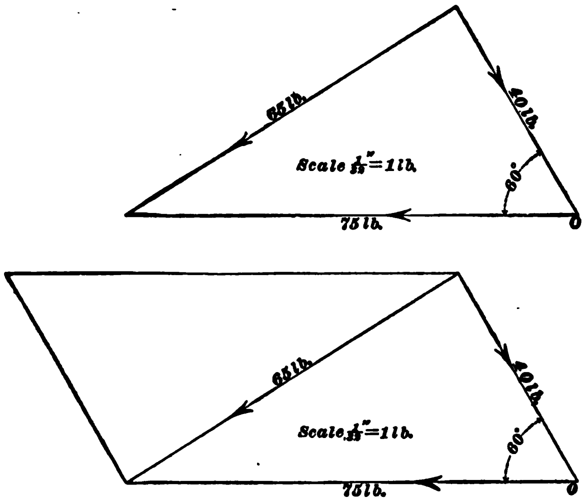

FIG. 24.

(383) $46 - 27 = 19$ lb., acting in the direction of the force of 46 lb. Ans.

(384) (*a*) $18 \times 60 \times 60 = 64{,}800$ miles per hour Ans
(*b*) $64{,}800 \times 24 = 1{,}555{,}200$ miles. Ans.

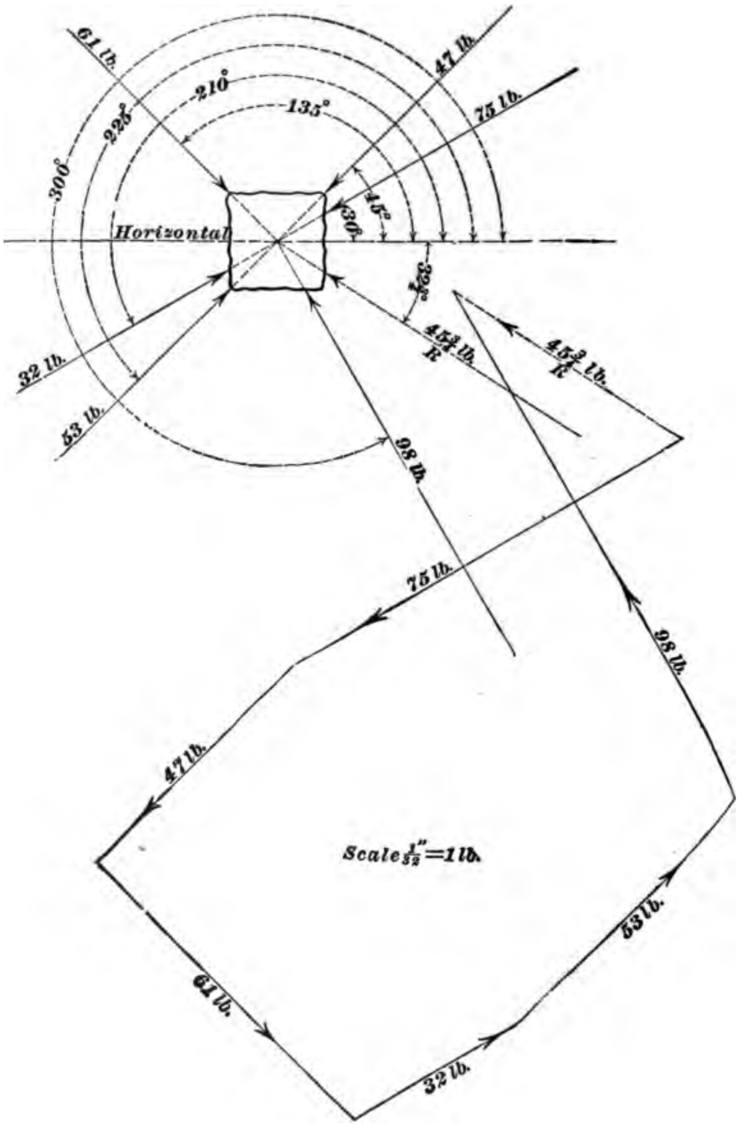

FIG. 25.

(385) (*a*) 15 miles per hour $= \frac{15 \times 5{,}280}{60 \times 60} = 22$ ft. per sec. As the other body is moving 11 ft. per sec., the distance between the two bodies in one second will be $22 + 11 = 33$ ft., and in 8 minutes the distance between them will be $33 \times 60 \times 8 = 15{,}840$ ft., which, divided by the number of feet in one mile, gives $\frac{15{,}840}{5{,}280} = 3$ miles. Ans.

(*b*) As the distance between the two bodies increases 33 ft. per sec., then, 825 divided by 33 must be the time required for the bodies to be 825 ft. apart, or $\frac{825}{33} = 25$ sec. Ans.

(386) See Fig. 25.

(387) (*a*) Although not so stated, the velocity is evidently considered with reference to a point on the shore. $10 - 4 = 6$ miles an hour. Ans.

(*b*) $10 + 4 = 14$ miles an hour. Ans.

(*c*) $10 - 4 + 3 = 9$, and $10 + 4 + 3 = 17$ miles an hour. Ans.

(388) See Fig. 26.

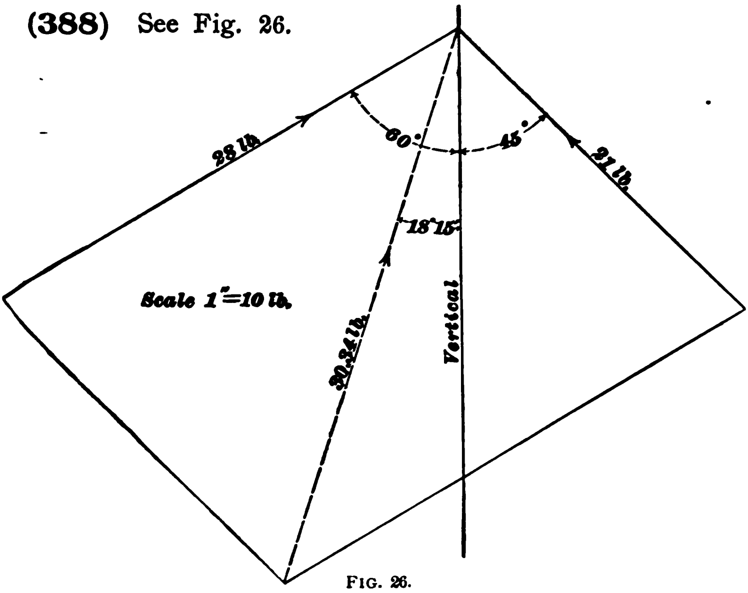

FIG. 26.

(389) See Fig. 27. By rules **2** and **4**, Art. **754,** $bc = 87 \sin 23° = 87 \times .39073 = 33.994$ lb., $ac = 87 \cos 23° = 87 \times .92050 = 80.084$ lb.

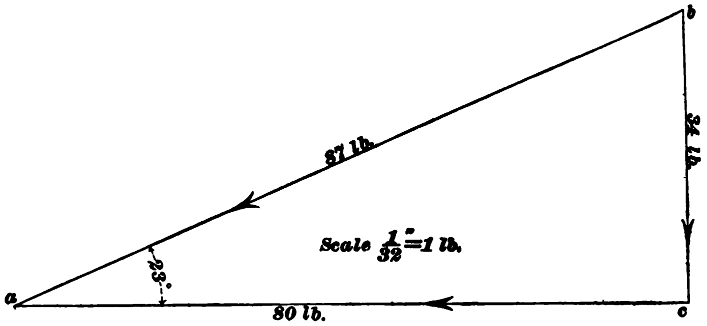

FIG. 27.

(390) See Fig. 28. (*b*) By rules **2** and **4**, Art. **754,** $bc = 325 \sin 15° = 325 \times .25882 = 84.12$ lb. Ans.

(*a*) $ac = 325 \cos 15° = 325 \times .96593 = 313.93$ lb. Ans.

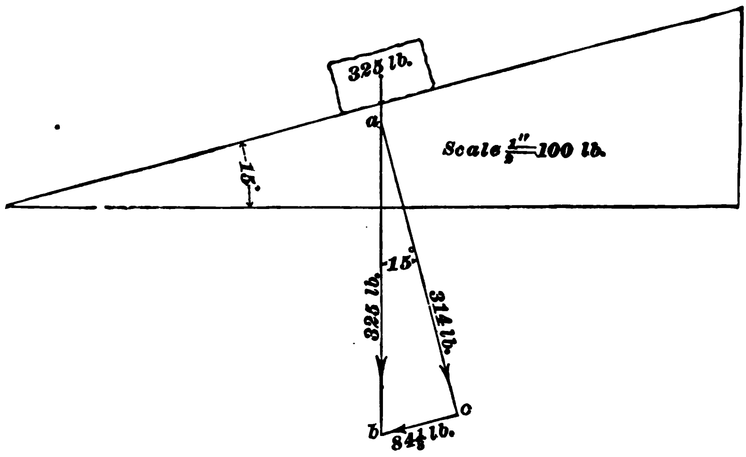

FIG. 28.

(391) Use formula **10.**

$$m = \frac{W}{g} = \frac{125}{32.16} = 3.8868. \text{ Ans.}$$

(392) Using formula **10,** $m = \frac{W}{g}$, $W = mg = 53.7 \times 32.16 = 1,727$ lb. Ans.

(393) (*a*) Yes. (*b*) 25. (*c*) 25. Ans.

(394) (*a*) Using formula **12,** $d^2 : R^2 :: W : w$, $d = \sqrt{\frac{R^2 W}{w}} = \sqrt{\frac{4,000^2 \times 141}{100}} = 4,749.736$ miles. $4,749.736 - 4,000 = 749.736$ miles. Ans.

(*b*) Using formula **11,** $R : d :: W : w$, $d = \frac{Rw}{W} = \frac{4,000 \times 100}{141} = 2,836.88$ miles. $4,000 - 2,836.88 = 1,163.12$ miles. Ans.

(395) (*a*) Use formula **18,**

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5,280}{32.16}} = 18.12 \text{ sec. Ans.}$$

(*b*) Use formula **13,** $v = gt = 32.16 \times 18.12 = 582.74$ ft. per sec., or, by formula **16,** $v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 5,280} = 582.76$ ft. per sec. Ans.

The slight difference in the two velocities is caused by not calculating the time to a sufficient number of decimal places, the actual value for t being 18.12065 sec.

(396) Use formula **25.** Kinetic energy $= Wh = \frac{Wv^2}{2g}$.

$$Wh = 160 \times 5,280 = 844,800 \text{ ft.-lb.}$$

$$\frac{Wv^2}{2g} = \frac{160 \times 582.76^2}{2 \times 32.16} = 844,799 \text{ ft.-lb. Ans.}$$

(397) (*a*) Using formulas **15** and **14,** $h = \frac{v^2}{2g} = \frac{2,360^2}{2 \times 32.16} = 86,592$ ft. $= 16.4$ miles. Ans. (*b*) $t = \frac{v}{g} =$ time required to go up or fall back. Hence, total time $= \frac{2v}{g}$ sec. $= \frac{2 \times 2,360}{60 \times 32.16} = 2.4461$ min. $= 2$ min. 26.77 sec. Ans.

(398) 1 hour = 60 min., 1 day = 24 hours; hence, 1 day = 60 × 24 = 1,440 min. Using formula **7**, $V = \frac{S}{t}$; whence, $V = \frac{8,000 \times 3.1416}{1,440} = 17.453+$ miles per min. Ans.

(399) (*a*) Use formula **25**.

Kinetic energy $= \frac{Wv^2}{2g} = \frac{400 \times 1,875 \times 1,875}{2 \times 32.16} = 21,863,339.55$ ft.-lb. Ans.

(*b*) $\frac{21,863,339.55}{2,000} = 10,931.67$ ft.-tons. Ans.

(*c*) See Art. **961**.

Striking force $\times \frac{6}{12} = 21,863,339.55$ ft.-lb.,

or striking force $= \frac{21,863,339.55}{\frac{6}{12}} = 43,726,679$ lb. Ans.

(400) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 200}{32.16}} =$ 3.52673 sec., when $g = 32.16$.

$t = \sqrt{\frac{2 \times 200}{20}} = 4.47214$ sec., when $g = 20$.

4.47214 − 3.52673 = 0.94541 sec. Ans.

(401) See Art. **910**.

(402) See Art. **963**.

(403) (*a*) See Art. **962**.

$D = \frac{m}{V} = \frac{W}{gv}$. $v = \frac{800}{1,728}$. Hence, $D = \frac{W}{gv} = \frac{500}{32.16 \times \frac{800}{1728}} =$ 33.582. Ans. (*b*) In Art. **962**, the density of water was found to be 1.941. (*c*) In Art. **963**, it is stated that the specific gravity of a body is the ratio of its density to the density of water. Hence, $\frac{33.582}{1.941} = 17.3 =$ specific gravity. If the weight of water be taken as 62.5 lb. per cu. ft., the specific gravity will be found to be 17.28. Ans.

(404) Assuming that it started from a state of rest, formula **13** gives $v = gt = 32.16 \times 5 = 160.8$ ft. per sec.

(405) Use formulas **17** and **13**. $h = \frac{1}{2} g t^2 = \frac{32.16}{2} \times 3^2 = 144.72$ ft., distance fallen at the end of third second.

$v = g t = 32.16 \times 3 = 96.48$ ft. per sec., velocity at end of third second.

$96.48 \times 6 = 578.88$ ft., distance fallen during the remaining 6 seconds.

$144.72 + 578.88 = 723.6$ ft. = total distance. Ans.

(406) See Art. **961.**

Striking force $\times \frac{\frac{1}{2}}{12} = 8 \times 8 = 64$. Therefore, striking force $= \frac{64}{\frac{\frac{1}{2}}{12}} = 1{,}536$ tons. Ans.

(407) See Arts. **901** and **902.**

(408) Use formula **19.**

Centrifugal force = tension of string $= .00034\ W R N^2 = .00034 \times (.5236 \times 4^3 \times .261) \times \frac{15}{12} \times 60^2 = 13.38+$ lb. Ans.

(409) $(80^2 - 70^2) \times .7854 \times 26 \times .261 \div 2 =$ weight of $\frac{1}{2}$ of rim.

$$R = \frac{80 - 10}{2 \times 12} = \frac{35}{12} \text{ ft.}$$

According to Art. **904,** $F = .00034\ W R N^2 \div 3.1416 = .00034 \times \frac{(80^2 - 70^2) \times .7854 \times 26 \times .261}{2} \times \frac{35}{12} \times 175^2 \div 3.1416 = 38{,}641$ lb. Ans.

(410) (*a*) Use formulas **11** and **12.** $R : d :: W : w$, or $W = \frac{w R}{d} = \frac{1 \times 4{,}000}{100} = 40$ lb. Ans.

(*b*) $d^2 : R^2 :: W : w$, or $w = \frac{4{,}000^2 \times 40}{4{,}100^2} = 38.072$ lb. Ans.

(411) See Art. **955.**

$$\frac{10{,}746 \times 354}{10 \times 33{,}000} = 11.5275 \text{ H. P. Ans.}$$

(412) Use formula **12.**

$d^2 : R^2 :: W : w$, or $d = \sqrt{\dfrac{4,000^2 \times 2}{\frac{3}{16}}} = 13,064$ mi., nearly.

$13,064 - 4,000 = 9,064$ miles. Ans.

(413) Use formula **18.** $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 50}{32.16}} =$ 1.7634 sec., nearly.

$1.7634 \times 140 = 246.876$ ft. Ans.

(414) $\dfrac{10}{30} = \dfrac{1}{3}$ sec. Use formula **17.**

$h = \dfrac{1}{2} g t^2 = \dfrac{1}{2} \times 32.16 \times \left(\dfrac{1}{3}\right)^2 = 1.78\frac{2}{3}$ ft. $= 1$ ft. 9.44 in. Ans.

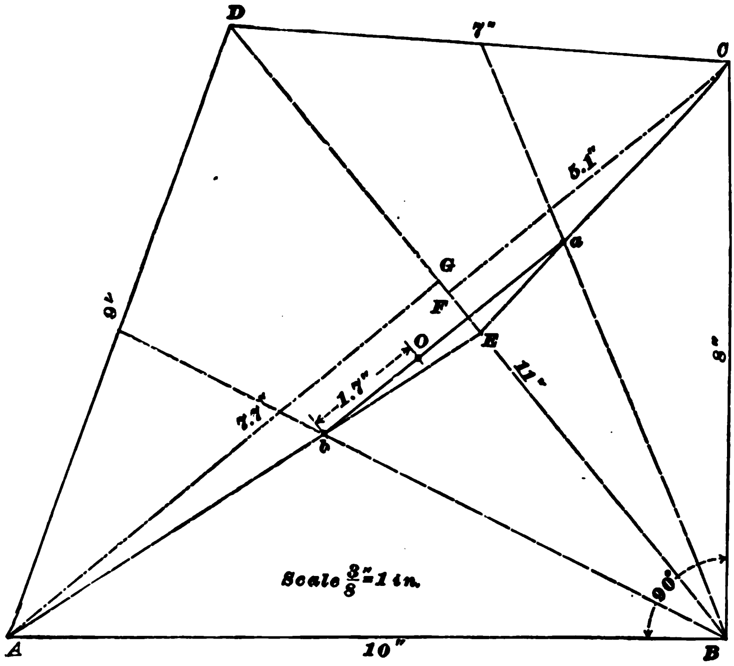

FIG. 29.

(415) See Arts. **906, 907.**

(416) See Arts. **908, 909.**

(417) No. It can only be counteracted by another equal couple which tends to revolve the body in an opposite direction.

(418) See Art. **914.**

(419) Draw the quadrilateral as shown in Fig. 29. Divide it into two triangles by the diagonal $B\ D$. The center of gravity of the triangle $B\ C\ D$ is found to be at a, and the center of gravity of the triangle $A\ B\ D$ is found to be at b (Art. **914**). Join a and b by the line $a\ b$, which, on being measured, is found to have a length of 4.27 inches. From C and A drop the perpendiculars $C\ F$ and $A\ G$ on the diagonal $B\ D$. Then, area of the triangle $A\ B\ D = \frac{1}{2}\ (A\ G \times B\ D)$, and area of the triangle $B\ C\ D = \frac{1}{2}\ (C\ F \times B\ D)$. Measuring these distances, $B\ D = 11''$, $C\ F = 5.1''$, and $A\ G = 7.7''$.

$$\text{Area } A\,B\,D = \frac{1}{2} \times 7.7 \times 11 = 42.35 \text{ sq. in.}$$

$$\text{Area } B\,C\,D = \frac{1}{2} \times 5.1 \times 11 = 28.05 \text{ sq. in.}$$

According to formula **20,** the distance of O, the center of gravity, from b is $\frac{28.05 \times 4.27}{28.05 + 42.35} = 1.7$. Therefore, the center of gravity is on the line $a\,b$ at a distance of $1.7''$ from b.

(420) See Fig. 30. The center of gravity lies at the geometrical center of the pentagon, which may be found as follows: From any vertex draw a line to the middle point of the opposite side. Repeat the operation for any other vertex, and the intersection of the two lines will be the desired center of gravity.

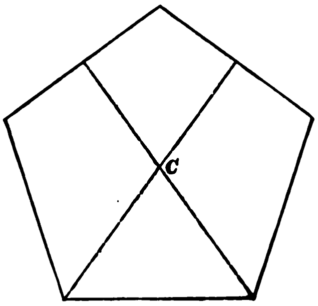

FIG. 30.

(421) See Fig. 31. Since any number of quadrilaterals can be drawn with the sides given, any number of answers can be obtained.

Draw a quadrilateral, the lengths of whose sides are equal to the distances between the weights, and locate a weight on each corner. Apply formula **20** to find the distance $C_1 W_1$; thus, $C_1 W_1 = \frac{9 \times 18}{9 + 21} = 5.4''$. Measure the distance $C_1 W_2$;

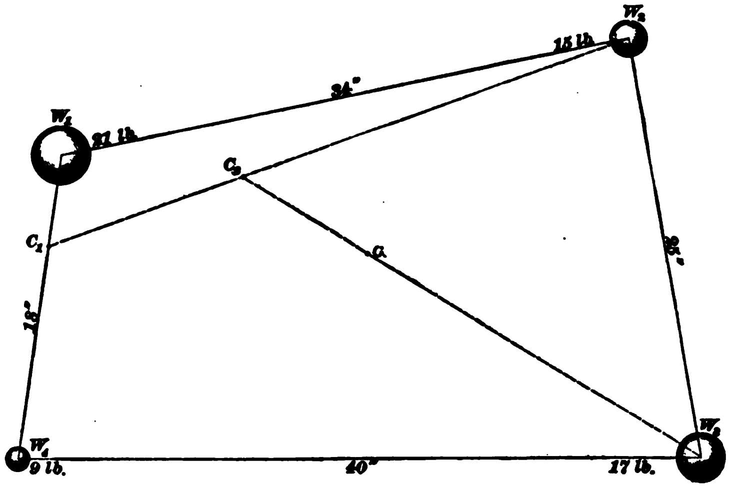

FIG. 31.

suppose it equals say 36''. Apply the formula again. $C_1 C_2 = \frac{15 \times 36}{15 + (9 + 21)} = 12''$. Measure $C_2 W_3$; it equals say 31.7''.

Apply the formula again. $C_2 C = \frac{17 \times 31.7}{17 + 15 + 9 + 21} = 8.7''$. C is center of gravity of the combination.

(422) Let $A\ B\ C\ D\ E$, Fig. 32, be the outline, the right-angled triangle cut-off being $E\ S\ D$. Divide the figure into two parts by the line $m\ n$, which is so drawn that it cuts off an isosceles right-angled triangle $m\ B\ n$, equal in area to $E\ S\ D$, from the opposite corner of the square.

The center of gravity of $AmnCDE$ is then at C_1, its geometrical center. $Bm = 4$ in.; angle $Bmr = 45°$; there-

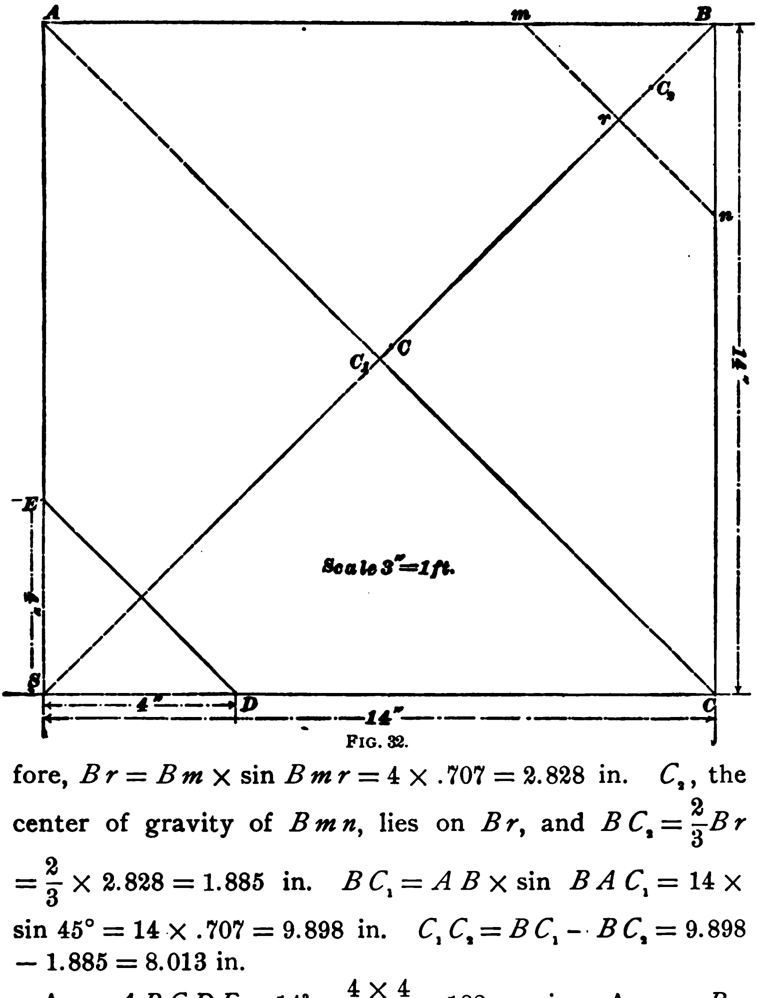

FIG. 32.

fore, $Br = Bm \times \sin Bmr = 4 \times .707 = 2.828$ in. C_2, the center of gravity of Bmn, lies on Br, and $BC_2 = \frac{2}{3}Br$ $= \frac{2}{3} \times 2.828 = 1.885$ in. $BC_1 = AB \times \sin BAC_1 = 14 \times \sin 45° = 14 \times .707 = 9.898$ in. $C_1C_2 = BC_1 - BC_2 = 9.898 - 1.885 = 8.013$ in.

Area $ABCDE = 14^2 - \frac{4 \times 4}{2} = 188$ sq. in. Area $mBn = \frac{4 \times 4}{2} = 8$ sq. in. Area $AmnCDE = 188 - 8 = 180$ sq. in.

The center of gravity of the combined area lies at C, at

a distance from C_1, according to formula **20** (Art. **911**), equal to $\frac{8 \times C_1 C_2}{180 + 8} = \frac{8 \times 8.013}{188} = .341$ in. $C_1 C = .341$ in. $B C = B C_1 - C_1 C = 9.898 - .341 = 9.557$ inches. Ans.

(423) (*b*) In one revolution the power will have moved through a distance of $2 \times 15 \times 3.1416 = 94.248'$, and the weight will have been lifted $\frac{1''}{4}$. The velocity ratio is then $94.248 \div \frac{1}{4} = 376.992$.

$376.992 \times 25 = 9{,}424.8$ lb. Ans.

(*a*) $9{,}424.8 - 5{,}000 = 4{,}424.8$ lb. Ans.

(*c*) $4{,}424.8 \div 9{,}424.8 = 46.95\%$. Ans.

(424) See Arts. **920** and **922.**

(425) Construct the prism $A\,B\,E\,D$, Fig. 33. From E, draw the line $E\,F$. Find the center of gravity of the

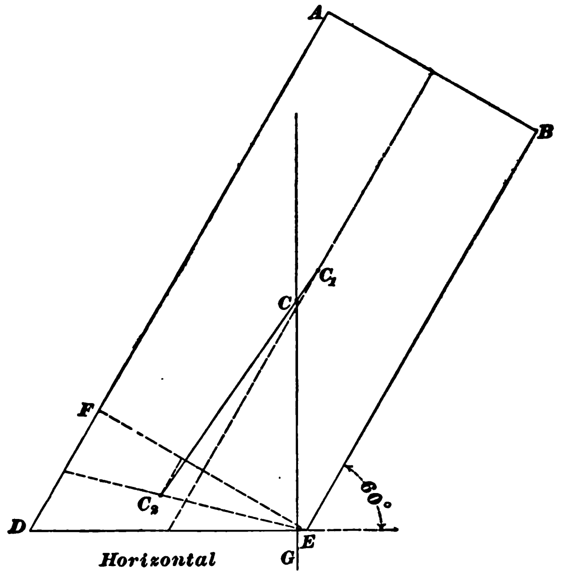

FIG. 33.

rectangle, which is at C_1, and that of the triangle, which is

at C_2. Connect these centers of gravity by the straight line $C_1 C_2$ and find the common center of gravity of the body by the rule to be at C. Having found this center, draw the line of direction $C G$. If this line falls within the base, the body will stand, and if it falls without, it will fall.

(426) (*a*) 5 ft. 6 in. = 66″. $66 \div 6 = 11$ = velocity ratio. Ans.

(*b*) $5 \times 11 = 55$ lb. Ans.

(427) $55 \times .65 = 35.75$ lb. Ans.

(428) Apply formula **20.** 5 ft. = 60″. $\frac{35 \times 60}{180 + 35} =$ 9.7674 in., nearly, = distance from the large weight. Ans.

(429) (*a*) $1{,}000 \div 50 = 20$, velocity ratio. Ans. See Art. **945.** (*b*) 10 fixed and 10 movable. Ans. (*c*) $50 \div 95 = 52.63\%$. Ans.

(430) $P \times \text{circumference} = W \times \frac{1}{8}$, or $60 \times 40 \times 3.1416 = W \times \frac{1}{8}$, or $W = 60 \times 40 \times 3.1416 \times 8 = 60{,}318.72$ lb. Since the efficiency of combination is 40%, the tension on the stud would be $.40 \times 60{,}318.72 = 24{,}127.488$ lb. Ans.

(431) (*a*) $\sqrt{20^2 + 5^2} = 20.616$ ft. = length of inclined plane.

$P \times$ length of plane $= W \times$ height, or $P \times 20.616 = 1{,}580 \times 5$.

$P = \frac{1{,}580 \times 5}{20.616} = 383.2$ lb. Ans. (*b*) In the second case, $P \times$ length of base $= W \times$ height, or $P \times 20 = 1{,}580 \times 5$; hence, $P = \frac{1{,}580 \times 5}{20} = 395$ lb. Ans.

(432) $W \times 2 = 42 \times 6$, or $W = \frac{42 \times 6}{2} = 126$ lb.

$126 + 42 = 168$ lb. $168 \times 1 = W' \times 12$, or $W' = \frac{168}{12} = 14$ lb. Ans.

(433) See Fig. 34. $P \times 14 \times 21 \times 19 = 2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725$, or

$$P = \frac{2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725}{14 \times 21 \times 19} = 3.032 \text{ lb.} \quad \text{Ans.}$$

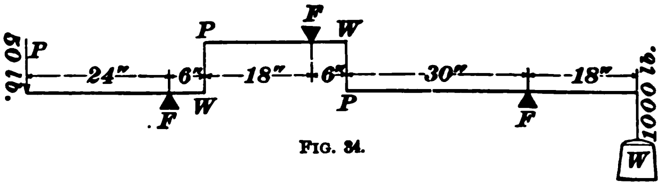

FIG. 34.

(434) See Fig. 35. (*a*) $35 \times 15 \times 12 \times 20 = 5 \times 3\frac{1}{2} \times 3 \times W$, or

$$W = \frac{35 \times 15 \times 12 \times 20}{5 \times 3\frac{1}{2} \times 3} = 2{,}400 \text{ lb.} \quad \text{Ans.}$$

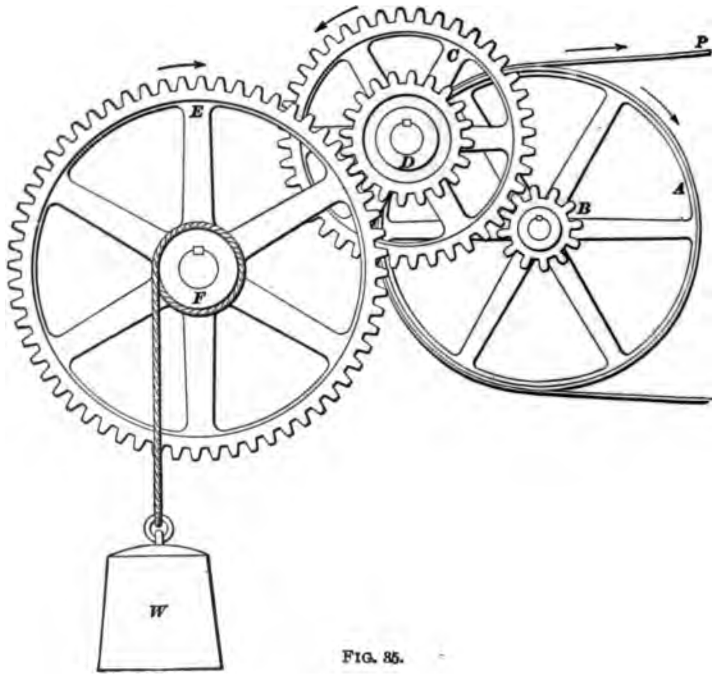

FIG. 35.

(*b*) $2,400 \div 35 = 68\frac{4}{7}$ = velocity ratio. Ans.

(*c*) $1,932 \div 2,400 = .805 = 80.5\%$. Ans.

(435) In Fig. 36, let the 12-lb. weight be placed at A, the 18-lb. weight at B, and the 15-lb. weight at D.

Use formula **20.**

$\frac{12 \times 15}{18 + 12} = 6'' =$ distance C_1B = distance of center of

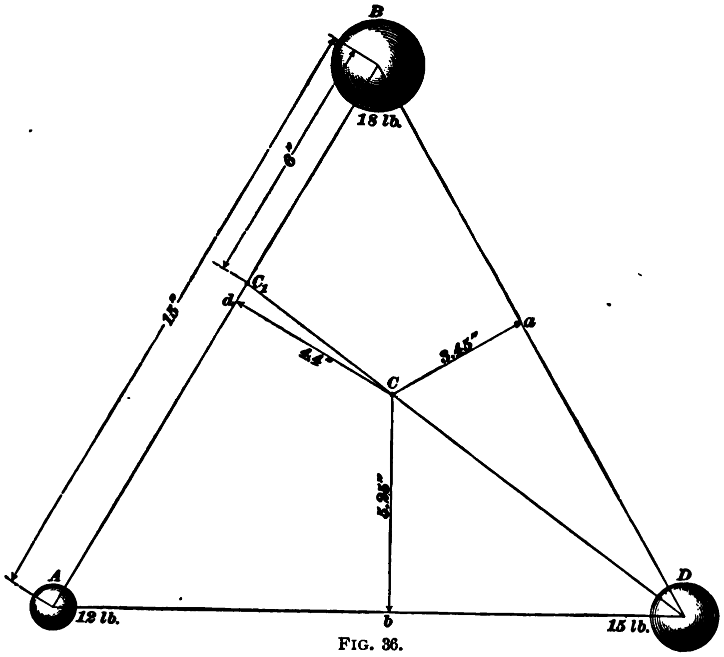

FIG. 36.

gravity of the 12 and 18-lb. weights from B. Drawing C_1D, $C_1C = \frac{15 \times C_1D}{(12 + 18) + 15} = \frac{1}{3}C_1D$. Measuring the distances of C from BD, DA, and AB, it is found that $Ca = 3.45''$, $Cb = 5.25''$, and $Cd = 4.4''$. Ans.

(436) (*a*) Potential energy equals the work which the body would do in falling to the ground $= 500 \times 75 =$ 37,500 ft.-lb. Ans.

(*b*) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 75}{32.16}} = 2.1597$ sec. $= .035995$ min., the time of falling.

$$\frac{37,500}{33,000 \times .035995} = 31.57 \text{ H. P. Ans.}$$

(437) $127 \div 62.5 = 2.032 =$ specific gravity. Ans.

(438) $\frac{62.5}{1,728} \times 9.823 = .35529$ lb. Ans.

(439) Use formula **21**. $W = \left(\frac{2PR}{R-r}\right)$, or $\frac{2 \times 60 \times 6.5}{6.5 - 5.75} \times .48 = 499.2$ lb. Ans. See Fig. 37.

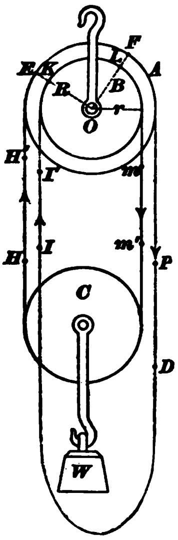

FIG. 37.

(440) See Art. **961**.

$$F \times \left(\frac{3}{8} \div 12\right) = \frac{Wv^2}{2g} = \frac{1.5 \times 25^2}{2 \times 32.16}, \text{ or}$$

$$F = \frac{\frac{1.5 \times 25^2}{2 \times 32.16}}{\frac{3}{8} \div 12} = 466.42 \text{ lb. Ans.}$$

(441) (*a*) $2,000 \div 4 = 500 =$ wt. of cu. ft. $500 \div 62.5 = 8 =$ specific gravity. Ans.

(*b*) $\frac{500}{1,728} = .28935$ lb. Ans.

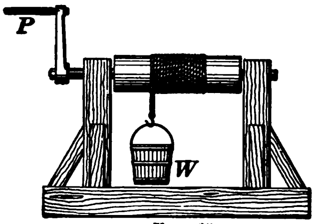

FIG. 38.

(442) See Fig. 38. $14.5 \times 2 = 29$. $30 \times 29 = W \times 5$, or $W = \frac{30 \times 29}{5} = 174$ lb. Ans.

(443) $75 \times .21 = 15.75$ lb. Ans.

(444) (*a*) $900 \times 150 = 135,000$ ft.-lb. Ans.

$$\frac{135,000}{15} = 9,000 \text{ ft.-lb. per min. Ans.}$$

(*b*) $\frac{9,000}{33,000} = \frac{3}{11}$ H.P. Ans.

(445) $900 \times .18 \times 2 = 324$ lb. = force required to overcome the friction. $900 + 324 = 1,224$ lb. = total force.

$$\frac{1,224 \times 150}{15 \times 33,000} = .37091 \text{ H. P.} \quad \text{Ans.}$$

(446) $18 \div 88 = .2045$. Ans.

(447) See Art. **962.** $D = \frac{W}{gV} = \frac{1,200}{32.16 \times 3} = 12.438$. Ans.

(448) See Fig. 39. $125 - 47.5 = 77.5$ lb. = downward pressure.

$77.5 \div 4 = 19.375$ lb. = pressure on each support. Ans.

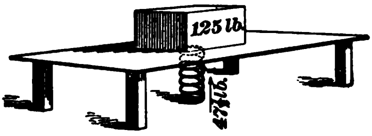

FIG. 39.

(449) See Fig. 40.

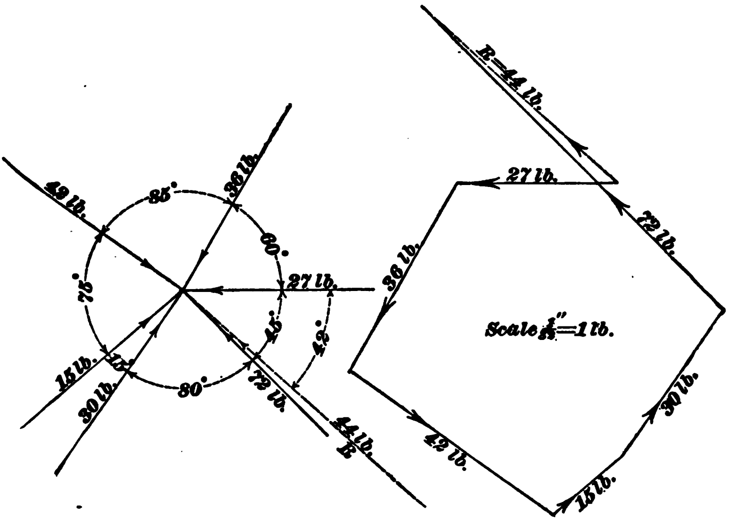

FIG. 40.

(450) See Fig. 41. $4.5 \div 2 = 2.25$.

$$\frac{12}{2.25} \times 6 \times 30 = 960 \text{ lb.} \quad \text{Ans.}$$

(451) (*a*) $960 \div 30 = 32$. Ans.

(*b*) $790 \div 960 = .8229 = 82.29\%$. Ans.

(452) (*a*) See Fig. 42. $475 + (475 \times .24) = 589$ lb. Ans.

(*b*) $475 \div 589 = .8064 = 80.64\%$. Ans.

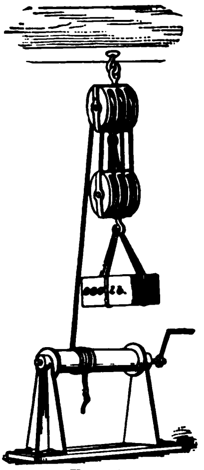

FIG. 41.

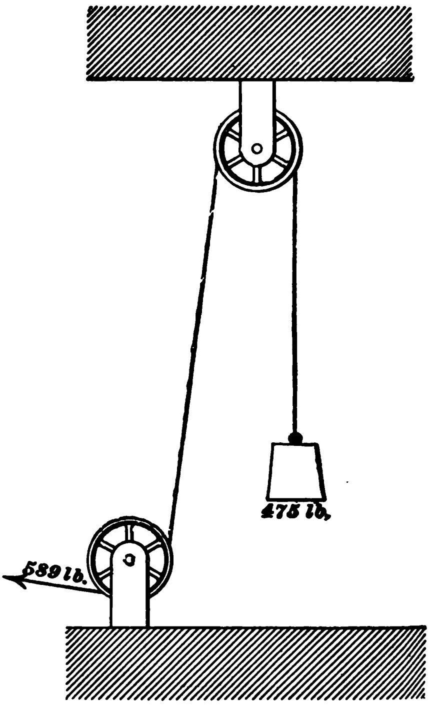

FIG. 42.

(453) (*a*) By formula **23,** $U = FS = 6 \times 25 = 150$ foot-pounds. Ans.

$$(b)\ 2\frac{1}{2} \text{ sec.} = \frac{2\frac{1}{2}}{60} = \frac{1}{24} \text{ min.}$$

Using formula **24,** Power $= \frac{FS}{T} = \frac{150}{\frac{1}{24}} = 3{,}600$ ft.-lb. per min. Ans.

HYDROMECHANICS.

(QUESTIONS 454–503.)

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(454) The area of the surface of the sphere is $20 \times 20 \times 3.1416 = 1,256.64$ sq. in. (See rule, Art. **817.**)

The specific gravity of sea water is 1.026. (See tables of Specific Gravity.) The pressure on the sphere per square inch is the weight of a column of water 1 sq. in. in cross-section and 2 miles long. The total pressure is, therefore, $1,256.64 \times 5,280 \times 2 \times .434 \times 1.026 = 5,908,971$ lb. Ans.

(455) $125 - 83.5 = 41.5$ lb. = loss of weight in water = weight of a volume of water equal to the volume of the sphere. (See Art. **987.**) 1 cu. in. of water weighs .03617 lb.; hence, 41.5 lb. of water must contain $41.5 \div .03617 = 1,147.4$ cu. in. = volume of the sphere. Ans.

(456) $$Q = \frac{225,000}{60 \times 60} = 62.5 \text{ gal. per sec.}$$

Substituting the values given in formula **51,**

$$d = 1.229 \sqrt[5]{\frac{2,800 \times 62.5^2}{26}} = 16.38'' +.$$

Substituting this value of d in formula **49,**

$$v_m = \frac{24.51 \times 62.5}{16.38^2} = 5.7095.$$

The value of f (from the table) corresponding to $v_m = 5.7095$ is .0216, using but four decimal places. Hence, applying formula **52,**

$$d = 2.57 \sqrt[5]{\frac{(.0216 \times 2,800 + \frac{1}{8} \times 16.38)62.5^2}{26}} = 16''. \text{ Ans.}$$

(479) $\frac{5^2 \times .7854}{144}$ = area of pipe in square feet. Using formula **31**,

$$Q = A\, v_m = \frac{5^2 \times .7854}{144} \times 7.2 = \text{discharge in cu. ft. per sec.}$$

$$\frac{5^2 \times .7854}{144} \times 7.2 \times 7.48 = \text{discharge in gal. per sec.}$$

$\frac{5^2 \times .7854}{144} \times 7.2 \times 7.48 \times 60 \times 60 \times 24 = 634{,}478$ gal. discharged in one day. Ans.

(480) 38,000 gallons per hour $\frac{38{,}000}{60 \times 60}$ gal. per sec. $= Q$.

Using formula **49**,

$$v_m = \frac{24.51\, Q}{d^2} = \frac{24.51 \times 38{,}000}{5.5^2 \times 60 \times 60} = 8.5526 \text{ ft. per sec. Ans.}$$

(481) Area of $2\frac{1}{2}$-in. circle = 4.9087 sq. in.; area of a 2-in. circle = 3.1416 sq. in. $(4.9087 - 3.1416) \times 12 = 21.2052$ cu. in. of brass.

$21.2052 \times .03617 = .767$ lb. = weight of an equal volume of water.

6 lb. 5 oz. = 6.3125 lb. $6.3125 \div .767 = 8.23$ Sp. Gr. of brass. Ans.

(482) (*b*) A column of water 1 ft. high and 1 in. square weighs .434 lb. $.434 \times 180 = 78.12$ lb. per sq. in. Ans.

(*a*) Projected area of 1 foot of pipe $= 6 \times 12 = 72$ sq. in. (See Art. **985.**) $72 \times 78.12 = 5{,}624.64$ lb., nearly. Ans.

(483) Use formula **42.**

(*a*) $Q_a = .41\, b \sqrt{2 g d^3} = .41 \times \frac{27}{12} \times \sqrt{2 \times 32.16 \times \left(\frac{36}{12}\right)^3} =$ 38.44 cu. ft. per sec. Ans.

(*b*) $Q = \frac{Q_a}{.615} = \frac{38.44}{.615} = 62.5$ cu. ft. per sec. Ans.

(484) (*a*) Area of pipe : area of orifice :: $6^2 : 1.5^2$; or, area of pipe is 16 times as large as area of orifice. Hence, using formula **35**,

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 45}{1 - \frac{(1.5^2 \times .7854)^2}{(6^2 \times .7854)^2}}} = 53.9 \text{ ft. per sec. Ans.}$$

(*b*) $2.304 \times 10 = 23.04$ ft. = height of column of water which will give a pressure of 10 lb. per sq. in. $45 + 23.04 = 68.04$ ft.

$$v = \sqrt{\frac{2 \times 32.16 \times 68.04}{1 - \frac{(1.5^2 \times .7854)^2}{(6^2 \times .7854)^2}}} = 66.28 \text{ ft. per sec. Ans.}$$

(485) Use formula **48.**

$Q = .0408\, d^2 v_m = .0408 \times 6^2 \times 7.5 = 11.016$ gal. per sec. Ans.

(486) $14^2 \times .7854 \times 27$ = volume of cylinder = volume of water displaced.

$14^2 \times .7854 \times 27 \times .03617 = 150$ lb., nearly, = weight of water displaced.

$(14^2 - 13^2) \times .7854 \times 27$ = volume of the cylinder walls.

$13^2 \times .7854 \times \frac{1}{4} \times 2$ = volume of the cylinder ends.

.261 lb. = weight of a cubic inch of cast iron, then,

$[(14^2 - 13^2) \times .7854 \times 27 + 13^2 \times .7854 \times \frac{1}{4} \times 2] \times .261 =$ 167 lb., nearly, = weight of cylinder. Since weight of cylinder is greater than the weight of the water displaced, it will sink. Ans.

(487) 2 lb. − 1 lb. 5 oz. = 11 oz., weight of water.

1 lb. 15.34 oz. − 1 lb. 5 oz. = 10.34 oz., weight of oil.

$10.34 \div 11 = .94$ = Sp. Gr. of oil. Ans.

(488) Head $= 41 \div .434 = 94.47$ ft. Using formula **36,** $v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 94.47} = 76.39$ ft. per sec. Ans. This is not the mean velocity, v_m.

(489) (*a*) Use formula **39.**

$$Q_a = .815\, A \sqrt{2gh}, \text{ or}$$

$$Q_a = .815 \times \frac{1.5^2 \times .7854}{144} \times \sqrt{2 \times 32.16 \times 94.47} \times 60 =$$

46.77 cu. ft. per min. Ans.

(*b*) See Art. **1005.**

The theoretical velocity of discharge is $v = \sqrt{2\,g\,h}$, and as $h = 41 \div .434 = 94.47$ ft., we have $v = \sqrt{2 \times 32.16 \times 94.47}$ $= 77.95$ ft. per sec.

Using formula **31,** $Q = A\,v$, and multiplying by 60 to reduce the discharge from cu. ft. per sec. to cu. ft. per min., we have

$$Q = \frac{1.5^2 \times .7854}{144} \times 77.95 \times 60 = 57.39 \text{ cu. ft. per min.} \quad \text{Ans.}$$

(*c*) $$\frac{Q_a}{Q} = \frac{46.77}{57.39} = .815. \quad \text{Ans.}$$

(**490**) (*b*) Use formulas **31** and **38.**

$$Q = A\,v = \frac{1.5^2 \times .7854}{144} \times 77.95 = .9566 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

(*a*) $Q_a = .615\,Q = .615 \times .9566 = .5883$ cu. ft. per sec. Ans.

(*c*) $$\frac{Q_a}{Q} = \frac{.5883}{.9566} = .615. \quad \text{Ans.}$$

(**491**) (*a*) $9 \times 5 \times .7854 = 35.343$ sq. in. = area of base.

$2^2 \times .7854 = 3.1416$ sq. in. = area of hole.

$\frac{3.1416}{35.343} = \frac{1}{11.25}$; hence, the area of the base is less than 20 times the area of the orifice, and formula **35** must be used.

$$v = \sqrt{\frac{2\,g\,h}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 6}{1 - \frac{(2^2 \times .7854)^2}{(9 \times 5 \times .7854)^2}}} = 19.722 \text{ ft. per sec.} \quad \text{Ans.}$$

(*b*) $.434 \times 6 = 2.604$, or say 2.6 lb. per sq. in. Ans.

(**492**) $6 \times 4 \times .7854 = 18.85$ sq. in. = area of upper surface.

$15^2 \times .7854 = 176.715$ sq. in. = area of base.

$\frac{132}{18.85} \times 176.715 = 1{,}237.5$ lb., pressure due to weight on upper surface.

$.03617 \times 24 \times 176.715 = 153.4$ lb., pressure due to water in vessel.

$1,237.5 + 153.4 = 1,390.9$ lb., total pressure. Ans.

(**493**) Use formula **31** or **32**.

Divide by 60×60 to get the discharge in gallons per second, and by 7.48 to get the discharge in cubic feet per second.

$$\text{Area in sq. ft.} = \frac{4^2 \times .7854}{144}.$$

$$v_m = \frac{Q}{A} = \frac{12,000 \times 144}{60 \times 60 \times 7.48 \times 4^2 \times .7854} = 5.106 \text{ ft. per sec.}$$

Ans.

(**494**) A sketch of the arrangement is shown in Fig. 45.

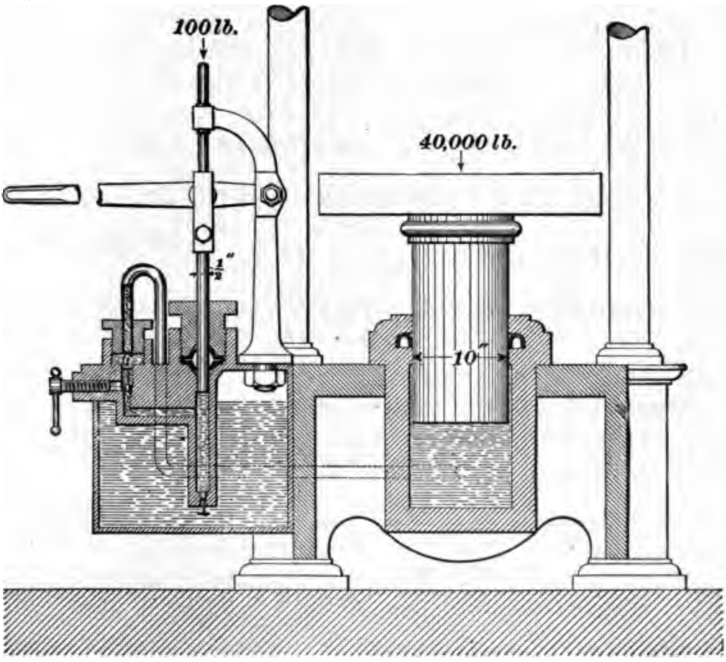

FIG. 45.

(*a*) Area of pump piston $= \left(\frac{1}{2}\right)^2 \times .7854 = .19635$ sq. in.

Area of plunger $= 10^2 \times .7854 = 78.54$ sq. in.

Pressure per square inch exerted by piston $= \frac{100}{.19635}$ lb.

Hence, according to Pascal's law, the pressure on the plunger is $\frac{100}{.19635} \times 78.54 = 40,000$ lb. Ans.

(*b*) Velocity ratio $= 1\frac{1}{2} : .00375 = 400 : 1$. Ans.

(*c*) According to the principle given in Art. **981**, $P \times 1\frac{1}{2}$ inches $= W \times$ distance moved by plunger, or $100 \times 1\frac{1}{2} = 40,000 \times$ required distance; hence, the required distance $= \frac{100 \times 1\frac{1}{2}}{40,000} = .00375$ in. Ans.

(495) (*a*) Use formula **44**, and multiply by 7.48 and 60 to reduce the discharge from cu. ft. per sec. to gal. per min.

$$Q_a = .41\, b \sqrt{2g}\,[\sqrt{h^3} - \sqrt{h_1^3}] \times 60 \times 7.48 =$$

$$.41 \times \frac{14}{12} \times \sqrt{64.32}\left[\sqrt{\left(9+\frac{20}{12}\right)^3} - \sqrt{9^3}\right] \times 60 \times 7.48 = 13,502 \text{ gal. Ans.}$$

(*b*) In the second case,

$$Q_a = .41 \times \frac{20}{12} \times \sqrt{64.32}\left[\sqrt{\left(9+\frac{14}{12}\right)^3} - \sqrt{9^3}\right] \times 60 \times 7.48 = 13,323 \text{ gal. Ans.}$$

(496) (*a*) Area of weir $= 14 \times 20 \div 144$ sq. ft. Use formula **32**, and divide by 60×7.48 to reduce gal. per min. to cu. ft. per sec.

$$v_m = \frac{Q}{A} = \frac{13,502 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.47 \text{ ft. per sec. Ans.}$$

(*b*) $v_m = \frac{13,323 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.27$ ft. per sec. Ans.

(497) (*a*) See Art. **997**.

$W = 2$ lb. $8\frac{1}{2}$ oz. $= 40.5$ oz.
$w =$ 12 oz.
$W' = 1$ lb. 11 oz. $= 27$ oz.

By formula **30**,

$$\text{Sp. Gr.} = \frac{W - w}{W' - w} = \frac{40.5 - 12}{27 - 12} = \frac{28.5}{15} = 1.9. \quad \text{Ans.}$$

(*b*) 15 oz. $= \frac{15}{16}$ lb. $= .9375$ lb. $.9375 \div .03617 = 25.92$ cu. in. = volume of water = volume of slate. Therefore, the volume of the slate = 25.92 cu. in. Ans.

(498) In Art. **1019** it is stated that the theoretical mean velocity is $\frac{2}{3}\sqrt{2gh}$. Hence, $v_m = \frac{2}{3}\sqrt{2 \times 32.16 \times 3} =$ 9.26 ft. per sec. Ans.

(499) (*a*) 4 ft. 9 in. = 4.75 ft. $19 - 4.75 = 14.25$.
Range $= \sqrt{4hy} = \sqrt{4 \times 4.75 \times 14.25} = 16.454$ ft. Ans.

(*b*) $19 - 4.75 = 14.25$ ft. Ans.

(*c*) $19 \div 2 = 9.5$. Greatest range $= \sqrt{4 \times 9.5^2} = 19$ ft. Ans. (See Art. **1009.**)

(500) Use formulas **46** and **50.**

$$v_m = 2.315\sqrt{\frac{hd}{fl}} = 2.315\sqrt{\frac{25 \times 5}{.025 \times 1,300}} = 4.5397 \text{ ft. per sec.}$$

From the table, $f = .0230$ for $v_m = 4$ and $.0214$ for $v_m = 6$. $.0230 - .0214 = .0016$. $6 - 4 = 2$.

$4.5397 - 4 = .5397$. Then, $2 : .5397 :: .0016 : x$, or $x =$ .0004. Hence, $.0230 - .0004 = .0226 = f$ for $v_m = 4.5397$.

$$Q = 60 \times 60 \times .09445 \times 5^2 \times \sqrt{\frac{25 \times 5}{.0226 \times 1,300 + \frac{1}{8} \times 5}} =$$

17,350 gal. per hr. Ans.

(501) Obtain the values by approximating to those given in Art. **1033.** Thus, for $v_m = 2$, $f = .0265$; for $v_m = 3$, $f =$ .0243; $.0265 - .0243 = .0022$. $2.37 - 2 = .37$. Hence, $1 : 37 :: .0022 : x$, or $x = .0008$. Then, $.0265 - .0008 = .0257 = f$ for $v_m = 2.37$. Ans.

For $v_m = 3$, $f = .0243$; for $v_m = 4$, $f = .0230$; $.0243 - .0230 = .0013$. $3.19 - 3 = .19$. Hence, $1 : .19 :: .0013 : x$, or $x =$ 0002. Then, $.0243 - .0002 = .0241 = f$ for $v_m = 3.19$. Ans.

For $v_m = 4, f = .0230$; for $v_m = 6, f = .0214$; $.0230 - .0214 = .0016$. $5.8 - 4 = 1.8$. $6 - 4 = 2$. Hence, $2 : 1.8 :: .0016 : x$, or $x = .0014$. Then, $.0230 - .0014 = .0216 = f$ for $v_m = 5.8$. Ans.

For $v_m = 6, f = .0214$; for $v_m = 8, f = .0205$; $.0214 - .0205 = .0009$. $7.4 - 6 = 1.4$. $8 - 6 = 2$. Hence, $2 : 1.4 = .0009 : x$, or $x = .0006$. Then, $.0214 - .0006 = .0208 = f$ for $v_m = 7.4$. Ans.

For $v_m = 8, f = .0205$; for $v_m = 12, f = .0193$; $.0205 - .0193 = .0012$. $9.83 - 8 = 1.83$. $12 - 8 = 4$. Hence, $4 : 1.83 :: .0012 : x$, or $x = .0005$. Then, $.0205 - .0005 = .02 = f$ for $v_m = 9.83$. Ans.

For $v_m = 8$, $f = .0205$; for $v_m = 12$, $f = .0193$; $.0205 - .0193 = .0012$. $11.5 - 8 = 3.5$. $12 - 8 = 4$. Hence, $4 : 3.5 :: .0012 : x$, or $x = .0011$. $.0205 - .0011 = .0194 = f$ for $v_m = 11.5$. Ans.

(502) Specific gravity of sea-water is 1.026. Total area of cube $= 10.5^2 \times 6 = 661.5$ sq. in. 1 mile $= 5,280$ ft. Hence, total pressure on the cube $= 661.5 \times 5,280 \times 3.5 \times .434 \times 1.026 = 5,443,383$ lb. Ans.

(503) $19^2 \times .7854 \times 80 = 22,682$ lb. Ans.

PNEUMATICS.

(QUESTIONS 504–553.)

(504) The force with which a confined gas presses against the walls of the vessel which contains it.

(505) (*a*) $4 \times 12 \times .49 = 23.52$ lb. per sq. in. Ans.
(*b*) $23.52 \div 14.7 = 1.6$ atmospheres. Ans.

(506) (*a*) A column of water 1 foot high exerts a pressure of .434 lb. per sq. in. Hence, $.434 \times 19 = 8.246$ lb. per sq. in., the required tension. A column of mercury 1 in. high exerts a pressure of .49 lb. per sq. in. Hence, $8.246 \div .49 = 16.828$ in. = height of the mercury column. Ans.

(*b*) Pressure above the mercury $= 14.7 - 8.246 = 6.454$ lb. per sq. in. Ans.

(507) Using formula **53**, $p_1 = \frac{p\,v}{v_1} = \frac{(14.7 \times 3) \times 1}{2.5} =$ 17.64 lb. Ans.

(508) (*c*) Using formula **61**,

$$V = \frac{.37052\,W\,T}{p} = \frac{.37052 \times 7.14 \times 535}{22.05} = 64.188 \text{ cu. ft. Ans.}$$

($T = 460° + 75° = 535°$, and $p = 14.7 \times 1.5 = 22.05$ lb. per sq. in.)

(*a*) $7.14 \div .08 = 89.25$ cu. ft., the original volume. Ans.

(*b*) If 1 cu. ft. weighs .08 lb., 1 lb. contains $1 \div .08 =$ 12.5 cu. ft. Hence, using formula **60**, $p\,V = .37052\,T$, or

$$T = \frac{p\,V}{.37052} = \frac{22.05 \times 12.5}{.37052} = 743.887°. \quad 743.887 - 460 =$$ 283.887°. Ans.

(509) Substituting in formula **59,** $p = 40$, $t = 120$, and $t_1 = 55$,

$$p_1 = 40\left(\frac{460 + 55}{460 + 120}\right) = \frac{40 \times 515}{580} = 35.517 \text{ lb. Ans.}$$

(510) Using formula **61,** $p\,V = .37052\,W\,T$, or

$$W = \frac{p\,V}{.37052\,T}. \quad T = 460° + 60° = 520°.$$

Therefore, $W = \dfrac{14.7 \times 1}{.37052 \times 520} = .076296$ lb. Ans.

(511) $175{,}000 \div 144 =$ pounds per sq. in.
$(175{,}000 \div 144) \div 14.7 = 82.672 =$ atmospheres. Ans.

(512) Extending formula **63** to include 3 gases, we have $P\,V = p_1 v_1 + p_2 v_2 + p_3 v_3$, or $40 \times P = 1 \times 12 + 2 \times 10 + 3 \times 8$.

Hence, $P = \dfrac{56}{40} = 1.4$ atmos. $= 1.4 \times 14.7 = 20.58$ lb. per sq. in. Ans.

(513) In the last example, $PV = 56$. In the present case, $P = \dfrac{23}{14.7}$ atmos. Therefore, $V = \dfrac{56}{P} = \dfrac{56}{\frac{23}{14.7}} = 35.79$ cu. ft. Ans.

(514) For $t = 280°$, $T = 740°$; for $t = 77°$, $T = 537°$.
$p\,V = .37052\,W\,T$, or $W = \dfrac{p\,V}{.37052\,T}$. (Formula **61.**)

$$\text{Weight of hot air} = \frac{14.7 \times 10{,}000}{.37052 \times 740} = 536.13 \text{ lb.}$$

$$\text{Weight of air displaced} = \frac{14.7 \times 10{,}000}{.37052 \times 537} = 738.81 \text{ lb.}$$

$738.81 - 536.13 = 202.68$. $202.68 - 100 = 102.68$ lb. Ans.

(515) According to formula **64,**

$$PV = \left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right) T, \text{ or}$$

$$20 \times 31 = \left(\frac{14.7 \times 13}{533} + \frac{30 \times 18}{513}\right) T = 1.411168\,T.$$

Therefore, $T = \frac{20 \times 31}{1.411168} = 439.35°$. Since this is less than 460°, the temperature is $460 - 439.35 = 20.65°$ below zero, or $-20.65°$. Ans.

(516) A hollow space from which all air or other gas (or gaseous vapor) has been removed. An example would be the space above the mercury in a barometer.

(517) One inch of mercury corresponds to a pressure of .49 lb. per sq. in.

$\frac{1}{40}$ inch of mercury corresponds to a pressure of $\frac{.49}{40}$ lb. per sq. in. $\frac{.49}{40} \times 144 = 1.764$ lb. per sq. ft. Ans.

(518) (*a*) $325 \times .14 = 45.5$ lb. = force necessary to overcome the friction. $6 \times 12 = 72'' =$ length of cylinder. $72 - 40 = 32 =$ distance which the piston must move. Since the area of the cylinder remains the same, any variation in the volume will be proportional to the variation in the length between the head and piston. By formula **53**, $p v = p_1 v_1$. Therefore, $p = \frac{p_1 v_1}{v} = \frac{14.7 \times 40}{72} = 8.1\frac{2}{3}$ lb. per sq. in. = pressure when piston is at the end of the cylinder. Since there is the atmospheric pressure of 14.7 lb. on one side of the piston and only $8.1\frac{2}{3}$ lb. on the other side, the force required to pull it out of the cylinder is $14.7 - 8.1\frac{2}{3} = 6.5\frac{1}{3}$ lb. per sq. in. Area of piston $= 40^2 \times .7854 = 1,256.64$ sq. in. Total force $= 1,256.64 \times 6.5\frac{1}{3} = 8,210.05$. Adding the friction, $8,210.05 + 45.5 = 8,255.55$ lb. Ans.

(*b*) Proceeding likewise in the second case, $p v = p_1 v_1$, or $p = \frac{p_1 v_1}{v} = \frac{14.7 \times 40}{6} = 98$ lb. $98 - 14.7 = 83.3$ lb. per sq. in.

$1,256.64 \times 83.3 + 45.5 = 104,723.612$ lb. Ans.

(519) [illegible] = [illegible] [illegible] [illegible] = [illegible] $- 4.3 = $ [illegible] [illegible] = [illegible] [illegible] = [illegible] By formula **53**,

$$[illegible] = \frac{[illegible]}{[illegible]} = [illegible] \text{ lb. per sq. in.} \quad \text{Ans.}$$

(520) Original weight $= W = $ [illegible] lb. $=$ [illegible] oz.; new weight [illegible] = [illegible] oz. According to formula **56**,

$$[illegible] = \frac{[illegible]\,V}{V} = \frac{[illegible] \times [illegible]}{[illegible]} = 40.[illegible] \text{ lb. per}$$

[illegible] [illegible] [illegible].

(521) According to formula **58**,

$$[illegible]\left(\frac{[illegible]}{[illegible]}\right) = \frac{[illegible]}{[illegible]}\left(\frac{460 - [illegible]}{460 - 260}\right) = [illegible] \text{ cu. ft.} \quad \text{Ans.}$$

(522) According to formula **61**, $p\,V = .37052\,W\,T$, or

$$W = \frac{p\,V}{.37052\,T} = \frac{[illegible] \times [illegible].25 \times 35}{.37052 \times 548} = [illegible] \text{ lb.} \quad \text{Ans.}$$

(523) Using formula **63**, $P\,V = p\,v + p_1\,v_1$, or $P \times 7.5 = [illegible] \times 2 \times 7.5 - [illegible] \times 7.5$, or $P = 49.4$ lb. per sq. in. Ans.

(524) 48", 36" and 24" = 4', 3', and 2', respectively. Hence $4 \times 3 \times 2 = 24$ cu. ft. = the volume of the block. The block will weigh as much more in a vacuum as the weight of the air it displaces. In example 510, it was found that 1 cu. ft. of air at a temperature of 60° weighed .076296 lb. $.076296 \times 24 + 1{,}200 = 1{,}201.83$ lb. Ans.

(525) (*a*) (See Art. **1088.**) $127 + 16 = 143$.

$$\frac{(\tfrac{9}{12})^2 \times .7854 \times 1 \times 125 \times 62.5 \times 143}{33{,}000} = 14.9563 \text{ H.P.}$$

$14.9563 \div .75 = 19.942$ H.P. Ans.

(*b*) Discharge in gallons per hour = volume of cylinder in cu. ft. × number of strokes per minute × 7.48 × 60 =

$\left(\frac{9}{12}\right)^2 \times .7854 \times 125 \times 7.48 \times 60 = 24{,}784.3$ gal. per hr. Ans.

(526) In this example, the number of times that the pump delivers water in 1 minute is $100 \div 2 = 50$; in the last example, 125. Hence, the number of gallons discharged per hour in this case will be $24{,}786 \times \frac{50}{125} = 9{,}914.4$ gal. Ans.

(527) See Art. **1043.**

Pressure in condenser $= \frac{30 - 23}{30} \times 14.7 = 3.43$ lb. per sq. in. Ans.

(528) $144 \times 14.7 = 2{,}116.8$ lb. per sq. ft. Ans.

(529) $.27 \div 3 = .09 =$ weight of 1 cu. ft. Using formula **56,**

$p\ W_1 = p_1\ W$, or $30\ W_1 = 65 \times .09$. $W_1 = .195$ lb. Ans.

(530) Using formula **61,**

$$p\ V = .37052\ W\ T, \text{ or } 30 \times 1 = .37052 \times .09 \times T.$$

$$T = \frac{30}{.37052 \times .09} = 899.6°. \quad 899.6° - 460° = 439.6°. \quad \text{Ans.}$$

(531) $460° + 32° = 492°$; $460° + 212° = 672°$; $460° + 62° = 522°$, and $460° + (-40°) = 420°$.

(532) Using formula **61,** $p\ V = .37052\ W\ T$, and substituting,

$(14.7 \times 10) \times 4 = .37052 \times 3.5 \times T$, or $T = \frac{14.7 \times 10 \times 4}{.37052 \times 3.5} =$ $453.417°$. $453.417 - 460° = -6.583°$. Ans.

(533) Using formula **63,** $V P = v p + v_1 p_1$, we find

$$P = \frac{15 \times 63 + 19 \times 14.7 \times 3}{25} = 71.316 \text{ lb.} \quad \text{Ans.}$$

(534) Using formula **60,** $p\ V = .37052\ T$, or $P = \frac{.37052 \times 540}{10} = 20$ lb. per sq. in., nearly. Ans.

(535) One inch of mercury represents a pressure of .49 lb. Therefore, the height of the mercury column is $12.5 \div .49 = 25.51$ in. Ans.

(536) Thirty inches of mercury corresponds to 34 ft. of water. (See Art. **1043.**) Therefore,

$$30'' : 34 \text{ ft.} :: 27'' : x \text{ ft., or } x = 30.6 \text{ ft.} \quad \text{Ans.}$$

A more accurate way is $(27 \times .49) \div .434 = 30.5$ ft.

(537) (*a*) $30 - 17.5 = 12.5$ in. = original tension of gas in inches of mercury. $30 - 5 = 25$ in. = new tension in inches of mercury.

$VP = vp + v_1 p_1$ (formula **63**), or $6.7 \times 25 = 6.7 \times 12.5 + v_1 \times 30$.

$$v_1 = \frac{6.7 \times 25 - 6.7 \times 12.5}{30} = 2.79\frac{1}{6} \text{ cu. ft. Ans.}$$

(*b*) To produce a vacuum of 0 inches,

$$v_1 = \frac{6.7 \times 30 - 6.7 \times 12.5}{30} = 3.908 \text{ cu. ft. Ans.}$$

(538) $11 + 25 = 36$, final volume of gas. $2.4 \div 36 = \frac{1}{15}$ lb. Ans.

(539) Using formula **59**,

$$p_1 = p\left(\frac{460 + t_1}{460 + t}\right) = 12 \times \left(\frac{460 + 300}{460 + 60}\right) = 17.54 \text{ lb. per sq. in. Ans.}$$

(540) $T = 460 + 212 = 672°$. Using formula **61**, $pV = .37052\ WT$, we have $14.7 \times 1 = .37052 \times W \times 672$, or $W = \frac{14.7}{.37052 \times 672} = .059039$ lb. Ans.

(541) (*a*) $\frac{20^2 \times .7854 \times 32}{1{,}728} = 5.8178$ cu. ft. = volume of cylinder.

$32 - 26 = 6$ in., length of stroke unfinished.

$$5.8178 \times \frac{6}{32} = 1.0908 \text{ cu. ft. Ans.}$$

(*b*) By formula **61**, taking the values of p, V, and T at the beginning of stroke,

$pV = .37052\ WT$, or $W = \frac{pV}{.37052\ T} = \frac{14.7 \times 5.8178}{.37052 \times 535} = .43143$ lb. Ans.

(*c*) Now, substituting in formula **61** the values of V, W, and T at time of discharge,

$p = \frac{.37052\ WT}{V} = \frac{.37052 \times .43143 \times 585}{1.0908} = 85.727$ lb. per sq. in. Ans.

(542) Using formula **63,** $VP = vp + v_1 p_1$, or $30 \times 35 = 19 \times 12 + 21 p_1$, or $p_1 = \frac{30 \times 35 - 19 \times 12}{21} = 39.14$ lb. per sq. in. Ans.

(543) Use formula **64.** $PV = \left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right)T$.

$$T = 460 + 72 = 532.$$

Therefore, $P = \frac{\left(\frac{13 \times 45}{520} + \frac{17 \times 60}{540}\right) 532}{60} = 26.723$ lb. per sq. in. Ans.

(544) See Art. **1043.** Since the barometer column stands at 30 inches for a perfect vacuum, the height when the air is admitted will be $30'' - 4\frac{1}{2}'' = 25\frac{1}{2}''$. Ans.

(545) Sp. Gr. of alcohol = .8. Therefore, $16 \times .434 \times .8$ = pressure exerted by the column of alcohol. $\frac{16 \times .434 \times .8}{.49} =$ 11.337 in. = height of a column of mercury that will give the same pressure as 16 ft. of alcohol = number of inches shown by the gauge. Ans.

(546) (*a*) $14.7 + 9 = 23.7$ lb. per sq. in. Using formula **53,**

$$v_1 = \frac{p\,v}{p_1} = \frac{14.7 \times 80}{23.7} = 49.62 \text{ in.} =$$

distance between piston and end of stroke. Since the area of the piston remains constant, the volume at any point of the stroke is proportional to the distance passed over by the piston. Hence, we may use the latter for the former in the formula. $80 - 49.62 = 30.38$ in. Ans.

(*b*) Area of piston $= 80^2 \times .7854$. The volume of air at point of discharge is $80^2 \times .7854 \times 49.62$ cu. in. =

$$\frac{80^2 \times .7854 \times 49.62}{1{,}728} = 144.34 \text{ cu. ft. Ans.}$$

(547) Using formula **56,** $p\,W_1 = p_1 W$, or $3.5 \times 14.7 \times 2 = p_1 \times 13$; hence, $p_1 = \frac{14.7 \times 3.5 \times 2}{13} = 7.915+$ lb. per sq. in. Ans.

(548) $60' - 50' = 10'$. Since the volumes are propor tional to the lengths of the spaces between the piston and the end of the stroke, we may apply formula **62,**

$$\frac{p\,V}{T} = \frac{p_1\,V_1}{T_1}; \text{ or } \frac{14.7 \times 60}{460 + 60} = \frac{p_1 \times 10}{460 + 130}.$$

Therefore, $p_1 = \frac{14.7 \times 60 \times 590}{520 \times 10} = 100.07$ lb. per sq. in. Ans.

(549) $T = 127° + 460° = 587°$. Using formula **60,** $p\,V = .37052\,T$, or $V = \frac{.37052 \times 587}{27} = 8.055$ cu. ft. Ans.

(550) $T = 100° + 460° = 560°$.
Substituting in formula **61,** $p\,V = .37052\,W\,T$, or

$$V = \frac{.37052\,W\,T}{p} = \frac{.37052 \times .5 \times 560}{\frac{4,000}{144}} = 3.735 \text{ cu. ft. Ans.}$$

(551) Use formula **64.** $P\,V = \left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right)T$.

$T = 110° + 460° = 570°$; $T_1 = 100° + 460° = 560°$; $T_2 = 130° + 460° = 590°$.

$$\text{Therefore, } V = \frac{\left(\frac{90 \times 40}{560} + \frac{80 \times 57}{590}\right)570}{120} = 67.248 \text{ cu. ft.}$$

Ans.

(552) The pressure exerted by squeezing the bulb may be found from formula **53,** in which p is 14.7, v, the original volume = 20 cu. in., and v_1, the new volume, = 5 cu. in. $p_1 = \frac{p\,v}{v_1} = \frac{14.7 \times 20}{5} = 58.8$ lb. The pressure due to the atmosphere must be deducted, since there is an equal pressure on the outside which balances it. $58.8 - 14.7 = 44.1$ lb. per sq. in. = pressure due to squeezing the bulb. $3^2 \times .7854 = 7.0686$ sq. in. = area of bottom. $7.0686 \times 44.1 = 311.725$ lb. $7.0686 \times .434 = 3.068$ lb. = pressure due to weight of water. $311.725 + 3.068 = 314.793$ lb. Ans.

(553) Use formula **58.**

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = 4\left(\frac{460 + 115}{460 + 40}\right) = 4.6 \text{ cu. ft. Ans.}$$

HEAT.

(QUESTIONS 554-618.)

(554) 9 hr. 25 min. = 565 min.

55 ÷ 2 = 27.5 min. = time it would take to heat 1 lb. of the substance to this temperature under the same conditions. 27.5 ÷ 565 = .04867 = specific heat of the substance. Ans.

(555) Increase in diameter = 4.001 − 3.9985 = .0025″. C_1 = .00000599, from Table 19. Hence, using formula **67**, $t = \frac{1}{L C_1} = \frac{.0025}{3.9985 \times .00000599} = 104.4°$, nearly. Then, 104.4 + 80 = 184.4°, nearly. Ans.

(556) (*a*) Substituting in formula **72** $W = 360$, $t_1 - t = 104.4$, and $s = .1165$; $n = W\ (t_1 - t)\ s = 360 \times 104.4 \times .1165 = 4{,}378.536$ B. T. U.

Since 12% of the total heat is lost by radiation, the above number of B. T. U. must be the number remaining after subtracting the heat units lost by radiation from the total number; or it is 100 − 12 = 88% of the total number. Hence, dividing by .88 we obtain 4,378.536 ÷ .88 = 4,975.6 B. T. U., total heat required. Ans.

(*b*) 4,975.6 × 778 = 3,871,017 ft.-lb. Ans.

(557) (*a*) $80^2 \times .7854 = 5{,}026.56$ sq. in. = area of piston.

$p\ v = p_1\ v_1$. At the beginning of the stroke $p = 14.7$, and the volume may be represented by the length of the cylinder, 80 in.; at the point of discharge p_1 is 120, therefore, $80 \times 14.7 = 120 \times v$, or $v = 9.8$ in., the portion of the stroke uncompleted at discharge.

$\frac{5{,}026.56 \times 9.8}{1{,}728} = 28.507$ cu. ft., volume of air discharged. Ans.

(*b*) By formula **80**, $L = 331.5744 p V \log \frac{p_1}{p} = 331.5744 \times$ $120 \times 28.507 \log \frac{120}{14.7} = 331.5744 \times 120 \times 28.507 \times .91186 =$ 1,034,289 ft.-lb. Ans.

(558) (*a*) Volume of cylinder = area of piston × length of stroke $= \frac{5,026.56}{144} \times \frac{80}{12} = 232.71$ cu. ft., nearly.

From formula **81**, $p_1 v_1^{1.41} = p_2 v_2^{1.41}$, or $v_2 = \sqrt[1.41]{\frac{p_1 v_1^{1.41}}{p_2}}$.

Substituting for p_1 and v_1 their values at the beginning of the stroke, and for p_2 its value at discharge, we have

$$v_2 = \sqrt[1.41]{\frac{14.7 \times 232.71^{1.41}}{120}};$$

$$\log v_2 = \frac{\log 14.7 + 1.41 \log 232.71 - \log 120}{1.41}, \text{ or}$$

$$\log v_2 = \frac{1.16732 + 1.41 \times 2.36682 - 2.07918}{1.41} = 1.72011.$$

$v_2 = 52.494$ cu. ft., the volume discharged. Ans.

(*b*) To find the work, use formula **84**,

$$L = 351.36 p V \left[1 - \left(\frac{V}{V_1}\right)^{.41}\right] =$$

$$351.36 \times 14.7 \times 232.71 \times \left[1 - \left(\frac{232.71}{52.494}\right)^{.41}\right].$$

$$\text{Log} \left(\frac{232.71}{52.494}\right)^{.41} = .41 (\log 232.71 - \log 52.494) =$$

$.41 (2.36682 - 1.72011) = .26515$. Therefore, $\left(\frac{232.71}{52.494}\right)^{.41} =$ 1.8414.

$1 - 1.8414 = -.8414$, the minus sign indicating that the work done by the air is negative; that is, that work is done upon the air instead of by it, or that the air is compressed.

$L = 351.36 \times 14.7 \times 232.71 \times .8414 = 1,011,317$ ft.-lb. Ans.

(559) See Arts. **1090** and **1135**.

(560) Using formula **65,**

(a) $t_f = \frac{9}{5} \times 2{,}917 + 32 = 5{,}282.6°$ F. Ans.

(b) $t_f = \frac{9}{5} \times 637 + 32 = 1{,}178.6$ F. Ans.

(c) $t_f = \frac{9}{5} \times (-260) + 32 = -436°$ F. Ans.

(561) See Table 21, Art. **1134,** and Table 22, Art. **1141.**

$-50 - (-37.8) \times 12 \times .0333 = 4.875$ = units of heat necessary to raise the temperature to the fusion point. $5.09 \times 12 = 61.08$ B. T. U. required to melt the mercury. $[662 - (-37.8)] \times .0333 \times 12 = 279.64$ B. T. U. required to raise the temperature to the point of vaporization. $157 \times 12 = 1{,}884$ B. T. U. required for vaporizing it. $4{,}875 + 61.08 + 279.64 + 1{,}884 = 2{,}229.595$ B. T. U. Ans.

(562) Using formula **71,** $p\,V = W R\,T$, or

$$W = \frac{p\,v}{R\,T} = \frac{18 \times 100}{5.34946 \times 540} = .6231 \text{ lb. Ans.}$$

(563) See Tables 21 and 22 for specific heat and temperature of fusion of copper. 7 lb. 5 oz. = 7.3125 lb. $2{,}100° - 78° = 2{,}022°$. $2{,}022 \times 7.3125 \times .0951 = 1{,}406.13$ B. T. U. Ans.

(564) By formula **73,** $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3} =$

$$\frac{2.25 \times .0314 \times 40 + 4 \times 65 + 1.75 \times .1298 \times 62}{2.25 \times .0314 + 4 + 1.75 \times .1298} = 64.43°. \text{ Ans.}$$

(565) Use formulas **69, 67,** and **68.** (a) $v = V C_3 t = 4 \times 4 \times 20 \times 1{,}200 \times .00002058 = 7.903$ cu. in. Ans.

(b) $l = L\,C_1 t = 20 \times 1{,}200 \times .00000686 = .16464$ in. Ans.

(c) $a = A\,C_2 t = 4 \times 4 \times 1{,}200 \times .00001372 = .2634$ sq. in. Ans.

(566) Using formula **71,**

$$p\,V = W R\,T, \text{ or } T = \frac{p\,V}{W R} = \frac{73 \times 61}{10 \times .38143} = 1{,}167.45°.$$

$$1{,}167.45° - 460° = 707.45°. \text{ Ans.}$$

(567) See Art. **1124.**

(568) By formula **73**, $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$, or

$$t_1 = \frac{(W_1 s_1 + W_2 s_2 + W_3 s_3) t - W_2 s_2 t_2 - W_3 s_3 t_3}{W_1 s_1} =$$

$$\frac{(1 \times .1298 + 3.25 \times 1 + 1.5 \times .0951)\, 128 - 3.25 \times 85 - 1.5 \times .0951 \times 85}{1 \times .1298} =$$

1,252°. Ans.

(569) Draw the lines $O\,X$ and $O\,Y$ (Fig. 46) perpendicular to each other. On $O\,X$ take any convenient distance, say $\frac{1}{2}$ inch, to represent one unit of volume (1 cu. ft.), and on $O\,Y$ take a distance of say 3 inches to represent the required pressure of 84 pounds. Our scale of pressure is,

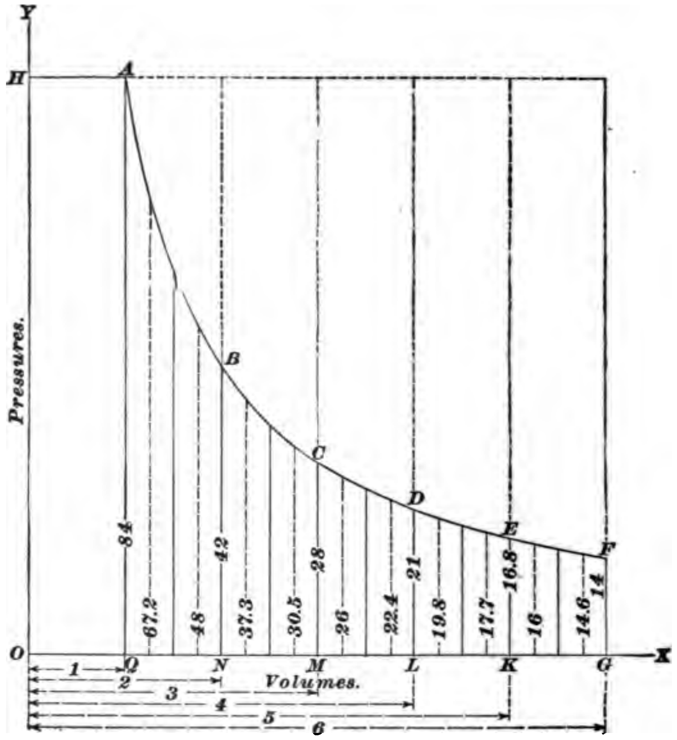

FIG. 46.

then, 28 pounds to the inch. The ordinates are now erected as shown in Art. **1156.** Thus, at A the volume is 1, and

the pressure 84 pounds; hence, $p\,v = 84$. But $p\,v = p_1\,v_1 = p_2\,v_2$, etc., $= 84$; hence, $p_1 = \frac{84}{v_1}$, $p_2 = \frac{84}{v_2}$, etc. When $v = 1$, $p = \frac{84}{1} = 84 = QA$.

When $v = 2$, $p = \frac{84}{2} = 42 = NB$.

When $v = 3$, $p = \frac{84}{3} = 28 = MC$.

When $v = 4$, $p = \frac{84}{4} = 21 = LD$.

When $v = 5$, $p = \frac{84}{5} = 16.8 = KE$.

When $v = 6$, $p = \frac{84}{6} = 14 = GF$.

A curve through A, B, C, D, E, and F will be the one required.

(570) Divide the space $Q\,G$ into 10 equal parts, as shown in the figure, and draw the ordinate at the middle of each space. The average ordinate will be found to be 29.95 lb. The volume represented by the length $Q\,G$ is 5 cu. ft. Hence, the work is $29.95 \times 5 \times 144 = 21{,}564$ foot-pounds. Ans.

Calculating the work by formula **79,**

$$L = 331.5744\,p\,V \log \frac{V_1}{V};$$

$$L = 331.5744 \times 84 \times 1 \times \log \frac{6}{1} = 21{,}673 \text{ ft.-lb.} \quad \text{Ans.}$$

(571) (*a*) $460 + 96 = 556°$ F. (*b*) $273\frac{1}{3} + 32 = 305\frac{1}{3}°$ C. (*c*) $273\frac{1}{3} + 180 = 453\frac{1}{3}°$ C. (*d*) $460 + 650 = 1{,}110°$ F. (*e*) $273\frac{1}{3} - 40 = 233\frac{1}{3}°$ C.

(572) $32° - 20° = 12°$. $12 \times 7 \times .504 = 42.336$ B. T. U. required to heat the ice to 32°. $142.65 \times 7 = 998.55$ B. T. U. required to melt the ice. $212° - 32° = 180°$. $180 \times 7 \times 1 = 1{,}260$ B. T. U. required to heat the *water* to 212°. $966.6 \times 7 = 6{,}766.2$ B. T. U. required to change 7 lb. of water at 212° into steam at 212°. $42.336 + 998.55 + 1{,}260 + 6{,}766.2 = 9{,}067.086$ B. T. U.

$$\frac{9{,}067.086 \times 778}{43 \times 33{,}000} = 4.971 \text{ horsepower.} \quad \text{Ans.}$$

(573) Using formula **84**, and substituting the values of p and V at beginning of compression, we have

$$L = 351.36\,p\,V\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right] =$$

$$351.36 \times 14.7 \times 1\left[1 - \left(\frac{1}{.25}\right)^{.41}\right].$$

$\text{Log}\left(\frac{1}{.25}\right)^{.41} = \log 4^{.41} = .41 \log 4 = .41 \times .60206 = .24684.$

The number whose logarithm is .24684 is 1.7654.

Therefore, $\left(\frac{1}{.25}\right)^{.41} = 1.7654.$ $1 - \left(\frac{1}{.25}\right)^{.41} = 1 - 1.7654 =$ $-.7654$, the minus sign indicating that the air is compressed. Hence, $L = 351.36 \times 14.7 \times .7654 = 3{,}953.28$ ft.-lb. Ans.

(574) On account of the great irregularity in outline of this figure, a division into ten parts will not give a sufficiently close approximation to the mean ordinate; hence, it is divided into 20 equal parts, as shown by the full lines.

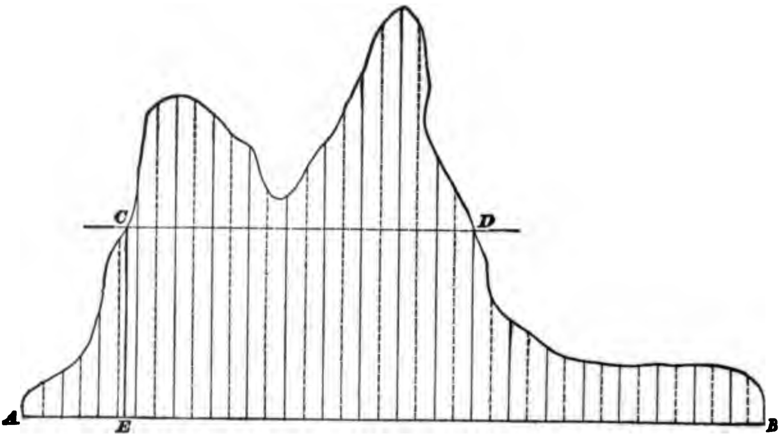

FIG. 47.

(See Fig. 47.) The dotted lines, situated midway between the full lines, represent the ordinates which are to be measured. The sum of all the dotted lines, divided by the number of divisions, gives .9365″ for the mean ordinate. Draw

the line $C\ D$ parallel to $A\ B$ at a distance from $A\ B$ equal to .9365″, and where it cuts the curve will be the points from which to draw the mean ordinate, as $C\ E$.

(575) Prolong $A\ B$ in both directions as shown in Fig. 48. Draw the tangents $h\ i$, $f\ A$, $e\ d$, and $c\ b$, perpendicular to $A\ B$. Since the outline $f\ A\ g$ is very nearly triangular, and is quite small compared with the rest of the figure, consider it a triangle, and draw $m\ n$ half way

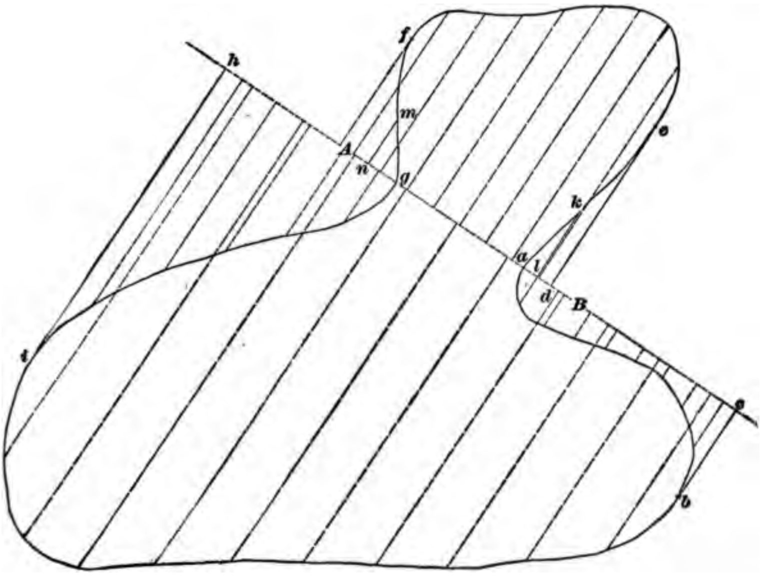

FIG. 48.

between A and g. Consider $m\ n$ as the mean ordinate. Then, $A\ g \times m\ n = .34 \times .28 = .0952$ sq. in. In a similar manner, area of $a\ d\ e = .17 \times .4 = .068$ sq. in. Dividing $A\ f\ e\ d$ into 8 equal parts, and drawing the ordinates at the middle points of these divisions (for convenience the full lines, similar to those in the last figure, have been omitted, and the ordinates at the middle points only have been drawn), the mean ordinate is found to be 1.228″. The length $A\ d = 1.31$″; hence, area of $A\ f\ e\ d = 1.31 \times 1.228 = 1.6087$ sq. in. Dividing $a\ c\ b$ into 8 equal parts, the mean

ordinate is found to be .154; the length $a\ c$ is 1.34; hence, area $a\ c\ b = 1.34 \times .154 = .2064$ sq. in. Dividing $h\ g\ i$ into 5 equal parts, the mean ordinate is found to be .82″; the length $h\ g$ is 1.07″; hence, area $h\ g\ i = 1.07 \times .82 = .8774$ sq. in. Dividing $h\ i\ b\ c$ into 10 equal parts, the mean ordinate is found to be 1.925″; its length $h\ c = 3.21$″; hence, area $h\ i\ b\ c = 3.21 \times 1.925 = 6.179$ sq. in.

$1.6087 + 6.179 = 7.7877$ sq in. = area $h\ i\ b\ c\ d\ e\ f\ A\ h$.

$.0952 + .068 + .2064 + .8774 = 1.247$ sq. in.

$7.7877 - 1.247 = 6.5407$ sq. in. = required area $g\ i\ b\ a\ e\ f\ g$. Ans.

(576) Using formula **73**, $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$. For water, $s_1 = 1$; therefore, $W_1 t + (W_2 s_2 + W_3 s_3)\ t = W_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3$, and

$$W_1 = \frac{W_2 s_2 t_2 + W_3 s_3 t_3 - t\,(W_2 s_2 + W_3 s_3)}{t - t_1}.$$

Substituting the given values for W and t and the values of s_2, s_3, etc., from Table 21, we have

$$W_1 = \frac{4 \times .426 \times 80 + .5 \times .0939 \times 73 - 75.61(4 \times .426 + .5 \times .0939)}{75.61 - 73}$$

$$= \frac{7.35802}{2.61} = 2.819 \text{ lb. Ans.}$$

(577) See Arts. **1138, 1139, 1131, 1132,** and **1152.**

(578) $K = \dfrac{W v^2}{2 g} = \dfrac{120 \times 1{,}200^2}{2 \times 32.16}$ = kinetic energy in foot-pounds.

(See Art. **957.**) $\dfrac{120 \times 1{,}200^2}{2 \times 32.16} \times .15$ = kinetic energy expended in heat. Now, dividing by 778 to reduce the foot-pounds to B.T.U., $\dfrac{120 \times 1200^2 \times .15}{2 \times 32.16 \times 778} = 517.975$ B.T.U. Ans.

(579) See Art. **1126.**

(580) Using formula **73**, $t = \dfrac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$. (Since s_2, the specific heat of water, is 1, it may be left out.)

Transposing,

$$s_1 = \frac{W_2(t_2 - t) + W_3 s_3(t_3 - t)}{W_1(t - t_1)} = \frac{1.875(91-86)+1.25\times.0562\times(91-86)}{5(86-40)} =$$

.0423. Ans.

(See example, Art. **1137.**)

(581) $T_1 = 460° + 450° = 910°$; $T_2 = 460° + 70° = 530°$.

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1} = \frac{910 - 530}{910} = 41.76\%. \text{ Ans.}$$

(582) Use formula **71,** $pV = WRT$. $T = 460° + 200 = 660°$; and R, from Table 20, is 5.34946; therefore, $W = \frac{pV}{RT} = \frac{20 \times 700}{5.34946 \times 660} = 3.9653$ lb. Ans.

(583) (See Arts. **1091** to **1098.**)

(*a*) $C : R = 100 : 80$, or $C = \frac{100}{80} R = \frac{5}{4} R$.

$$\text{Hence, } 44 \times \frac{5}{4} = 55° \text{ C. Ans.}$$

(*b*) $F : R = 180 : 80$, or $F = \frac{180}{80} R = \frac{9}{4} R$.

$$\text{Hence, } 44 \times \frac{9}{4} + 32 = 131° \text{ F. Ans.}$$

(584) By formula **81,** $p_1 v_1^{1.41} = p_2 v_2^{1.41}$. The volume of 1 lb. of air at atmospheric pressure, and having a temperature of 60°, is $\frac{1}{.076296} = 13.107$ cu. ft. (See question 510.)

Substituting in the above formula, $14.7 \times 13.107^{1.41} = 235 \times v_2^{1.41}$, or

$$v_2 = \sqrt[1.41]{\frac{14.7 \times 13.107^{1.41}}{235}} = 1.8356 \text{ cu. ft. Ans.}$$

From formula **71,** remembering that $W = 1$, we have $pV = RT$, or $T = \frac{pV}{R} = \frac{235 \times 1.8356}{.37052} = 1,164.2°$. $1,164.2° - 460° = 704.2°$. Ans.

(585) (See Arts. **1107, 1054,** and **1099.**)

(586) (*a*) Cubical contents of cylinder $= (10^2 - \overline{9\frac{1}{2}}^2) \times .7854 \times 72 = 816.48$ cu. in. $= \frac{816.48}{1,728}$ cu. ft. Specific gravity of copper is 8.79. Then, the weight of the cylinder is $\frac{816.48}{1,728} \times 62.5 \times 8.79 = 259.58$ lb. Specific heat of copper, from Table 21, is .0951. Substituting the values of n, W, and s, in formula **72**, $n = W(t_1 - t)\,s$, we have $7,000 = 259.58\,(t_1 - t) \times .0951$, or $t_1 - t = \frac{7,000}{259.58 \times .0951} = 283.56° =$ increase of temperature.

By formula **67**, $l = L\ C_1\ t = 72 \times .00000955 \times 283.56 = .195$ in. Ans.

(*b*) By formula **69**, $v = V\ C_3\ t = 816.48 \times .00002864 \times 283.56 = 6.63$ cu. in. Ans.

(*c*) By formula **67**, $l = L\ C_1\ t = 10 \times .00000955 \times 283.56 = .027$ in. Ans.

(587) From Table 19, the coefficient of expansion for gases is .00203252. Substituting in formula **70**,

$$V_2 = \left[\frac{1 + C_3\,(t_2 - 32)}{1 + C_3\,(t_1 - 32)}\right] \quad V_2 = \left[\frac{1 + .00203252(390 - 32)}{1 + .00203252(65 - 32)}\right] \times$$

$12 = 19.428$ cu. ft. $19.428 - 12 = 7.428$ cu. ft. Ans.

The same answer may be obtained by using formula **58.**

(588) Foot-pounds per minute $= 75 \times 2 \times 20 \times 2 \times .18$.
Foot-pounds per hour $= 75 \times 2 \times 2 \times 20 \times .18 \times 60$.

Heat units per hour $= \frac{75 \times 2 \times 2 \times 20 \times .18 \times 60}{778} = 83.29$ B. T. U. Ans.

(589) (*a*) Using formula **87**, $T_1 = T\left(\frac{V}{V_1}\right)^{.41} = 610 \times \left(\frac{1}{4.5}\right)^{.41} = -130.76°$. Ans.

(*b*) Substituting in formula **71**, the initial values of W, R, T, and V, we obtain $p = \frac{W\,R\,T}{V} = \frac{.5 \times .37052 \times 610}{.9} = 125.565$ lb. per sq. in., the initial pressure. Ans.

(*c*) Using formula **86**,

$$p_1 = p\left(\frac{T_1}{T}\right)^{.\text{[illegible]}} = 125.565 \times \left(\frac{329.24}{610}\right)^{\text{[illegible]}} = 15.06 \text{ lb. per sq. in.}$$ Ans.

(590) $12^3 \times .5236 = 904.78$ cu. in. = volume of sphere. Sp. Gr. of zinc = 6.86; therefore, $\frac{904.78}{1{,}728} \times 62.5 \times 6.86 =$ 224.5 lb. = weight of sphere. (See example 586.) Using formula **73**, $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2} = \frac{8 \times 212 + 224.5 \times .0956 \times 70}{8 + 224.5 \times .0956}$ $= 108.49°$. $108.49° - 70° = 38.49°$ = increase in temperature of the zinc sphere after dipping in the boiling water.

Using formula **69**, $v = V C_3 t = 904.78 \times .00004903 \times 38.49 = 1.71$ cu. in., nearly. Ans.

(591) By formula **67**, $l = L\ C_1\ t$. $C_1 = .00000599$; $L = 900 \times 12$; $t = 90° - 28°$; therefore, $1 = 900 \times 12 \times .00000599 \times (90 - 28) = 4.01$ in. Ans.

(592) According to formula **72**, $n = W(t_1 - t)s$, or $s = \frac{n}{W(t_1 - t)} = \frac{n}{W} = \frac{5}{26} = .1923$. Ans.

(593) See Art. **1161**.

(594) (*a*) Temperature of vaporization of sulphur, from Table 22, is 228.3°.

Specific heat of sulphur, from Table 21, is .2026.

Latent heat of fusion of sulphur, from Table 22, is 13.26.

$228.3 - 40 = 188.3$. $13 \times .2026 \times 188.3 = 495.94454$ B. T. U. to raise sulphur to melting point. $13.26 \times 13 = 172.38$ B. T. U. to melt the sulphur. $(495.94454 + 172.38) \times 778 \times 519{,}956.5$ ft.-lb., total work required to perform both of the above operations. Ans.

(*b*) $\frac{519{,}956.5}{10 \times 33{,}000} = 1.5756$ horsepower. Ans.

(595) $124 \times 3 = 372$ B. T. U., heat given up by the turpentine in changing from gas to liquid. $75 \times 4 = 300$ B. T. U., heat contained in the water. $300 + 372 = 672$

B. T. U., heat contained in the mixture at the instant the turpentine has become liquid. $672 \div 4 = 168° =$ temperature of water at the instant that the turpentine has become a liquid.

Applying formula **73**, $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2} =$

$$\frac{.3 \times .426 \times 313 + 168 \times 4}{3 \times .426 + 4} = 203.11°, \text{ final temperature. Ans.}$$

(596) Specific heat of melted lead, from Table 21, $= .0402$.
Specific heat of solid lead, from Table 21, $= .0314$.
Temperature of fusion of lead, from Table 22, $= 626°$.
Latent heat of fusion of lead, from Table 22, $= 9.67$.

$$626° - 46° = 580°. \quad 800° - 626° = 174°.$$

$25 \times .0314 \times 580 = 455.3$ B. T. U., heat required to raise the lead to melting point. $9.67 \times 25 = 241.75$ B. T. U., heat required to melt the lead. $25 \times .0402 \times 174 = 174.87$ B. T. U., heat required to raise the temperature of the melted lead from fusion point to 800°. $455.3 + 241.75 + 174.87 = 871.92$ B. T. U., total heat required. $871.92 \times 778 = 678{,}353.76$ ft.-lb. Ans.

(597) $32° - 10° = 22°$. $22 \times 10 \times .504 = 110.88$ B. T. U., heat required to raise ice from 10° to 32°. Latent heat of fusion of ice, from Table 22, is 142.65; then $142.65 \times 10 = 1{,}426.5$ B. T. U., heat units to melt the ice. $1{,}426.5 + 110.88 = 1{,}537.38$ B. T. U., total heat units to be taken from the mixture to melt the ice. Hence, applying formula **73**,

$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 t_3 + W_4 t_4 + W_5 s_5 t_5 - 1{,}537.38}{W_1 s_1 + W_2 s_2 + W_3 + W_4 + W_5 s_5} =$$

$$\frac{11.5 \times .1298 \times 180 + 42 \times .0939 \times 240 + 10 \times 32 + 50 \times 120 + 20 \times .0314 \times 80 - 1{,}537.38}{11.5 \times .1298 + 42 \times .0939 + 10 + 50 + 20 \times .0314}$$

$= 91.55°$. Ans.

(598) (*a*) $p_1 v_1^{1.41} = p_2 v_2^{1.41}$, or $p_2 = \frac{140 \times 3^{1.41}}{16^{1.41}} = 13.215$

$$\text{Area} = \frac{p_1 v_1 - p_2 v_2}{.41} = \frac{140 \times 3 - 13.215 \times 16}{.41} = 508.69.$$

$$508.69 \div 20 = 25.434 \text{ sq. in. Ans.}$$

(*b*) $25.434 \div 13 = 1.9565$. Ans.

(*c*) $1.9565 \times 20 = 39.13$ lb. per sq. in. Ans.

(599) Use formula **66,** $t_c = (t_f - 32)\frac{5}{9}$.

(*a*) $t_c = (-10 - 32) \times \frac{5}{9} = -23\frac{1}{3}°$ C. Ans.

(*b*) $t_c = (25 - 32) \times \frac{5}{9} = -3\frac{8}{9}°$ C. Ans.

(*c*) $t_c = (2{,}200 - 32) \times \frac{5}{9} = 1{,}204\frac{4}{9}°$ C. Ans.

(600) This must be answered by the student.

(601) Using formula **80,** $L = 331.5744\,p\,v \log \frac{p_1}{p} =$
$331.5744 \times 15 \times 10 \times \log \frac{85}{15} = 37{,}467.74$ ft.-lb. Ans.

(602) 520 H. P. $= 520 \times 33{,}000$ ft.-lb. per minute $=$
$520 \times 33{,}000 \times 60$ ft.-lb. per hour $=$
$\frac{520 \times 33{,}000 \times 60}{778}$ B. T. U. $= 1{,}323{,}393.31$ B. T. U. Ans

(603) Let l = the required latent heat;
W_1 = given weight of zinc;
W_2 = given weight of water;
s_1 = specific heat of zinc;
s_2 = specific heat of water ($= 1$);
t_1 = temperature of melting zinc;
t_2 = temperature of water.

Then, the total heat of the mixture is $W_1 s_1 t_1 + W_2 s_2 t_2 + W_1 l$, and from formula **73,** the temperature

$$t = \frac{W_1 s_1 t_1 + W_2 t_2 + W_1 l}{W_1 s_1 + W_2}, \text{ or}$$

$$l = \frac{t(W_1 s_1 + W_2) - W_1 s_1 t_1 - W_2 t_2}{W_1} =$$

$$\frac{102\frac{1}{3}(4 \times .0956 + 10) - 4 \times .0956 \times 680 - 10 \times 60}{4} = 50.61.$$

Ans.

(604) See Arts. **1117** and **1120.**

(605) (*a*) and (*b*) See Art. **1118.**

(c) A non-conductor is one which will not conduct heat. There really is no such substance, but some substances are such bad conductors that they are termed non-conductors.

(606) When answering this question, consult Art. **1123**.

(607) In Art. **1130** it is stated that 1 calorie = 3.96 B. T. U. Hence, (a) 798 B. T. U. = 798 ÷ 3.96 = 201.515 calories. Ans.

(b) 40 × 3.96 = 158.4 B. T. U. Ans.

(608) (a) First calculate the weight by formula **71**.

$$W = \frac{p\,V}{R\,T} = \frac{18 \times 7.68}{.33552 \times 500} = .824 \text{ lb., nearly.}$$

Units of heat required $= s_p\, W\,(T_1 - T) = s_v\, W\,(t_1 - t)$ $= .15507 \times .824 \times (416 - 40) = 48.0444$ B. T. U.

Therefore, 48.0444 × 778 = 37,379 ft.-lb., nearly. Ans.

(b) .21751 × .824 (416 − 40) × 778 = 52,429 ft.-lb., nearly. Ans.

(609) See Arts. **1144** to **1147**.

(610) See Arts. **1155** and **1164**.

(611) Using formula **87**,

$T_1 = T\left(\frac{V}{V_1}\right)^{.41} = 528\left(\frac{3.72}{1.2}\right)^{.41} = 839.64° = 380°$, nearly, above 0° F. Ans.

(612) (See Arts. **1149** and **1177**.)

(613) Use formula **81**, $p\,v^{1.41} = p_1\, v_1^{1.41}$.

$$14.7 \times 48^{1.41} = p_1 \times (48 - 38)^{1.41}, \text{ or}$$

$$p_1 = 14.7\left(\frac{48}{10}\right)^{1.41} = 134.24 \text{ lb. per sq. in. Ans.}$$

Now, using formula **86**,

$T_1 = 500\left(\frac{134.24}{14.7}\right)^{.29078} = 951°$, nearly, $= 491°$ above 0° F. Ans.

STEAM AND STEAM ENGINES.

(QUESTIONS 614–688.)

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(614) See Arts. **1191** and **1192.**

(615) See Arts. **1189** to **1195.**

(616) See Arts. **1198** to **1201.**

(617) Log $p = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2} = 6.1007 - \frac{2,719.78}{290 + 460} - \frac{400,215}{(290 + 460)^2} = 1.76284$. Hence, $p = 57.92$ lb. per sq. in. Ans.

(618) Using formula **91,**

$H = 1,081.94 + 305\,t = 1,081.94 + .305 \times 318 = 1,178.93$ B. T. U. Ans.

(619) See Arts. **1217** and **1218.**

(620) Using formula **92,**

$$p\,v^{\frac{17}{16}} = 475, \text{ or } v^{\frac{17}{16}} = \frac{475}{p} = \frac{475}{81}.$$

Hence,

$$\tfrac{17}{16} \log v = \log 475 - \log 81 = 2.67669 - 1.90849 = .76820$$

$$\log v = .76820 \times \frac{16}{17} = .72301.$$

$$v = 5.2846 \text{ cu. ft.} = \text{volume of one pound.}$$

$$5.2846 \times 6 = 31.7076 \text{ cu. ft. Ans.}$$

(621) Using formula **92,**

$$p\,v^{\frac{17}{16}} = 475, \text{ or } p = \frac{475}{v^{\frac{17}{16}}} = \frac{475}{v^{\frac{17}{16}}}.$$

Hence, $\log p = \log 475 - \frac{17}{16} \log 7 = 1.77877$.

$p = 60.086$ lb. per sq. in. Ans.

(622) (*a*) Looking in the table of Properties of Saturated Steam, it is found that for a temperature of 320.094° the pressure is 90 pounds, and for the next lower pressure of the table, 88 pounds, the temperature is 318.510°. Hence, the difference in temperature for a difference of one pound in pressure is

$$\frac{320.094 - 318.510}{2} = .792°.$$

$320.094° - 320° = .094°$ = actual difference.

Then 1 lb. : .792° :: x lb. : .094, or $x = .12$ lb., nearly = difference in pressure between 320° and 320.094°.

$90 - .12 = 89.88$ lb. per sq. in., absolute pressure.

$89.88 - 14.7 = 75.18$ lb. per sq. in., gauge pressure. Ans.

(*b*) 110 lb. gauge pressure $= 110 + 14.7 = 124.7$ lb., absolute.

Temperature corresponding to 125 lb. = 344.136°.

Temperature corresponding to 120 lb. = 341.058°.

Difference, per pound difference in pressure =

$$\frac{344.136° - 341.058°}{5} = .616°.$$

$125 - 124.7 = .3$ lb., difference in pressure; $.616 \times .3 = .185°$.

$344.136 - .185 = 343.951°$. Ans.

(*c*) 70 lb., gauge pressure, $= 70 + 14.7 = 84.7$ lb., absolute.

Heat of liquid for 84 lb. = 285.414 B. T. U.

Heat of liquid for 86 lb. = 287.096 B. T. U.

$$\text{Difference per pound} = \frac{287.096 - 285.414}{2} = .841 \text{ B. T. U.}$$

$.841 \times .7 = .589$. $285.414 + .589 = 286.003$. B. T. U. Ans.

Total heat at 86 lb. = 1,178.592 B. T. U.

Total heat at 84 lb. = 1,178.091 B. T. U.

$$\text{Difference per pound} = \frac{1,178.592 - 1,178.091}{2} = .25 \text{ B. T. U.}$$

$.25 \times .7 = .175$. $1,178.091 + .175 = 1,178.266$ B. T. U. Ans.

(*d*) Latent heat per pound at 66 lb. pressure = 904.443 B. T. U.

Latent heat per pound at 68 lb. pressure = 903.02 B. T. U.

$$\text{Difference per pound} = \frac{904.443 - 903.02}{2} = .712 \text{ B. T. U.}$$

$904.443 - .712 = 903.731$ B. T. U., latent heat per pound at 67 lb. pressure.

$903.731 \times 3 = 2,711.193$ B. T. U. Ans.

(623) (*a*) The isothermal of saturated steam is a straight line parallel to the axis of volumes.

(*b*) The equilateral hyperbola.

(624) From the table of the Properties of Saturated Steam, the weight of one cubic foot of steam at a pressure of 44 lb. is .106345 lb., and at a pressure of 42 lb. it is .101794 lb.

$$\text{Difference per pound} = \frac{.106345 - .101794}{2} = .002276 \text{ lb.}$$

The weight of a cubic foot at 43 lb. pressure is

$.101794 + .002276 = .10407$ lb.

Weight of 38 cu. ft. $= .10407 \times 38 = 3.95466$ lb. Ans.

(625) See Arts. **1214** and **1215**.

(626) See Art. **1206**.

To raise 1 pound of water from 55° to 140° requires $140 - 55 = 85$ B. T. U. Then, to raise 300 pounds of water

through the same range of temperature requires 85 × 300 = 25,500 B. T. U.

60 pounds gauge pressure = 60 + 14.7 = 74.7 pounds absolute.

Total heat of 1 pound of steam at 74 pounds pressure = 1,175.431 B. T. U.

Total heat of 1 pound of steam at 76 pounds pressure = 1,175.985 B. T. U.

Difference per pound pressure $= \dfrac{1,175.985 - 1,175.431}{2} =$ .277 B. T. U.

.277 × .7 = .194. 1,175.431 + .194 = 1,175.625 B. T. U., total heat of 1 pound of steam at an absolute pressure of 74.7 pounds per sq. in.

The total heat (above 32°) of a pound of water at 140° is 140 − 32 = 108 B. T. U.

Hence, each pound of steam on being mixed with water lowers in temperature to 140°, and in so doing gives up 1,175.625 − 108 = 1,067.625 B. T. U.

The required amount of steam is

$$\frac{25,500}{1,067.625} = 23.885 \text{ lb.} \quad \textbf{Ans.}$$

(627) The temperature 256° does not appear in the table of the Properties of Saturated Steam. The next lower temperature is found to be 254.002°; and the next higher, 257.523°. The difference is 257.523 − 254.002 = 3.521°. The difference between the lower temperature and 256° is 256 − 254.002 = 1.998°. The volume of a pound of steam at the lower temperature is 12.68 cu. ft.; and at the higher temperature, 11.98 cu. ft. The difference is 12.68 − 11.98 = .7 cu. ft.

The difference between the volume at the lower temperature and the volume at 256° may now be found by proportion

$$3.521 : 1.998 :: .7 : x, \text{ or } x = .397.$$

Hence, the required volume of one pound of steam at 256° is

$$12.68 - .397 = 12.283 \text{ cu. ft. Ans.}$$

$$12.283 \times 4\frac{1}{3} = 53.226 \text{ cu. ft. Ans.}$$

(628) See Art. **1192.** Specific heat of superheated steam is .48. Hence, to raise 1 pound of superheated steam from 310° to 342° requires

$$.48 \times (342 - 310) = 15.36 \text{ B. T. U.}$$

To raise 6 pounds through the same range of temperature requires

$$15.36 \times 6 = 92.16 \text{ B. T. U. Ans.}$$

(629) (*a*) $\log p = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^2}$

$$= 6.1007 - \frac{2{,}719.78}{254 + 460} - \frac{400{,}215}{(254 + 460)^2}$$

$$= 1.50643.$$

$$p = 32.094 \text{ lb. per sq. in. Ans.}$$

From the tables, $p = 32$ lb. per sq. in. for a temperature of 254°.

(*b*) $\log p = 6.1007 - \frac{2{,}719.78}{377 + 460} - \frac{400{,}215}{(377 + 460)^2} = 2.27999,$

or $p = 190.54$ lb. per sq. in. Ans.

From the tables, $p = 189.212$ lb. per sq. in. for a temperature of 377°.

(630) See Arts. **1220** and **1221.**

(631) (*a*) $H = 1{,}081.94 + .305\ t$. From the table, $t = 385.759°$.

Hence, $H = 1{,}081.94 + .305 \times 385.759 = 1{,}199.596$ B. T. U. Ans.

From the tables, $H = 1{,}199.597$.

(*b*) $p = 88$; $t = 318.51°$.

Hence, $H = 1{,}081.94 + .305 \times 318.51 = 1{,}179.086$ B. T. U. Ans.

From the tables, $H = 1{,}179.085$.

(*c*) $p = 37$; $t = 260.883 + \frac{264.093 - 260.883}{2} = 262.488°$.

$H = 1{,}081.94 + .305 \times 262.488 = 1{,}161.999$ B.T.U. Ans.

From the tables, $H = 1{,}161.999$.

(632) See Art. **1293.**

(633) See Fig. 49. The lines $O\,X$ and $O\,Y$ are drawn at right angles. The scale of volumes chosen is 20 cu. ft. to the inch, and the scale of pressures 20 lb. to the inch. The vol-

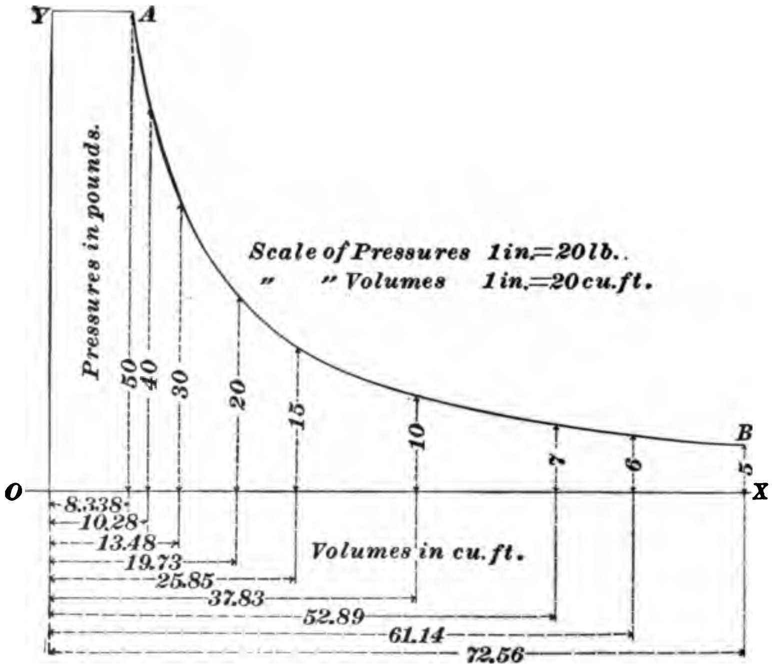


FIG. 49.

umes corresponding to 50, 40, 30 lb., etc., are found from the steam table, and laid off along $O\,X$. The pressures are then laid off vertically, as shown, forming the required curve $A\,B$.

(634) See Art. **1224.**

(635) (*a*) See Arts. **1252** to **1254**.

(*b*) Clearance $i = .07$; apparent cut-off $k_1 = \frac{3}{8} = .375$.

Then, by formula **97**, the real cut-off =

$$k = \frac{k_1 + i}{1 + i} = \frac{.375 + .07}{1 + .07} = \frac{.445}{1.07} = .416. \quad \text{Ans.}$$

By formula **96**, ratio of expansion, $e = \frac{1}{k} = \frac{1}{.416} = 2.4.$ Ans.

(636) By formula **97**, $k = \frac{k_1 + i}{1 + i} = \frac{.625 + .06}{1 + .06} = \frac{.685}{1.06}.$

By formula **96**, $e = \frac{1}{k} = \frac{1}{\frac{.685}{1.06}} = \frac{1.06}{.685} = 1.547.$

By formula **99**, M. E. P. $= \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p.$

$$P = 60 + 14.7 = 74.7 \text{ lb.}; \; p = 17 \text{ lb.}$$

Then, M. E. P. $= \frac{.9 \times 74.7\,(1 + 2.3 \log 1.547)}{1.547} - .9 \times 17 =$ 47.1 lb. per sq. in. Ans.

(637) (*a*) Using formula **98**,

I. H. P. $= \frac{PLAN}{33{,}000} = \frac{47.1 \times \frac{18}{12} \times 11^2 \times .7854 \times 210 \times 2}{33{,}000} =$ 85.45 H. P. Ans.

(*b*) $85.45 \times .83 = 70.92$ actual H. P. Ans.

(*c*) $85.45 - 70.92 = 14.53$ friction H. P. Ans.

(638) See Arts. **1232**, **1236**, **1244**, and **1245**.

(639) (*a*) See Arts. **1262** and **1276**.

(*b*) The three principal uses are the following:

1. To find the I. H. P. of the engine.
2. To detect defects in valve setting, and to serve as a guide in setting the valves.
3. To determine, approximately, the steam consumption of the engine.

(640) (*a*) Work per stroke per sq. in. of piston =

$$3.47 \times \frac{6}{12} \times 40 = 69.4 \text{ ft.-lb.}$$

Area of piston = $16^2 \times .7854 = 201.0624$ sq. in.

Then, total work per stroke = $69.4 \times 201.0624 = 13{,}953.73$ ft.-lb. Ans.

(*b*) 120 rev. per min. = $120 \times 2 = 240$ strokes per min.

$$\text{Then, H. P.} = \frac{13{,}953.73 \times 240}{33{,}000} = 101.48. \quad \text{Ans.}$$

(641) Admission, cut-off, release, and compression are all too late, and the back pressure is excessive. Admission, release, and compression may be made earlier by increasing the angular advance; cut-off may be made earlier by adding lap to the valve; the back pressure can be reduced only by reducing the resistance to the passage of the exhaust steam; that is, by enlarging the exhaust port or the exhaust pipe, or both.

(642) Assuming a mechanical efficiency of 80%, the I. H. P. of the engine must be $\frac{120}{.80} = 150$.

The probable M. E. P. may be found from formula **99.**

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p =$$

$$\frac{.9 \times 84.7\,(1 + 2.3 \log 4)}{4} - .9 \times 3 = 42.75 \text{ lb. per sq. in.}$$

Assume 550 feet per minute as a fair piston speed. We have, then, $P = 42.75$, and $L\,N$ = piston speed = 550. Substituting, in formula **98,** I. H. P. = $\frac{P\,L\,A\,N}{33{,}000}$, $150 = \frac{42.75 \times 550 \times A}{33{,}000}$, or $A = \frac{150 \times 33{,}000}{42.75 \times 550} = 210.52$ sq. in., the area of the piston. The diameter is, consequently, $\sqrt{\frac{210.52}{.7854}} =$ $16\frac{1}{2}''$, nearly. Assuming the stroke to be 42 in., the number of revolutions, from formula **100,** is

$$R = \frac{6\,S}{L} = \frac{6 \times 550}{42} = 78.57.$$

This is a fair number of revolutions for a Corliss engine. Hence, an engine $16\frac{1}{2}'' \times 42''$, making $78\frac{1}{2}$ rev. per min., will do the required work.

(643) (*a*) 60 pounds, gauge pressure, $= 60 + 14.7 =$ 74.7 lb., absolute; 2 lb. above atmosphere $= 2 + 14.7 =$ 16.7 lb., absolute.

Temperature of steam at 74.7 lb. pressure is, from the table of the Properties of Saturated Steam,

$$306.526 + \left(\frac{308.344 - 306.526}{2}\right).7 = 307.162^\circ.$$

Temperature of steam at 16.7 is

$216.347 + (219.452 - 216.347)\,.7 = 218.521^\circ.$ Then,

$$T_1 = 307.162^\circ + 460^\circ = 767.162^\circ$$

and $$T_2 = 218.521^\circ + 460^\circ = 678.521^\circ.$$

Thermal efficiency $= \dfrac{T_1 - T_2}{T_1} = \dfrac{767.162 - 678.521}{767.162} =$ $.1155 = 11.55\%$. Ans.

(*b*) In this case, the absolute pressure of the entering steam is $90 + 14.7 = 104.7$ pounds per sq. in., and the pressure of the exhaust steam 3 pounds per sq. in., absolute.

The temperature corresponding to the former pressure is from the table $= 331.169 - \left(\dfrac{331.169 - 327.625}{5}\right).3 = 330.956^\circ.$

The temperature corresponding to the latter pressure is 141.654°. Hence, T_1 is $330.956^\circ + 460^\circ = 790.956^\circ$, and T_2 is $141.654^\circ + 460^\circ = 601.654^\circ$.

Thermal efficiency $= \dfrac{790.956 - 601.654}{790.956} = .2393 = 23.93\%.$ Ans.

(644) (*a*) Using formula **100,**

$$L = \frac{6\,S}{R} = \frac{6 \times 540}{150} = 21.6''. \quad \text{Ans.}$$

(*b*) $R = \dfrac{6\,S}{L} = \dfrac{6 \times 900}{2\frac{1}{2} \times 12} = 180$ rev. per min. Ans

(645) The M. E. P. is found as shown in Fig. 50.

The middle ordinates are drawn as explained in Art. **1159.**

These ordinates are measured and multiplied by the scale of the spring, which reduces them from inches to pounds.

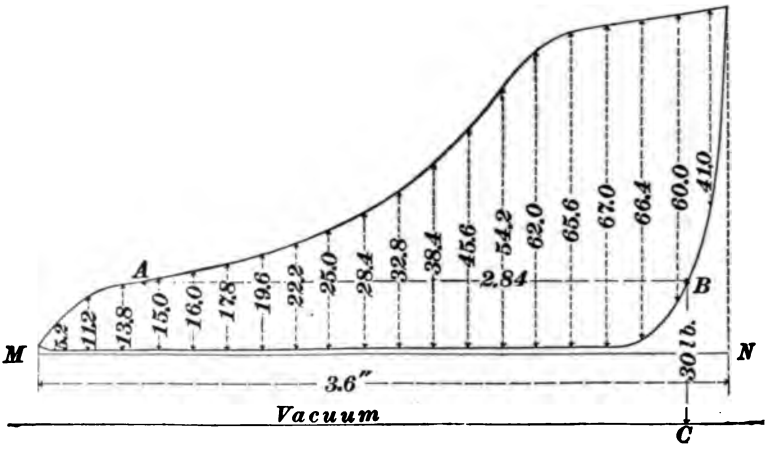


FIG. 50.

In the figure the sum of the 20 ordinates is 707.2 pounds. The mean ordinate, or, what is the same thing, the M. E. P., is, therefore, $\frac{707.2}{20} = 35.36$ lb. Draw the vacuum line at a distance of $\frac{14.7}{40} = .3675''$ below the atmospheric line MN. Choose the point A near the point of release, and draw AB parallel to MN. The height BC of this line above the vacuum line is $\frac{3''}{4}$; therefore, the absolute pressure at A or B is $\frac{3}{4} \times 40 = 30$ pounds. The length $AB = l = 2.84''$, and the length of the diagram L is $3.6''$. The weight of a cu. ft. of steam at 30 pounds pressure, absolute, is .074201 lb.

Substituting these values in formula **101**,

$$Q = \frac{13,750\, l\, W}{PL} = \frac{13,750 \times 2.84 \times .074201}{35.36 \times 3.6} =$$

22.76 lb. per I. H. P. per hour. Ans.

(646) (*a*) See Arts. **1293** and **1294.**

(*b*) See Arts. **1314** and **1315.**

(647) Area of high-pressure cylinder $= 19^2 \times .7854$ sq. in.

Area of low-pressure cylinder $= 32^2 \times .7854 = 804.25$ sq. in.

Ratio of area of high-pressure cylinder to area of low-pressure cylinder $= \frac{19^2 \times .7854}{32^2 \times .7854} = \frac{19^2}{32^2} = \frac{361}{1,024}$.

M. E. P. of high-pressure cylinder reduced to low-pressure cylinder $= 52 \times \frac{361}{1,024} = 18.332$ lb. per sq. in.

Total M. E. P. reduced to low-pressure cylinder $=$
$18 + 18.332 = 36.332$ lb. per sq. in.

(*a*) Substituting now in formula **98,**

$$\text{I. H. P.} = \frac{PLAN}{33,000} = \frac{36.332 \times \frac{30}{12} \times 804.25 \times 120 \times 2}{33,000} = 531.27.$$

Ans.

(*b*) Since the stroke is the same for each cylinder, the ratio of the work done by the two cylinders is proportional to the ratio of the two M. E. P.'s reduced to the low-pressure cylinder. Hence, the ratio of the work done in the high-pressure cylinder to that done in the low-pressure cylinder is $18.332 : 18 = 1.0184$. Ans.

(648) (*a*) See Art. **1276.**

(*b*) 1. Decrease the angular advance.

2. Increase the angular advance.

3. Lower the boiler pressure or decrease the number of revolutions.

4. Raise the boiler pressure or increase the number of revolutions.

(649) (*a*) The points desired are shown in Fig. 51. To locate the point of cut-off, prolong the steam and expansion

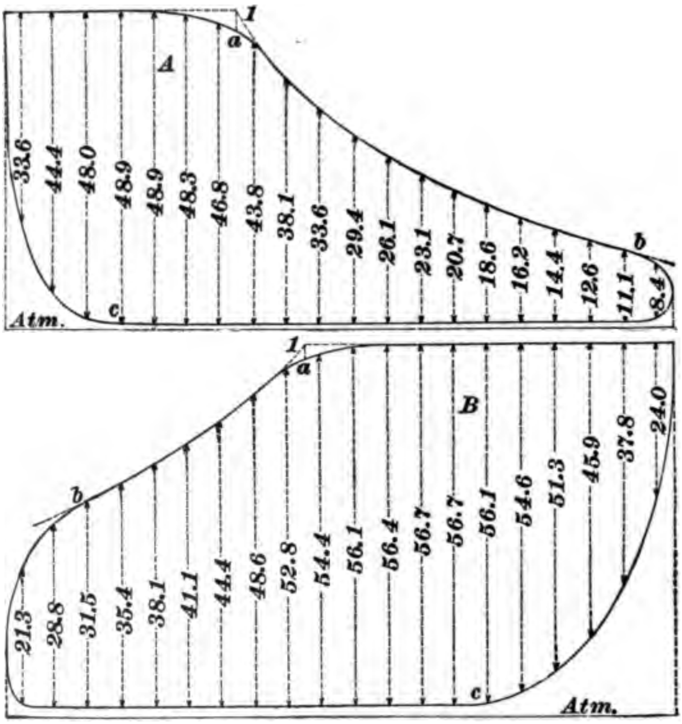


FIG. 51.

lines till they intersect, as at *1*. From *1* drop the perpendicular *1 a*, and *a* will be the point of cut-off.

The point of release may be located by prolonging the expansion curve, and noting the point where the actual curve departs from it, as shown at *b*. The point of compression *c* is easily located.

(*b*) The M. E. P. of the two diagrams are found as shown in the figure. The length of each diagram is divided into 20 equal parts, and ordinates are erected at the middle points of the divisions. The lengths of these ordinates multiplied by the scale of the spring used, in this case, 1" = 30 lb., added together and divided by 20 gives the required M. E. P. The

M. E. P. of diagram A is found to be 30.75 lb. per sq. in., and the M. E. P. of diagram B is found to be 44.6 lb. per sq. in. Ans.

(650) Using formula **106,**

$S = .0944\,W = .0944 \times 675 \times 26\frac{1}{2} = 1{,}688.58$ sq. ft. Ans

(651) Area of piston $= .7854 \times 30^2 = 706.86$ sq. in. The piston speed is, from formula **100,** $S = \frac{LR}{6} = \frac{48 \times 80}{6} = 640$ ft. per min.; hence, the velocity of the crank-pin ($= v_0$) is $\frac{640 \times 3.1416}{2} = 1{,}005.3$ ft. per min. $= \frac{1{,}005.3}{60} = 16.755$ ft. per sec. The ratio n between the radius of the fly-wheel and length of crank is $\frac{10}{2} = 5$. Substituting now in formula **107,**

$$W = \frac{AHg}{n^2 E V_0^2} = \frac{706.86 \times 88 \times 32.16}{5^2 \times \frac{1}{50} \times 16.755^2} = 14{,}251.92 \text{ lb. Ans.}$$

(652) (*a*) Use formula **98.**

$\text{I. H. P.} = \frac{PLAN}{33{,}000}$, or $A = \frac{33{,}000 \text{ I. H. P.}}{PLN} = \frac{33{,}000 \times 1{,}200}{42 \times \frac{18}{12} \times 70 \times 2} = 1{,}924.2$ sq. in., area of low-pressure piston. Diameter of low-pressure cylinder $= \sqrt{\frac{1{,}924.2}{.7854}} = 49\frac{1}{2}''$. Ans.

(*b*) Since the stroke is the same for both cylinders, their volumes are proportional to the areas of their pistons. Hence, using formula **103,**

$\frac{V}{v} = \frac{\text{area of low-pressure piston}}{\text{area of high-pressure piston}} = \frac{E}{2.72}$, or area of high-pressure piston $= \frac{2.72 \times 1{,}924.2}{5.5} = 951.6$ sq. in.

The corresponding diameter is

$$\sqrt{\frac{951.6}{.7854}} = 34\frac{13}{16}'', \text{ or say } 34\frac{7}{8}''. \quad \text{Ans.}$$

Using formula **104,**

$$\frac{V}{v} = \frac{\text{area of low-pressure piston}}{\text{area of high-pressure piston}} = \sqrt{E} = \sqrt{5.5}.$$

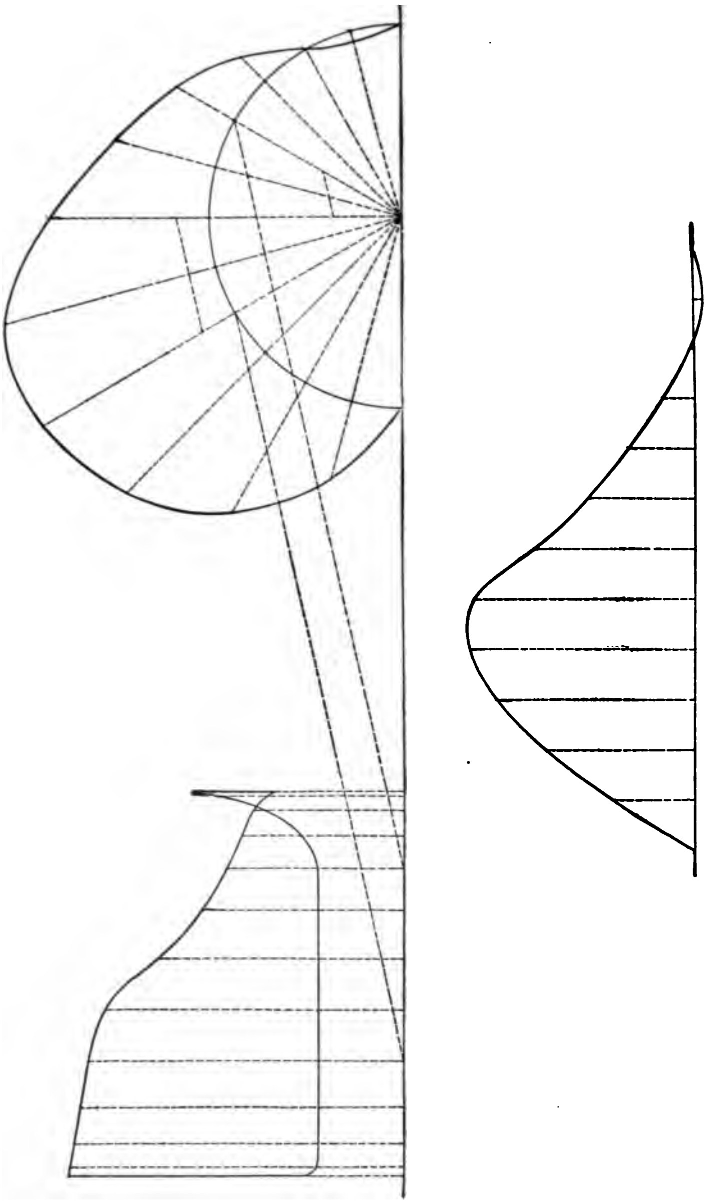


FIG. 52.

Hence, area of high-pressure piston $= \frac{1,924.2}{\sqrt{5.5}} = 820.48$ sq. in.

The corresponding diameter is $32\frac{5''}{16}$, say $32\frac{3''}{8}$. Ans.

(653) (*a*) By formula **102**, $E = \frac{e\,V}{v}$, or $e = \frac{E\,v}{V}$; but, by formula **103**, $\frac{V}{v} = \frac{E}{2.72}$, or $E = 2.72\frac{V}{v}$. Substituting this value of E in **102**, we have, $e = 2.72\frac{V}{v}\,\frac{v}{V} = 2.72$. Hence, the real cut-off $= \frac{1}{e} = \frac{1}{2.72} = .368$. Ans.

(*b*) From formula **104**, $\frac{V}{v} = \sqrt{E}$, or $\frac{v}{V} = \frac{1}{\sqrt{E}}$.

Substituting this value of $\frac{v}{V}$ in formula **102**, above,

$$e = \frac{E}{\sqrt{E}} = \sqrt{E} = \sqrt{5.5} = 2.345.$$

Real cut-off $= \frac{1}{e} = \frac{1}{2.345} = .426$. Ans.

(654) See Art. **1330**.

(655) The construction is shown in Fig. 52. It is precisely similar to the construction shown in Art. **1313**, and need not be further described.

(656) The total heat of steam at $4\frac{1}{2}$ pounds pressure is, from the steam tables,

$$\frac{1,128.641 + 1,131.462}{2} = 1,130.051 \text{ B. T. U.}$$

Now, using formula **105**,

$$W = \frac{H - t_2 + 32}{t_1 - t_2} = \frac{1,130.051 - 130 + 32}{130 - 55} = 13.76 \text{ lb. Ans.}$$

(657) See Art. **1291**.

(658) (*b*) Using formula **98**, I. H. P. $= \frac{PLAN}{33,000}$, or

$$LA = \frac{33,000 \text{ I. H. P.}}{PN} = \frac{33,000 \times 42}{36.3 \times 155 \times 2}$$

L = stroke in feet.

$12\,L$ = stroke in inches.

From the conditions of the problem, $12\,L = 1\frac{1}{3}d$, when d is taken in inches, or $L = \frac{1}{9}d$. $A = .7854\,d^2$. Substituting these values of L and A,

$$\frac{1}{9}d \times .7854\,d^2 = \frac{.7854\,d^3}{9} = \frac{33,000 \times 42}{2 \times 36.3 \times 155},$$

or $d = \sqrt[3]{\frac{33,000 \times 42 \times 9}{36.3 \times 155 \times 2 \times .7854}} = 11.22''$ nearly, or, say $11\frac{1''}{4}$. Ans.

(a) Stroke $= 1\frac{1}{3}d = 11\frac{1}{4} \times 1\frac{1}{3} = 15''$, nearly. Ans.

(c) Using formula **100**,

$$S = \frac{LR}{6} = \frac{15 \times 155}{6} = 387.5 \text{ ft. per min.} \quad \text{Ans.}$$

(659) (a) Area of piston, $.7854 \times 22^2 = 380.1336$ sq. in. Total pressure on piston $= 380.1336 \times 72 = 27,369.62$ lb. At the beginning of the stroke, the piston rod, connecting-rod, and crank lie in the same straight line, and, consequently, the total pressure on the piston is transmitted to the crank-shaft.

(b) In this case the total pressure on the piston is $380.1336 \times 65 = 24,708.68$ lb., which is also the horizontal pressure on the crank-pin.

Then, the pressure on the crank-shaft is $24,708.68 \times \cos 60° = 12,354.34$ lb. Ans.

(c) The tangential pressure is $24,708.68 \times \sin 60° = 24,708.68 \times .86603 = 21,398.46$ lb. Ans.

(660) See Art. **1311.**

The mean ordinate should be $34.6 \times \frac{2}{3.1416} = 22.03$ lb. per sq. in. Ans.

(661) Diameter of drivers $= 80'' = \frac{80}{12} = 6\frac{2}{3}$ ft.

Circumference of drivers $= 6\frac{2}{3} \times \pi = 20.944$ ft.

60 miles per hour = 1 mile per min.

One mile contains 5,280 ft. Hence, the number of revolutions of the driver per min. is $\frac{5,280}{20.944}$.

Area of piston $= .7854 \times 19^2 = 283.53$.

Now, using formula **98**,

$$\text{I.H.P.} = \frac{PLAN}{33,000} = \frac{52.5 \times \frac{24}{12} \times 283.53 \times \frac{5,280}{20.944} \times 2}{33,000} = 454.86.$$

Since there are two cylinders, the total horsepower $= 454.86 \times 2 = 909.72$ I. H. P. Ans.

(662) (*a*) As shown in Fig. 53, the diameter of the path described by the eccentric is the travel of the valve, 5''.

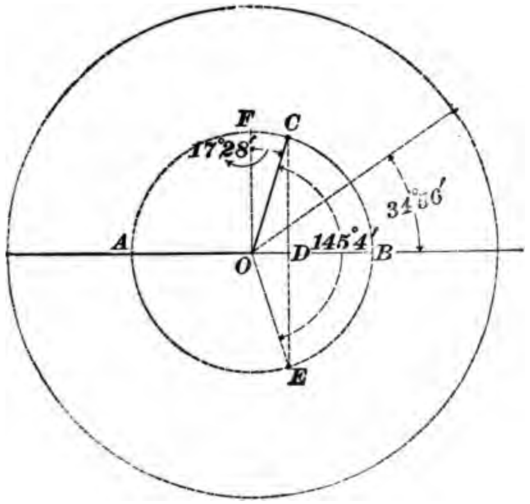


FIG. 53.

The eccentric crank must be in a position $O\ C$, so that, upon

dropping the perpendicular $C D$ upon $A B$, the distance $O D$ will equal the lap $\frac{3''}{4}$. Then, $\cos C O D = \frac{O D}{O C} = \frac{\frac{3}{4}}{2\frac{1}{2}} = .3$.

The angle whose cosine is .3 is 72° 32′, nearly. Hence, $C O D = 72° 32'$, and the angular advance $= C O F = 90° - C O D = 90 - 72° 32' = 17° 28'$.

(*b*) When cut-off occurs the valve must be in the same position as it was at the beginning of the stroke, but moving in the opposite direction. Hence, the center of the eccentric must be at E vertically below C, and must have passed through the angle $C O E = 2\ C O B = 2 \times 72° 32'' = 145° 4'$. Therefore, the crank has moved 145° 4′ from the dead-center position, and now makes an angle of $\frac{145° 4'}{2} = 72° 32'$, with $A B$, the diameter through the dead points.

(663) (*a*) Efficiency $= \frac{12.325}{15.36} = .8024 = 80.24\%$. Ans.

(*b*) Area of piston $= .7854 \times 9^2 = 63.617$ sq. in.

Stroke $= 12'' = 1$ foot.

Using formula **98**, I.H.P. $= \frac{PLAN}{33,000}$, or

$$P = \frac{\text{I.H.P.} \times 33,000}{LAN} = \frac{15.36 \times 33,000}{1 \times 63.617 \times 240 \times 2} = 16.6 \text{ lb.}$$

(664) See Arts. **1223** and **1224.**

(665) The construction is shown in Fig. 54. Choose a suitable length for the diagram, say 3 inches. Draw the atmospheric line $M N$. Choose a scale of pressures, say 40 pounds per inch, and draw the vacuum line $O X$ parallel to MN and $\frac{14.7}{40} = .3675''$ below it. On $O X$ lay off the length of the diagram $A B = 3''$. This length represents to some scale the volume of the cylinder, and 7% of this length must then represent to the same scale the clearance. Therefore,

from A, lay off $A\,O$ equal to 7% of $A\,B = 3 \times .07 = .21''$, and from O draw $O\,Y$ perpendicular to $O\,X$. Lay off $M\,C = \frac{70}{40} = 1\frac{3}{4}''$ to represent the boiler pressure, and through C draw $C\,E$ parallel to $O\,B$. The number of expansions is 3. Hence, lay off $C\,D = \frac{1}{3}\,O\,B$, and D will be the point of cut-off. Through D, draw the equilateral hyperbola $D\,F$,

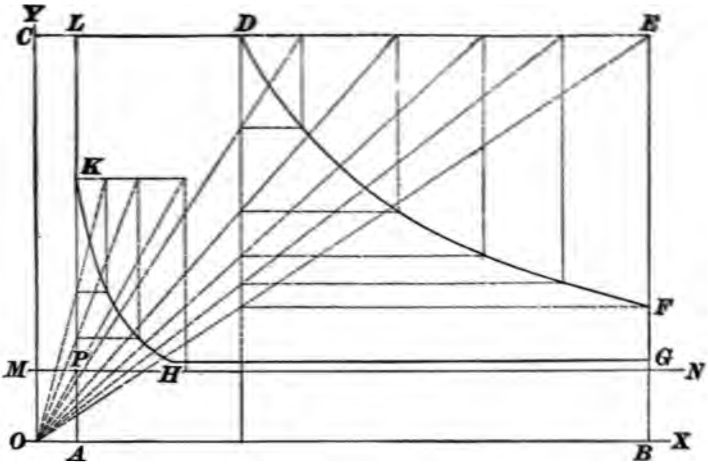


FIG. 54.

as explained in Art. **1161**, taking O as the point from which to draw the radial lines. $D\,F$ is the expansion line. The back pressure is 2 lb. above the atmosphere. Hence, the line $G\,H$ drawn parallel to $M\,N$ and $\frac{2}{40} = \frac{1''}{20}$ above 1 will represent the back-pressure line of the diagram. Since the steam is compressed to 40 lb., the point where compression ends may be found by measuring $P\,K = \frac{40}{40} = 1''$ above $M\,N$. Through K, the equilateral hyperbola $K\,H$ is drawn in the same manner as $D\,F$. Draw $K\,L$ perpendicular to $M\,N$, and the diagram is finished.

(666) See Art. **1308.**

(667) To obtain an idea of the condensation of the steam in the cylinder. There is generally more condensation at cut-off than at release, on account of reevaporation, and if the steam consumption be calculated from both points, the difference will tell more or less, approximately, the amount of condensation at cut-off.

(668) The data assumed will, of course, depend largely upon circumstances. Suppose the engine to be non-condensing, and to run with a piston speed of 600 feet per minute, and use steam at a boiler pressure of 75 lb. The mechanical efficiency will probably be about 80%, which requires the engine to develop $\frac{240}{.80} = 300$ I. H. P. Assuming the number of expansions to be $2\frac{1}{2}$, the probable M. E. P. is, from formula **99**, M. E. P. $= \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p = \frac{.9 \times 89.7\,(1 + 2.3 \log 2.5)}{2.5} - .9 \times 17 = 46.55.$

Now, using formula **98**, I. H. P. $= \frac{PLAN}{33{,}000}$, or $A = \frac{33{,}000 \text{ I. H. P.}}{PLN}$. But $LN =$ piston speed $= 600$ ft.

Hence, $A = \frac{33{,}000 \times 300}{46.55 \times 600} = 354.46$ sq. in. = area of piston.

The diameter corresponding to this area is $21\frac{1}{4}''$.

For this diameter a fair stroke would be 30″, the number of revolutions being from formula **100**,

$$R = \frac{6\,S}{L} = \frac{6 \times 600}{30} = 120.$$

The engine would be made $21\frac{1}{4}'' \times 30''$, and run at 120 rev. per min. Ans.

(669) The scale of the spring should be in general not less than $\frac{1}{2}$ of the boiler pressure.

(*a*) $\frac{54}{2} = 27$; hence, a 30 spring should be used.

(*b*) $\frac{115}{2} = 57\frac{1}{2}$; hence, a 60 spring should be used.

(*c*) $1.834'' \times 40 = 73.36$ lb. per sq. in. Ans.

(670) Weight of condensing water per pound of steam $= \frac{13,580}{906}$.

Total heat of steam at an absolute pressure of 7 pounds is, from the table, 1,135.908. Substituting in formula **105**, $W = \frac{H - t_3 + 32}{t_1 - t_2}$, or $t_3 = H + 32 - W(t_1 - t_2)$, we obtain

$$t_3 = 1,135.908 + 32 - \frac{13,580}{906}(120 - 52) = 148.66°. \text{ Ans.}$$

(671) The areas of the three pistons are proportional to the squares of their respective diameters. Hence, the M. E. P. of the high-pressure cylinder reduced to the low-pressure cylinder is $72 \times \frac{27^2}{66^2} = 12.05$ lb. per sq. in. The M. E. P. of the intermediate cylinder reduced to the low-pressure cylinder is $40 \times \frac{42^2}{66^2} = 16.2$ lb. per sq. in.

The total M. E. P. reduced to the low-pressure cylinder is, consequently, $12.05 + 16.2 + 16.5 = 44.75$ lb. per sq. in.

Area of low-pressure piston $= .7854 \times 66^2 = 3,421.2$ sq. in.

(*a*) Substituting, now, in formula **98**,

$$\text{I. H. P.} = \frac{PLAN}{33,000} = \frac{44.75 \times 4 \times 3,421.2 \times 70 \times 2}{33,000} = 2,598. \text{ Ans.}$$

(*b*) The work done in each cylinder is proportional to the M. E. P. of the cylinder reduced to the low-pressure cylinder. Hence, the percentage of the total work done in the high-pressure cylinder is $\frac{12.05}{44.75} = .269 = 26.9\%$. Ans. The percentage of the work done in the intermediate cylinder is $\frac{16.2}{44.75} = .362 = 36.2\%$. Ans. Lastly, the percentage of the work done in the low-pressure cylinder is $\frac{16.5}{44.75} = .369 = 36.9\%$. **Ans.**

(672) See Art. **1287.**

(673) The number of expansions is determined from formulas **97** and **96.**

$$k = \frac{k_1 + c}{1 + c} = \frac{.25 + .025}{1 + .025} = \frac{.275}{1.025}.$$

$$r = \frac{1}{k} = \frac{1}{\frac{.275}{1.025}} = \frac{1.025}{.275} = 3.727.$$

(a) The M. E. P. may now be found from formula **99.**

$$\text{M. E. P.} = \frac{.9P(1 + 2.3 \log r)}{r} - .9p =$$

$$\frac{.9 \times 98.7(1 + 2.3 \log 3.727)}{3.727} - .9 \times 3 = 52.455 \text{ lb. per sq. in.}$$

Ans.

(b) Using formula **98,** $\text{I. H. P.} = \frac{PLAN}{33,000}$, or $A = \frac{33,000 \times \text{I. H. P.}}{PLN} = \frac{33,000 \times 120}{52.455 \times 500} = 151$ sq. in. The required diameter is, therefore, $\sqrt{\frac{151}{.7854}} = 13\frac{7}{8}''$. Ans.

(674) Since the gauge pressure is 93 lb., the absolute pressure is $93 + 14.7 = 107.7$ lb. Substituting the logarithm of 107.7 in formula **90,**

$$2.03222 = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2}.$$

Clearing of fractions and transposing,

$$4.06848\ T^2 - 2,719.78\ T = 400,215.$$

As this is an affected quadratic, it may be solved by the rule given in algebra; hence, dividing by the coefficient of T^2, $T^2 - 668.5\ T = 98,369.66$. Completing the square $T^2 - 668.5\ T + 111,723.2 = 210,092.86$. Extracting square root, $T - 334.25 = 458.359$, or $T = 792.609°$. Therefore, $792.609 - 460 = 332.609°$. Ans.

(675) See rule in Art. **1246,** and the answer to question 662.

(676) The volume discharged in 1 stroke is

$$\frac{18^2 \times .7854 \times 24}{1,728} \text{ cu. ft.},$$

and the volume discharged in 1 minute is

$$\frac{18^2 \times .7854 \times 24 \times 175 \times 2}{1,728}.$$

Formula **93** gives the work in foot-pounds which is done in 1 stroke; hence, if multiplied by the number of strokes per minute and divided by 33,000, the result will be the horsepower. The pressure is 62.4 lb. Substituting these values of the pressure and volume, in formula **93**, and dividing by 33,000, we have

$$\frac{144 \times 62.4 \times 18^2 \times .7854 \times 24 \times 175 \times 2}{1,728 \times 33,000} = 336.825 \text{ horsepower. Ans.}$$

(677) Dividing the diagram into 10 equal parts, and measuring the ordinates drawn at the middle of these divisions, the sum of their lengths for diagram A is 7.16″, for diagram B, 7.24″, and for diagrams C and D, 4.68″ and 4.72″, respectively. Dividing these sums for A and B by 10 and multiplying by 60, the scale of spring, we have,

for A, $7.16 \times \frac{60}{10} = 42.96$ lb. per sq. in.;

for B, $7.24 \times \frac{60}{10} = 43.44$ lb. per sq. in.

Adding and dividing by 2, the mean effective pressure for the high-pressure cylinder is found to be $\frac{42.96 + 43.44}{2} = 43.2$ lb. per sq. in.

The M. E. P. for the low-pressure cylinder is,

for C, $4.68 \times \frac{30}{10} = 14.04$ lb. per sq. in.;

for D, $4.72 \times \frac{30}{10} = 14.16$ lb. per sq. in.

Adding and dividing by 2, the M. E. P. for the low-pressure cylinder is $\frac{14.04 + 14.16}{2} = 14.1$ lb. per sq. in.

Reducing the M. E. P. of the high-pressure cylinder to the area of the low-pressure piston, we have $43.2 \times \frac{13^2 \times .7854}{20^2 \times .7854} = 18.252$ lb. per sq. in.

Hence, $50.6 - 18.252 = 32.352$ lb. per sq. in. = the M. E. P. if the work was all done in the low-pressure cylinder.

Therefore, I. H. P. $= \frac{PLAN}{33,000} =$

$$\frac{32.352 \times \frac{15}{12} \times 20^2 \times .7854 \times 230 \times 2}{33,000} = 177.1 \text{ I. H. P. Ans.}$$

(678) Fig. 55 shows diagram A and diagram D drawn as mentioned in Art. **1305**; that is, the scale of pressures

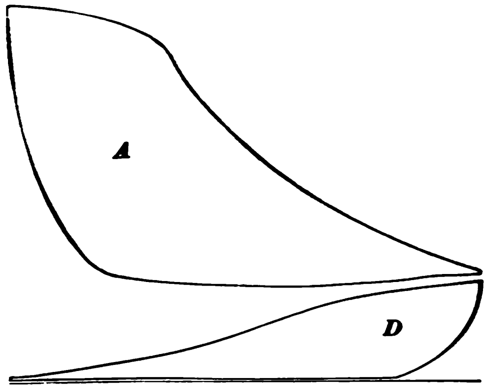


FIG. 55.

on diagram D has been reduced to 60, the same as on dia gram A. In Fig. 56 the length of diagram A has been reduced in the proportion of the ratio of the volume of the high-pressure cylinder to that of the low-pressure cylinder. By reading Arts. **1304** and **1305**, the student should have no trouble in drawing the diagrams by aid of the above figures.

(679) Using a scale divided into thirtieths of an inch, and locating for this case a point $\frac{28''}{30}$ above the vacuum line

(which should be drawn previous to this), we draw through this point a line parallel to the atmospheric line. The length of the portion of this line included between the

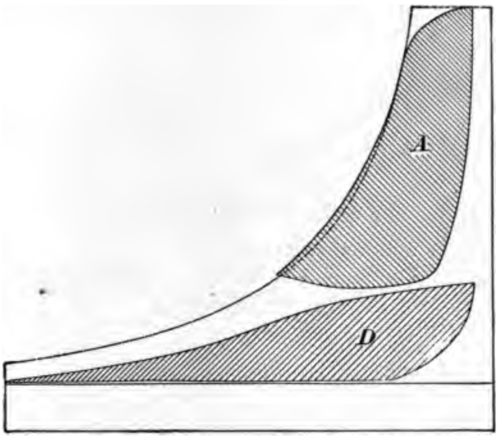


FIG. 56.

bounding lines of the diagram is 2.95″, and the absolute pressure which it represents is 28 lb. The whole length of the diagram is $3\frac{1}{2}''$. The weight of a cubic foot of steam at this pressure is, from the table of the Properties of Saturated Steam, .069545 lb. The M. E. P. was found to be 30.75 lb. per sq. in., in example 649.

Substituting these values in formula **101**,

$Q = \frac{13,750 \times 2.95 \times .069545}{30.75 \times 3.5} = 26.21$ lb. per I. H. P. per hour. Ans.

(680) For (*a*) and (*b*) see Art. **1292.** (*c*) No. The stroke may be so short that the piston speed will be low, although the rotative speed is high.

(681) The absolute pressure $= 123 + 14.7 = 137.7$ lb. Logarithm of $137.7 = 2.13893$. Substituting in formula **90**,

$$2.13893 = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2}.$$

[illegible]

[illegible]

[illegible] the diagram shown [illegible] is not drawn to scale. [illegible] equal parts and measuring the [illegible] the middle of those parts, the sum of [illegible] and the average length [illegible] The length of a line par- [illegible] measured between the point [illegible] Hence, area of [illegible] = .[illegible]3 [illegible] Dividing [illegible] equal parts, [illegible] = mean [illegible] length from point [illegible] to line [illegible] = .3[illegible] [illegible] = [illegible] sq. in. Subtracting area [illegible] from area [illegible] − [illegible] = [illegible] sq. in. The [illegible] Hence the mean pressure urging the piston ahead is [illegible] = [illegible] lb. per sq. in. Ans.

(688) This example is worked like the previous examples where the horsepower is to be found from the diagrams. Dividing the length of the diagrams into ten equal parts, and measuring the middle ordinates, their sum for diagram

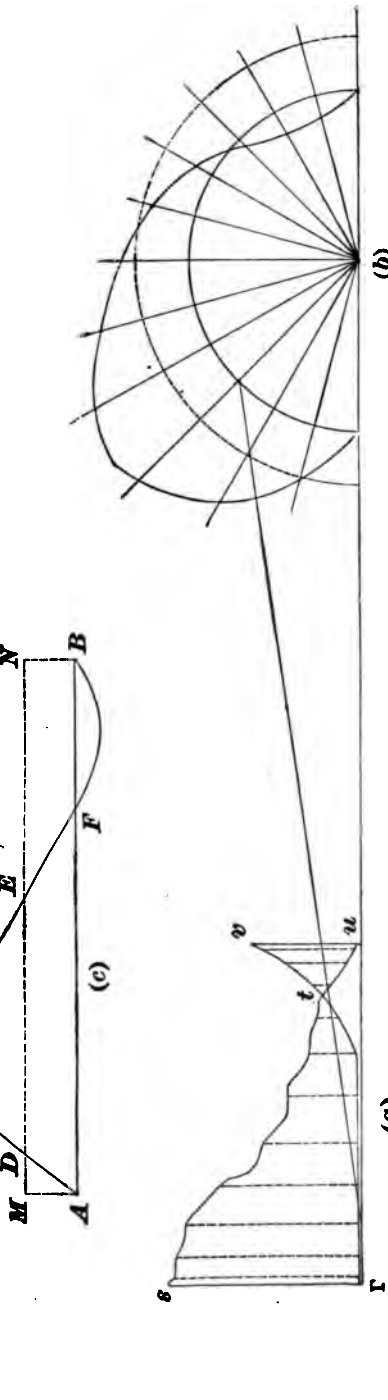


FIG. 57

A is found to be 6.04″, and for B, 6.62″. The mean ordinate for $A = \frac{6.04}{10} = .604$, and for B, $\frac{6.62}{10} = .662$. Hence, the total M. E. P. $= \frac{.604 + .662}{2} \times 60 = 37.98$ lb. per sq. in.

$$\text{I. H. P.} = \frac{37.98 \times 1 \times 13^2 \times .7854 \times 300 \times 2}{33,000} = 91.66. \quad \text{Ans.}$$

(686) Drawing a line parallel to the atmospheric line, and at a distance from the vacuum line corresponding to a pressure of 40 lb., absolute, the length included between the bounding lines of diagram A is 1.46″, and the length between the bounding lines of diagram B is 1.72″. The length of both diagrams is 2.6″. The weight of a cubic foot of steam at an absolute pressure of 40 lb. is .097231 lb. The M. E. P. for card $A = .604 \times 60 = 36.24$ lb., and for card B, $.662 \times 60 = 39.72$ lb. (See example 685.) Substituting in formula **101**, we have, for card A,

$$Q = \frac{13,750 \times 1.46 \times .097231}{36.24 \times 2.6} = 20.716 \text{ lb.},$$

and for card B,

$$Q = \frac{13,750 \times 1.72 \times .097231}{39.72 \times 2.6} = 22.267 \text{ lb.}$$

Taking the average of both cards,

$$Q = \frac{20.716 + 22.267}{2} = 21.49 \text{ lb. per I. H. P. per hour.} \quad \text{Ans.}$$

(687) In Fig. 57 is shown the solution to this question. The cut is not drawn to scale, and the student should redraw it full size. By referring to Art. **1313,** the student should have no difficulty.

Find the areas of $A\ C\ F$ and $F B$; subtract the smaller from the larger, and divide by the length $A\ B$; the result is the height $A\ M$. Draw $M N$ parallel to $A\ B$; also the dotted semicircle, at a distance from the full semicircle equal to $A\ M$.

(688) Compute the area of $D\ C\ E\ D$ in the usual manner; for the present case it equals .78 sq. in. The stroke

of the engine $= 12'' = 1$ ft. and the crank-pin travels in one stroke a distance $= \frac{1 \times 3.1416}{2} = 1.5708$ feet. The length $A\,B = 4.08''$. Consequently, $1''$ of length on $A\,B = \frac{1.5708}{4.08} =$ .385 ft. of crank-pin travel. Since the vertical scale of pressures is $1'' = 60$, H, in formula **107**, $= .78 \times 60 \times .385 =$ 18.018 lb. per sq. in. of piston area. Substituting in formula **107**, we have $W = \frac{A\,H\,g}{n^2\,E\,v_c^2}$

$$= \frac{13^2 \times .7854 \times 18.018 \times 32.16}{5^2 \times \frac{1}{50} \times \left(\frac{300 \times 2 \times 1 \times 3.1416}{2 \times 60}\right)^2} = 623 \text{ lb. Ans.}$$

STRENGTH OF MATERIALS.

(QUESTIONS 689–748.)

(689) See Arts. **1336, 1339,** and **1338.**

(690) See Arts. **1344, 1345, 1352,** and **1354.**

(691) See Art. **1347.**

(692) Use formula **110.**

$$E = \frac{Pl}{Ae}; \text{ therefore, } e = \frac{Pl}{AE}.$$

$A = .7854 \times 2^2$; $l = 10 \times 12$; $P = 40 \times 2{,}000$; $E = 25{,}000{,}000.$

Therefore, $e = \dfrac{40 \times 2{,}000 \times 10 \times 12}{.7854 \times 4 \times 25{,}000{,}000} = .12223''$. Ans.

(693) Using formula **110,**

$E = \dfrac{Pl}{Ae} = \dfrac{7{,}000 \times 7\frac{1}{2}}{.7854 \times (\frac{1}{2})^2 \times .009} = 29{,}708{,}853.2$ lb. per sq. in. Ans.

(694) Using formula **110,**

$E = \dfrac{Pl}{Ae}$, or $P = \dfrac{AeE}{l} = \dfrac{1\frac{1}{2} \times 2 \times .006 \times 15{,}000{,}000}{9 \times 12} =$ 2,500 lb. Ans.

(695) By formula **110,**

$E = \dfrac{Pl}{Ae}$, or $l = \dfrac{AeE}{P} = \dfrac{.7854 \times 3^2 \times .05 \times 1{,}500{,}000}{2{,}000} = 265.07''$. Ans.

(696) Using a factor of safety of 4 (see Table 28), formula **108** becomes

$$P = \frac{A\,S_1}{4}, \text{ or } A = \frac{4\,P}{S_1} = \frac{4 \times 6 \times 2{,}000}{55{,}000} = .8727272 \text{ sq. in.}$$

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{.8727272}{.7854}} = 1.054''. \quad \text{Ans.}$$

(697) From Table 23, the weight of a piece of cast iron 1″ square and 1 ft. long is 3.125 lb.; hence, each foot of length of the bar makes a load of 3.125 lb. per sq. in. The breaking load—that is, the ultimate tensile strength—is 20,000 lb. per sq. in. Hence, the length required to break the bar is $\frac{20{,}000}{3.125} = 6{,}400$ ft. Ans.

(698) Let t = the thickness of the bolt head;
d = diameter of bolt.
Area subject to shear $= \pi\, d\, t$.

Area subjected to tension $= \frac{1}{4}\pi\, d^2$.

$S_1 = 55{,}000$. $S_2 = 50{,}000$.

Then, in order that the bolt shall be equally strong in both tension and shear, $\pi\, d\, t\, S_2 = \frac{1}{4}\pi\, d^2\, S_1$,

$$\text{or } t = \frac{\pi\, d^2\, S_1}{4\,\pi\, d\, S_2} = \frac{d\,S_1}{4\,S_2} = \frac{\frac{3}{4} \times 55{,}000}{4 \times 50{,}000} = .206''. \quad \text{Ans.}$$

(699) Using a factor of safety of 15 for brick, formula **108** gives

$$P = \frac{A\,S_2}{15}.$$

$A = (2\frac{1}{2} \times 3\frac{1}{2})$ sq. ft. $= 30 \times 42 = 1{,}260$ sq. in.; $S_2 = 2{,}500$.

Therefore, $P = \frac{1{,}260 \times 2{,}500}{15} = 210{,}000$ lb. $= 105$ tons. Ans.

(700) The horizontal component of the force P is $P \cos 30° = 3{,}500 \times .866 = 3{,}031$ lb. The area A is $4\,a$, the ultimate shearing strength, S_s, 600 lb., and the factor of safety, 8.

Hence, from formula **108,**

$$P = \frac{A S_3}{8} = \frac{4 a S_3}{8} = \frac{a S_3}{2}. \quad a = \frac{2P}{S_3} = \frac{2 \times 3{,}031}{600} = 10.1''. \text{ Ans.}$$

(701) See Art. **1366.**

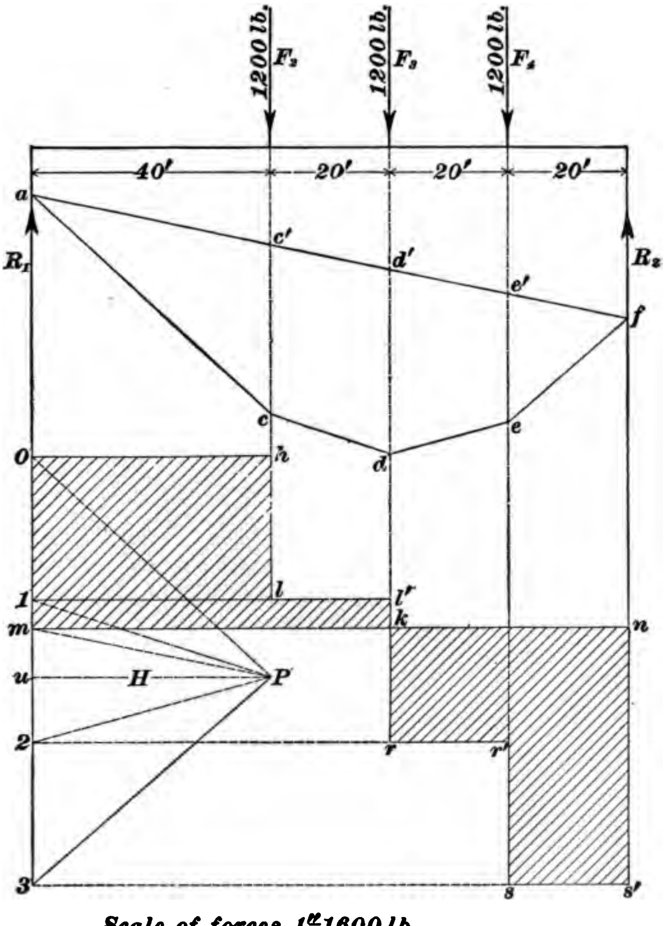


FIG. 58.

(702) Using formula **111,** with the factor of safety 4

$$p d = \frac{2 t S_1}{4} = \frac{t S_1}{2}, \text{ or } t = \frac{2 p d}{S_1} = \frac{2 \times 120 \times 48}{55{,}000}.$$

Since 40% of the plate is removed by the rivet holes, 60% remains, and the actual thickness required is

$$\frac{t}{.60} = \frac{2 \times 120 \times 48}{.60 \times 55,000} = .349''. \quad \text{Ans.}$$

(703) Using a factor of safety of 6, in formula **111**,

$$pd = \frac{2 t S_1}{6} = \frac{t S_1}{3}.$$

Hence, $t = \frac{3pd}{S_1} = \frac{3 \times 6 \times 200}{20,000} = .18''$. Ans.

(704) Using formula **114**, with a factor of safety of 10,

$$p = \frac{9,600,000\, t^{2.18}}{10\, l d} = 960,000 \frac{t^{2.18}}{l d}.$$

Hence, $t = \sqrt[2.18]{\frac{p l d}{960,000}} = \sqrt[2.18]{\frac{130 \times 12 \times 12 \times 3}{960,000}} = .272''$. Ans.

(705) From formula **113**,

$$p = \frac{S t}{r + t}, \text{ or } t = \frac{p r}{S - p} = \frac{2,000 \times \frac{1}{2}}{2,800 - 2,000} = \frac{4,000}{800} = 5''. \quad \text{Ans.}$$

(706) See Fig. 58. (*a*) Upon the load line, the loads *O*, *1*, *1-2*, and *2-3* are laid off equal, respectively, to F_2, F_3, and F_4; the pole P is chosen, and the rays drawn in the usual manner; the pole distance $H = 2,000$ lb. The equilibrium polygon is constructed by drawing $a\,c$, $c\,d$, $d\,e$, and $e\,f$ parallel to $P\,O$, $P\,1$, $P\,2$, and $P\,3$, respectively, and finally drawing the closing line $f\,a$ to the starting point a. $P\,m$ is drawn parallel to the latter line, dividing the load line into the reactions $m\,O = R_1$, and $3\,m = R_2$. The shear axis $m\,n$ is drawn through m, and the shear diagram $O\,h\,l \ldots\, s'\,n\,m\,O$ is constructed in the usual manner. To the scale of forces $m\,O = 1,440$ lb., and $3\,m = 2,160$ lb. To the scale of distances the maximum vertical intercept $y = d'\,d = 31.2$ ft., which, multiplied by H, $= 31.2 \times 2,000 = 62,400$ ft.-lb. $= 748,800$ in.-lb. Ans.

(*b*) The shear at a point 30 ft. from the left support $= O\,m = 1,440$ lb. Ans.

(*c*) The maximum shear $= n\,s' = -2,160$ lb. Ans.

(707) See Fig. 59. Draw the force polygon *O 1 2 3 4 5 O* in the usual manner, *O 1* being equal to and parallel to F_1, *1-2* equal to and parallel to F_2, etc. *O 5* is the resultant.

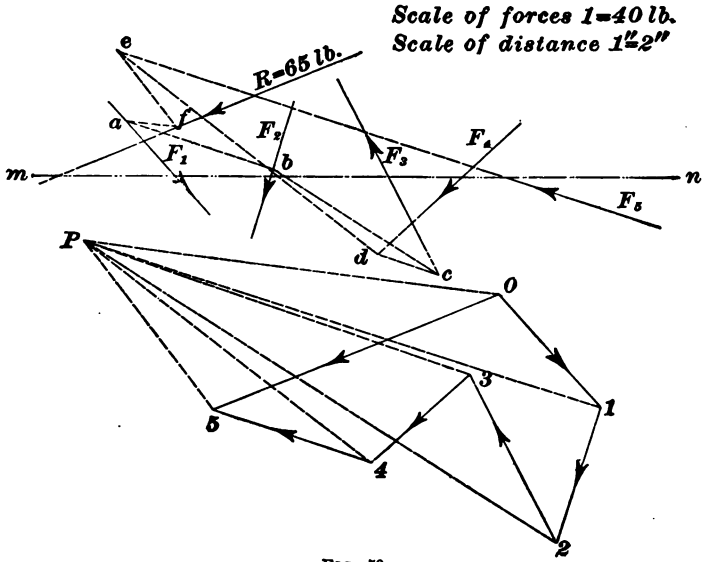
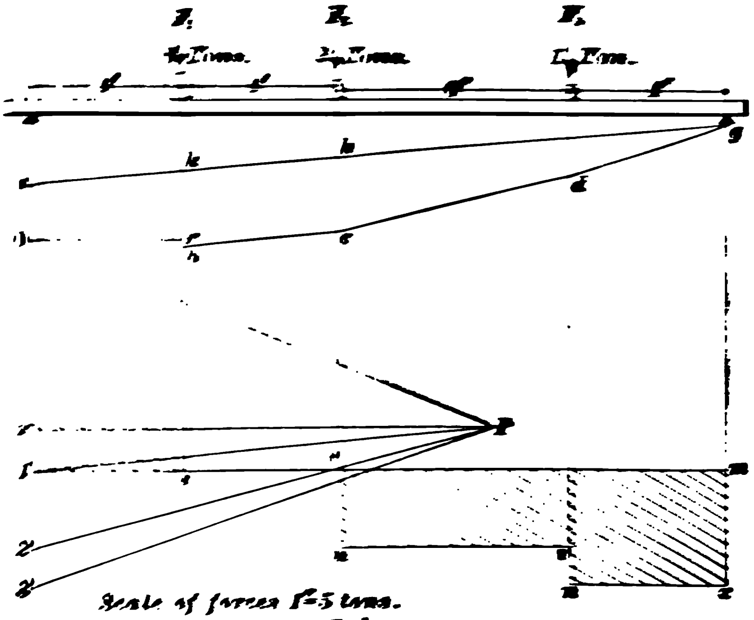


FIG. 59.

Choose the pole *P*, and draw the rays *P O*, *P 1*, *P 2*, etc. Choose any point, *a* on F_1, and draw through it a line parallel to the ray *P 1*. From the intersection *b* of this line with F_2, draw a line parallel to *P 2*; from the intersection *c* of the latter line with F_3 produced draw a parallel to *P 3*, intersecting F_4 produced in *d*. Finally, through *d*, draw a line parallel to *P 4*, intersecting F_5 produced in *e*. Now, through *a* draw a line parallel to *P O*, and through *e* a line parallel to *P 5*; their intersection *f* is a point on the resultant. Through *f* draw the resultant *R* parallel to *O 5*. It will be found by measurement that $R = 65$ lb., that it makes an angle of $22\frac{1}{2}°$ with *m n*, and intersects it at a distance of $1\frac{1}{4}'$ from the point of intersection of F_1 and *m n*.

(708) See Fig. 60. The construction is entirely similar to those given in the text. *O 1*, *1-2*, and *2-3* are laid off to represent F_1, F_2, and F_3; the pole *P* is chosen, and the rays

[illegible] are drawn the lines of the [illegible] The closing line $g\ s$ is [illegible] Consequently, [illegible] is the left reaction and [illegible] the right reaction, the former being [illegible] tons



Scale of forces 1" = 5 tons.
Scale of distance 1" = 5'
FIG. [illegible]

and the latter 3 tons. The shear diagram is drawn in the usual manner; it has the peculiarity of being zero between F_1 and F_2.

(7[illegible]) The maximum moment occurs when the shear line crosses the shear axis. In the present case the shear line and shear axis coincide with $s\ t$, between F_1 and F_2; hence, the bending moment is the same (and maximum) at F_1 and F_2, and at all points between. This is seen to be true from the diagram, since $k\ h$ and $b\ c$ are parallel. Ans.

(b) By measurement, the moment is found to be $24 \times 12 = 288$ inch-tons. Ans.

(c) $288 \times 2{,}000 = 576{,}000$ inch-pounds. Ans.

(710) See Arts. **1375** to **1379.**

(711) See Fig. 61. The force polygon *0 1 2 3 4 0* is drawn as in Fig. 59, *0 4* being the resultant. The equilibrium polygon *a b c d g a* is then drawn, the point *g* lying on the resultant. The resultant *R* is drawn through *g*, parallel to and equal to *0 4*. A line is drawn through *C*, parallel to *R*. Through *g* the lines *g e* and *g f* are drawn parallel, respectively, to *P 0* and *P 4*, and intersecting the parallel to *R*, through *C* in *e* and *f*; then, *e f* is the intercept, and *P u*, perpendicular to *0 4*, is the pole distance. $Pu = 33$ lb.; $ef = 1.32''$. Hence, the resultant moment is $33 \times 1.32 = 43.6$ in.-lb. Ans.

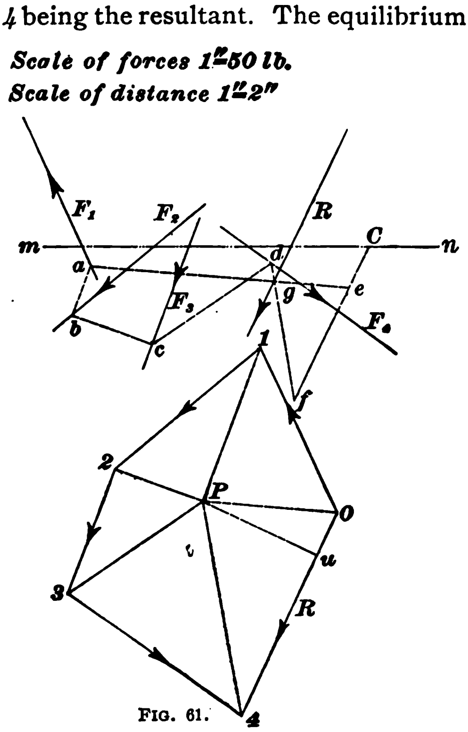


FIG. 61.

(712) The maximum bending moment, $M = W\frac{l_1 l_2}{l}$ (see Fig. 6 of table of Bending Moments) $= 4 \times 2,000 \times \frac{14 \times 8}{22} =$ $40,727\frac{3}{11}$ ft.-lb. $= 488,727$ in.-lb. Then, according to formula **117,**

$$\frac{S_4 I}{fc} = 488,727.$$

$$\frac{I}{c} = \frac{488,727 f}{S_4} = \frac{488,727 \times 8}{9,000} = 434.424.$$

But, $\frac{I}{c} = \frac{\frac{1}{12} b d^3}{\frac{1}{2} d} = \frac{1}{6} b d^2$, and, according to the conditions of the problem, $b = \frac{1}{2} d$.

Therefore, $\frac{I}{c} = \frac{1}{6} b d^2 = \frac{1}{12} d^3 = 434.424.$

$$d^3 = 5{,}213.088.$$

$$\left. \begin{array}{l} d = 17\tfrac{1}{3}''. \\ b = 8\tfrac{2}{3}''. \end{array} \right\} \text{Ans.}$$

(713) The beam, with the moment and shear diagrams, is shown in Fig. 62. On the line, through the left reaction,

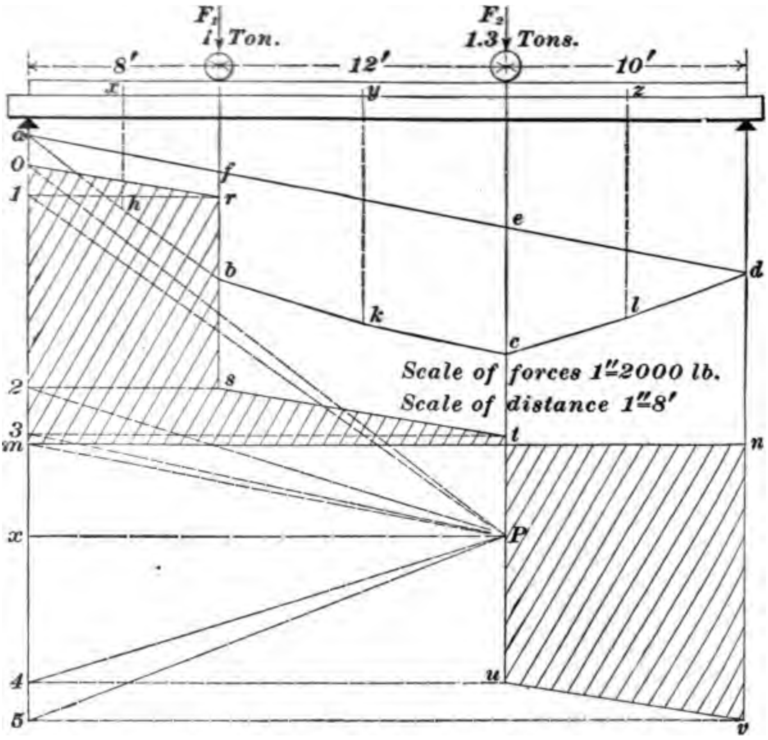


FIG. 62.

are laid off the loads in order. Thus, $O\,1 = 40 \times 8 = 320$ lb., is the uniform load between the left support and F_1; *1-2* is

$F_1 = 2{,}000$ lb.; *2-3* $=40 \times 12 = 480$ lb., is the uniform load between F_1 and F_2; *3-4* $= 2{,}000 \times 1.3 = 2{,}600$ lb., is F_2, and *4-5* $= 40 \times 10 = 400$ lb., is the uniform load between F_2 and the right support. The pole P is chosen and the rays drawn. Since the uniform load is very small compared with F_1 and F_2, it will be sufficiently accurate to consider the three portions of it concentrated at their respective centers of gravity x, y, and z. Drawing the equilibrium polygon parallel to the rays, we obtain the moment diagram $a\ h\ b\ k\ c\ l\ d\ a$. From P, drawing $P\,m$ parallel to the closing line $a\,d$, we obtain the reactions $O\,m$ and $m\,5$, equal, respectively, to 2,930 and 2,870 lb. Ans. The shear axis $m\,x$, and the shear diagram $O\ r\ s\ t\ u\ v\ n\ m$, are drawn in the usual manner. The greatest shear is $O\,m$, 2,930 lb. The shear line cuts the shear axis under F_2. Hence, the maximum moment is under F_2. By measurement, $e\,c$ is 64″, and $P\,x$ is 5,000 lb.; hence, the maximum bending moment is $64 \times 5{,}000 = 320{,}000$ in.-lb. Ans.

(714) From the table of Bending Moments, the greatest bending moment of such a beam is $\frac{w\,l^2}{8}$, or, in this case, $\frac{w \times 240^2}{8}$.

By formula **117**,

$$M = \frac{w \times 240^2}{8} = \frac{S_4 I}{f\,c} = \frac{45{,}000}{4} \times \frac{280}{12 \div 2}.$$

Therefore, $w = \dfrac{45{,}000 \times 280 \times 8}{240 \times 240 \times 4 \times 6} = 72.92$ lb. per inch of length $= 72.92 \times 12 = 875$ lb. per foot of length. Ans.

(715) From the table of Bending Moments, the maximum bending moment is

$$\frac{W\,l}{4} = \frac{W \times 96}{4} = 24\,W.$$

From formula **117**,

$$M = 24\,W = \frac{S_4 I}{f\,c}.$$

$I = \frac{\pi}{64}(d^4 - d_1^4) = 56.945$; $c = \frac{1}{2}\,d = \frac{6\frac{1}{2}}{2} = 3\frac{1}{4}$; $S_4 = 38{,}000$; $f = 6$.

Hence [illegible]

$$[illegible] = [illegible] \quad \text{Ans.}$$

(716) (a) From the table of Bending Moments,

$$M = \frac{wl^2}{8} = \frac{w \times 192^2}{8}.$$

From formula **117**,

$$M = \frac{w \times 192^2}{8} = \frac{S_e I}{fc}.$$

$$S_e = 7,200;\ f = 8;\ I = \frac{1}{12} b d^3 = \frac{2,000}{12};\ c = \frac{1}{2} d = 5.$$

$$\text{Then,}\quad \frac{w \times 192^2}{8} = \frac{7,200}{8} \times \frac{2,000}{12 \times 5}.$$

$$w = \frac{7,200 \times 2,000 \times 8}{8 \times 12 \times 5 \times 192 \times 192} =$$

6.51 lb. per in. $= 6.51 \times 12 = 78.12$ lb. per ft. **Ans.**

(b) $I = \frac{1}{12} b d^3 = \frac{10 \times 2^3}{12} = \frac{80}{12}.\quad c = 1''.$

$$\frac{w \times 192^2}{8} = \frac{7,200}{8} \times \frac{80}{12 \times 1}.$$

$$w = \frac{7,200 \times 80 \times 8}{8 \times 12 \times 192 \times 192} = 1.3 \text{ lb. per in.}$$

$\therefore 1.3 \times 12 = 15.6$ lb. per ft. Ans.

(717) (a) From the table of Bending Moments, the deflection of a beam uniformly loaded is $\frac{5}{384}\frac{Wl^3}{EI}$. In example 714, $W = 875 \times 20 = 17,500$ lb.; $l = 240''$, $E = 25,000,000$, and $I = 280$.

Hence, deflection $s = \frac{5 \times 17,500 \times 240^3}{384 \times 25,000,000 \times 280} = .45$ in. Ans.

(b) From the table of Bending Moments, $s = \frac{1}{48}\frac{Wl^3}{EI}$.

In example 715, $W = 4,624$ lb.; $l = 96$ in.; $E = 15,000,000$, and $I = 56.945$.

Hence, $s = \dfrac{4,624 \times 96^3}{48 \times 15,000,000 \times 56.945} = .1''$, nearly. Ans.

(c) $s = \dfrac{5}{384}\dfrac{Wl^3}{EI}$. In example 716 (a), $W = 78.12 \times 16$

$l = 192$; $E = 1,500,000$, and $I = \dfrac{2,000}{12}$.

Hence, $s = \dfrac{5 \times 78.12 \times 16 \times 192^3}{384 \times 1,500,000 \times \frac{2000}{12}} = .461''$. Ans.

(718) Area of piston $= \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi \times 14^2$.

$$W = \text{pressure on piston} = \frac{1}{4}\pi \times 14^2 \times 80.$$

From the table of Bending Moments, the maximum bending moment for a cantilever uniformly loaded is

$\dfrac{wl^2}{2} = \dfrac{Wl}{2} = \dfrac{\frac{1}{4}\pi \times 14^2 \times 80 \times 4}{2} = \dfrac{S_4}{f}\dfrac{I}{c}$. (See formula **117.**)

$$\frac{S_4}{f} = \frac{45,000}{10} = 4,500. \quad \frac{I}{c} = \frac{\frac{1}{64}\pi d^4}{\frac{1}{2}d} = \frac{1}{32}\pi d^3.$$

$$\text{Hence, } \frac{\frac{1}{4}\pi \times 14^2 \times 80 \times 4}{2} = \frac{4,500\,\pi d^3}{32},$$

$$\text{or } d^3 = \frac{14^2 \times 80 \times 4 \times 32}{4 \times 2 \times 4,500} = 55.75.$$

$$d = \sqrt[3]{55.75} = 3.82''. \quad \text{Ans.}$$

(719) Substituting in formula **119,** $S_2 = 90,000$; $A = 6^2 \times .7854$; $f = 6$; $l = 14 \times 12 = 168$; $g = 5,000$; $I = \dfrac{\pi}{64} \times 6^4$, we obtain

$$W = \frac{S_2 A}{f\left(1 + \dfrac{Al^2}{gI}\right)} = \frac{90,000 \times 6^2 \times .7854}{6\left(1 + \dfrac{6^2 \times .7854 \times 168^2}{5,000 \times \dfrac{3.1416 \times 6^4}{64}}\right)} = 120,872 \text{ lb. Ans.}$$

(720) For timber, $S_2 = 8,000$ and $f = 8$; hence, $\dfrac{S_2}{f} = \dfrac{8,000}{8} = 1,000.$

[illegible] **109.**

$$[illegible] = 1{,}000 \ [illegible]$$

[illegible] $= [illegible]$ necessary area of a [illegible] the given load. Since the [illegible] assume [illegible] be 6″ square. Then $A = 36$, and [illegible] $= 108$.

Formula **119** gives

$$[illegible], \quad \text{or} \ \frac{S_2}{f} = \frac{P}{A}\left(1 - \frac{[illegible]}{gI}\right)$$

$[illegible] = [illegible]$, $l = 360$, and $g = 3{,}000$.

$$\frac{S_2}{f} = [illegible]\left(1 - \frac{[illegible] \times 360^2}{3{,}000 \times [illegible]}\right) = 5{,}900, \text{ nearly.}$$

[illegible] much too large, the column must be made larger. Trying [illegible] square, $A = 81$, $I = 546\frac{3}{4}$.

$$\text{Then, } \frac{S_2}{f} = [illegible]\left(1 - \frac{81 \times 360 \times 360}{3{,}000 \times 546\frac{3}{4}}\right) = 1{,}279.$$

This value of $\frac{S_2}{f}$ is much nearer the required value, 1,000.

Trying 10″ square, $A = 100$, $I = \frac{10{,}000}{12} = 833\frac{1}{3}$.

$$\frac{S_2}{f} = \frac{14{,}000}{100}\left(1 - \frac{100 \times 360 \times 360}{3{,}000 \times 833\frac{1}{3}}\right) = 866, \text{ nearly.}$$

Since this value of $\frac{S_2}{f}$ is less than 1,000, the column is a little too large; hence, it is between 9 and 10 inches square. $9\frac{5}{8}''$ will give 997.4 lb. as the value of $\frac{S_2}{f}$; hence, the column should be $9\frac{5}{8}''$ square.

This problem may be more readily solved by formula **120**, which gives

$$[illegible] \frac{7}{9} \times \frac{[illegible]000 \times 8}{800} [illegible] \frac{7 \times 2000 \times 8}{8000}\left(\frac{7 \times 2000 \times 8}{4 \times 8000} + \frac{12 \times 360^2}{3000}\right) =$$

$$[illegible] 11(3.5 + 518.4) [illegible] 93.479 = 9.61'' = 9\frac{5}{8}'', \text{ nearly.}$$

(721) Here $W = 21{,}000$; $f = 10$; $S_2 = 150{,}000$; $g = 6{,}250$; $l = 7.5 \times 12 = 90''$. For using formula **121**, we have

$$\frac{.3183\ Wf}{S_2} = \frac{.3183 \times 21{,}000 \times 10}{150{,}000} = .4456.$$

$$\frac{16\, l^2}{g} = \frac{16 \times 8100}{6250} = 20.7360.$$

Therefore,

$$d = 1.4142\sqrt{.4456 + \sqrt{.4456\,(.4456 + 20.7360)}} = 1.4142\sqrt{.4456 + 3.0722} = 2.65'', \text{ or say } 2\tfrac{5}{8}''.$$

(722) For this case, $A = 3.1416$ sq. in.; $l = 4 \times 12 = 48''$; $S_2 = 55{,}000$; $f = 10$; $I = .7854$; $g = 20{,}250$. Substituting these values in formula **119**,

$$W = \frac{S_2 A}{f\left(1 + \frac{A\,l^2}{g\,I}\right)} = \frac{55{,}000 \times 3.1416}{10\left(1 + \frac{3.1416 \times 48^2}{20{,}250 \times .7854}\right)} = \frac{5{,}500 \times 3.1416}{1.4551}.$$

Steam pressure = 60 lb. per sq. in.

Then, area of piston $= .7854\, d^2 = \frac{W}{60} = \frac{5{,}500 \times 3.1416}{1.4551 \times 60}$.

$$\text{Hence, } d^2 = \frac{5{,}500 \times 3.1416}{.7854 \times 1.4551 \times 60} = 252, \text{ nearly,}$$

and $d = \sqrt{252} = 15\tfrac{7}{8}''$, nearly. Ans.

(723) (*a*) The strength of a beam varies directly as the width and square of the depth and inversely as the length. Hence, the ratio between the loads is

$$\frac{6 \times 8^2}{10} : \frac{4 \times 12^2}{16} = 16 : 15, \text{ or } 1\tfrac{1}{15}. \quad \text{Ans.}$$

(*b*) The deflections vary directly as the cube of the lengths, and inversely as the breadths and cubes of the depths.

Hence, the ratio between the deflections is

$$\frac{10^3}{6 \times 8^3} : \frac{16^3}{4 \times 12^3} = .549. \quad \text{Ans.}$$

[illegible] Table II [illegible]

[illegible] Ans.

[illegible] Ans.

[illegible]

[illegible] formula **124** [illegible]

[illegible] Ans.

[illegible] formula **123**.

[illegible] (Table II.)

[illegible] Ans.

[illegible] formula **126**.

[illegible] Ans.

[illegible] Table II.)

728. (a) Using formula **127**.

[illegible] $= 1.400$ [illegible] Ans.

(b) Using formula **128**,

$$P = 500\, C^2;\ C^2 = \frac{P}{500} = \frac{5,000}{500} = 10.$$

$$C = \sqrt{10} = 3.162''.$$

$$d = \frac{1}{3} C = 1.054''. \quad \textbf{Ans.}$$

(c) Using formula **129**,

$$P = 1,000\, C^2;\ C^2 = \frac{P}{1,000} = \frac{6\frac{2}{3} \times 2,000}{1,000} = 13\tfrac{1}{3}.$$

$$C = \sqrt{13\tfrac{1}{3}} = 3.651''. \quad \textbf{Ans.}$$

(729) (*a*) Using formula **130**,

$$P = 12{,}000\,d^2 = 12{,}000 \times \left(\frac{7}{8}\right)^2 = 9{,}187.5 \text{ lb.} \quad \text{Ans.}$$

(*b*) Formula **131** gives $P = 18{,}000\,d^2$.

Therefore, $d = \sqrt{\dfrac{P}{18{,}000}} = \sqrt{\dfrac{8{,}000}{18{,}000}} = \sqrt{\dfrac{4}{9}} = .667''$. Ans.

(730) The deflection is by formula **118**,

$s = a\dfrac{Wl^3}{EI} = \dfrac{1}{192}\dfrac{Wl^3}{EI}$, the coefficient being found from the table of Bending Moments.

Transposing, $W = \dfrac{192\,s\,E\,I}{l^3}$; $l = 120$; $E = 30{,}000{,}000$; $I = .7854$; $s = \dfrac{1}{8}$.

Then, $W = \dfrac{192 \times 30{,}000{,}000 \times .7854}{8 \times 120^3} = 327.25$ lb. Ans.

(731) (*a*) The maximum bending moment is, according to the table of Bending Moments, $\dfrac{Wl}{4} = \dfrac{6{,}000 \times 60}{4} = 90{,}000$ inch-pounds.

By formula **117**,

$$M = 90{,}000 = \frac{S_4}{f}\frac{I}{c}.$$

$$S_4 = 120{,}000;\ f = 10. \quad \frac{I}{c} = \frac{\frac{\pi d^4}{64}}{\frac{1}{2}d} = \frac{\pi d^3}{32}.$$

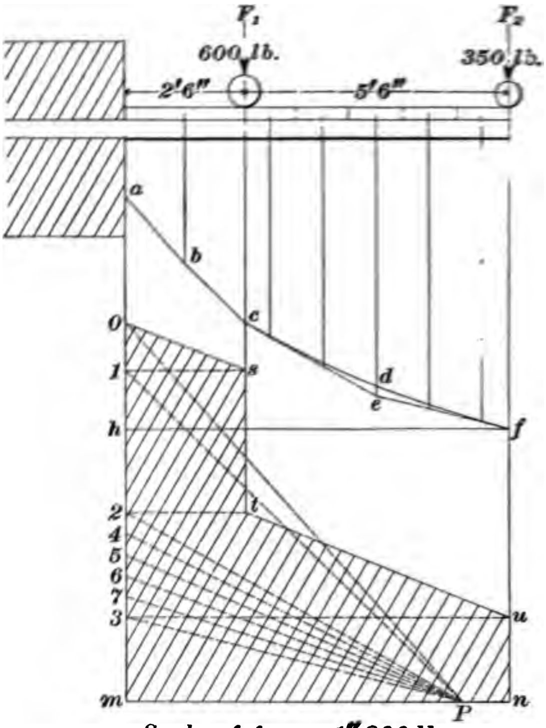
$$\text{Hence, } 90{,}000 = \frac{120{,}000}{10}\,\frac{\pi d^3}{32},$$

$$\text{or } d = \sqrt[3]{\frac{90{,}000 \times 10 \times 32}{120{,}000 \times 3.1416}} = 4.244'' = 4\tfrac{1}{4}'', \text{ nearly.}$$

(*b*) Using formula **123**,

$$d = c_1\sqrt[4]{\frac{H}{N}} = 4.7\sqrt[4]{\frac{75}{80}} = 4\tfrac{5}{8}'', \text{ nearly.} \quad \text{Ans.}$$

(732) (*a*) The graphic solution is shown in Fig. 63. On the vertical through the support *O 1* is laid off equal to the uniform load between the support and F_1; *1-2* is laid off



Scale of forces 1″ = 800 lb.
Scale of distance 1″ = 4′

FIG. 63.

to represent F_1. *2-3* represents to the same scale the remainder of the uniform load, and *3 m* represents F_2. The pole *P* is chosen and the rays drawn. The polygon *a b c e f h* is then drawn, the sides being parallel, respectively, to the corresponding rays. If the uniform load between F_1 and F_2 be considered as concentrated at its center of gravity, the polygon will follow the broken line *c e f*. It will be better in this case to divide the uniform load into several parts, *2-4*, *4-5*, *5-6*, etc., thus obtaining the line of the polygon

c d f. To draw the shear diagram, project the point *1* across the vertical through F_1, and draw $O\,s$. Next project the point *2* across to *t*, and *3* across to *u*, and draw $t\,u$. $O\,s\,t\,u\,n\,m$ is the shear diagram. The maximum moment is seen to be at the support, and is equal to $a\,h \times P\,m$. To the scale of distances, $a\,h = 58.8$ in., while $P\,m = H = 1{,}400$ lb. to the scale of forces. Hence, the maximum bending moment is $58.8 \times 1{,}400 = 82{,}320$ in.-lb. Ans.

(*b*) From formula **117**,

$$M = \frac{S_4}{f}\frac{I}{c} = 82{,}320. \quad S_4 = 12{,}500; \; f = 8.$$

$$\text{Therefore, } \frac{I}{c} = \frac{82{,}320 \times 8}{12{,}500} = 52.68.$$

$$\text{But, } \frac{I}{c} = \frac{\frac{1}{12} b\,d^3}{\frac{1}{2} d} = \frac{b\,d^2}{6}. \text{ and } d = 2\tfrac{1}{2}\,b, \text{ or } b = \frac{2\,d}{5}.$$

$$\text{Hence, } \frac{I}{c} = \frac{b\,d^2}{6} = \frac{d^3}{15} = 52.68. \quad d^3 = 52.68 \times 15 = 790.2.$$

$$d = \sqrt[3]{790.2} = 9.245''. \quad b = \frac{2\,d}{5} = 3.7'', \text{ nearly. Ans.}$$

(733) Referring to the table of Moments of Inertia,

$$I = \frac{(b\,d^2 - b_1\,d_1^2)^2 - 4\,b\,d\,b_1\,d_1(d - d_1)^2}{12\,(b\,d - b_1\,d_1)} =$$

$$\frac{[8 \times 10^2 - 6 \times (8\tfrac{1}{2})^2]^2 - 4 \times 8 \times 10 \times 6 \times 8\tfrac{1}{2}\,(10 - 8\tfrac{1}{2})^2}{12\,(8 \times 10 - 6 \times 8\tfrac{1}{2})} =$$

$$280.466.$$

$$c = \frac{d}{2} + \frac{b_1\,d_1}{2}\left(\frac{d - d_1}{b\,d - b_1\,d_1}\right) =$$

$$\frac{10}{2} + \frac{6 \times 8\tfrac{1}{2}}{2}\left(\frac{10 - 8\tfrac{1}{2}}{8 \times 10 - 6 \times 8\tfrac{1}{2}}\right) = 6.319.$$

(*a*) From the table of Bending Moments, the maximum bending moment is $\frac{W\,l}{4}$.

$$S_4 = 120{,}000; \; f = 7; \; l = 35 \times 12 = 420 \text{ in.}$$

Using formula **117**, $M = \frac{W\,l}{4} = \frac{S_4\,I}{f\,c}$, or

$$W = \frac{4\,S_4\,I}{l\,f\,c} = \frac{4 \times 120{,}000 \times 280.466}{420 \times 7 \times 6.319} = 7{,}246 \text{ lb. Ans.}$$

[illegible] and the maximum bending moment [illegible]. Hence from formula **117**,

$$M = [illegible] = [illegible], \text{ or } w = [illegible]$$

Therefore [illegible] $= \dfrac{[illegible]}{[illegible]} = \dfrac{[illegible] \times 230,000 \times 350.446}{[illegible] \times 3 \times [illegible]} =$ [illegible] Ans.

734. [illegible] According to formula **115**,

[illegible] $= \sqrt{3} = 1.732$ Ans.

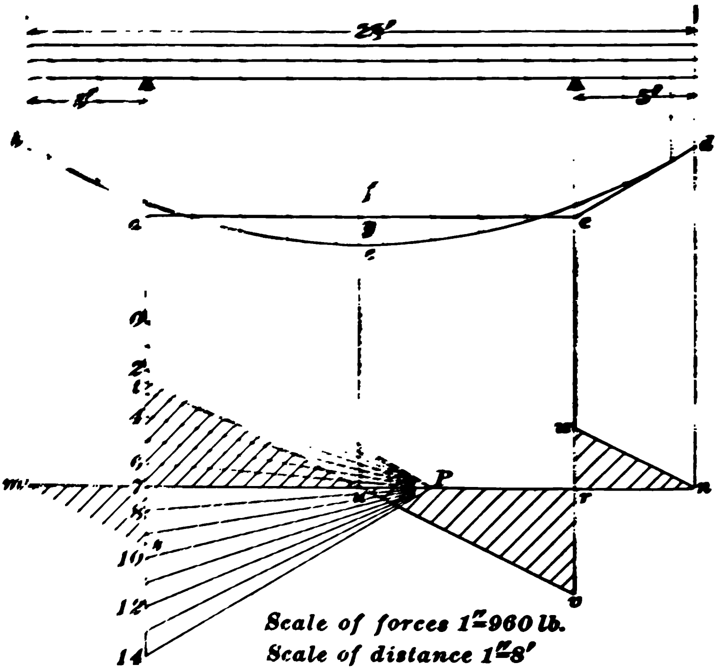


Fig. 64.

(b) From the table of Moments of Inertia, $I = \frac{1}{12} b d^3 = 72$; $A = bd = 24$. Dividing, $\dfrac{\frac{1}{12} b d^3}{b d} = \dfrac{72}{24}$, or $\frac{1}{12} d^2 = 3$. $d^2 = 36$; $d = 6''$ and $b = 4''$. Ans.

(*c*) As above, $r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4}$. Ans.

(735) Using formula **112**, $p\,d = 4\,t\,S$, we have $t = \frac{p\,d}{4\,S}$. Using a factor of safety of 6,

$$p\,d = \frac{4\,t\,S}{6}, \text{ or } t = \frac{6\,p\,d}{4\,S} = \frac{6 \times 100 \times 8}{4 \times 20{,}000} = .06'' \quad \text{Ans.}$$

(736) The graphic solution is shown in Fig. 64. The uniform load is divided into 14 equal parts, and lines drawn through the center of gravity of each part. These loads are laid off on the line through the left reaction, the pole P chosen, and the rays drawn. The polygon $b\,c\,d\,e\,f\,a$ is then drawn in the usual manner. The shear diagram is drawn as shown. The maximum shear is either $t\,7$ or $r\,v = 540$ lb. The maximum moment is shown by the polygon to be at $f\,c$ vertically above the point u, where the shear line crosses the shear axis. The pole distance $P\,7$ is 1,440 lb. to the scale of forces, and the intercept $f\,c$ is 14 inches to the scale of distances. Hence, the bending moment is 20,160 in.-lb.

(737) From formula **117**,

$$M = \frac{S_4}{f}\frac{I}{c} = 20{,}160. \quad S_4 = 9{,}000;\ f = 8.$$

$$\text{Then, } \frac{I}{c} = \frac{20{,}160 \times 8}{9{,}000} = 17.92.$$

$$\text{But, } \frac{I}{c} = \frac{\frac{1}{12}b\,d^3}{\frac{1}{2}d} = \frac{1}{6}b\,d^2 \text{ for a rectangle.}$$

$$\text{Hence, } \frac{1}{6}b\,d^2 = 17.92, \text{ or } b\,d^2 = 107.52.$$

Any number of beams will fulfil this condition.

$$\text{Assuming } d = 6'',\ b = \frac{107.52}{36} = 3'', \text{ nearly.}$$

$$\text{Assuming } d = 5'',\ b = \frac{107.52}{25} = 4.3''.$$

(738) Using the factor of safety 10, in formula **114**,

$$p = \frac{9{,}600{,}000}{10}\,\frac{t^{2.18}}{l\,d} = \frac{960{,}000 \times .2^{2.18}}{108 \times 2.5} = 106.43 \text{ lb. Ans}$$

(739) Using formula **130,**

$P = 12{,}000\, d^2$, or $d = \sqrt{\frac{P}{12{,}000}} = \sqrt{\frac{5 \times 2{,}000}{12{,}000}} = .913''$. Ans.

(740) The radius r of the gear-wheel is 24″. Using formula **123,** $d = c\sqrt[3]{Pr} = .297\sqrt[3]{350 \times 24} = 2.84''$. Ans.

(741) Area of cylinder $= .7854 \times 12^2 = 113.1$ sq. in.
Total pressure on the head $= 113.1 \times 90 = 10{,}179$ lb.

Pressures on each bolt $= \frac{10{,}179}{10} = 1{,}017.9$ lb.

Using formula **108,**

$P = A\,S$, or $A = \frac{P}{S} = \frac{1{,}017.9}{2{,}000} = .5089$ sq. in., area of bolt.

Diameter of bolt $= \sqrt{\frac{.5089}{.7854}} = .8''$, nearly. Ans.

(742) (a) The graphic solution is clearly shown in Fig. 65. On the vertical through F_1, the equal loads F_1 and F_2

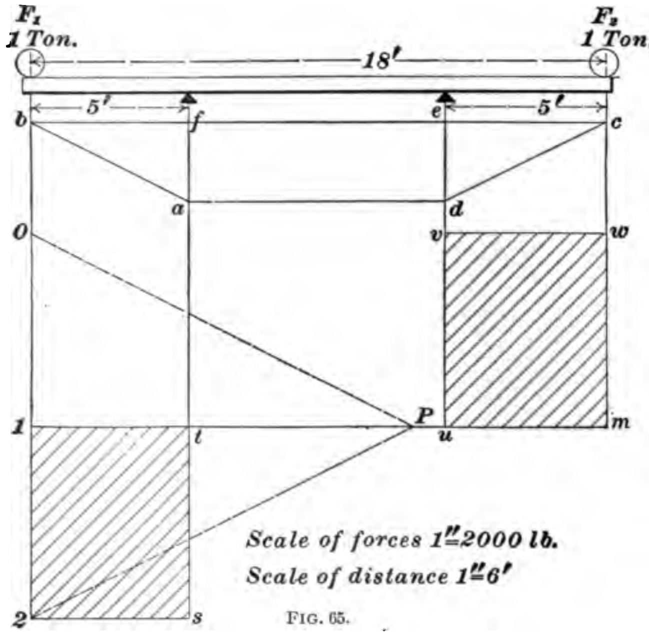


FIG. 65.

are laid off to scale, $O\ 1$ representing F_1 and 1-2 representing

F_2. Choose the pole P, and draw the rays $P\,O$, $P\,1$, $P\,2$. Draw $a\,b$ between the left support and F_1 parallel to $P\,O$; $b\,c$ between F_1 and F_2 parallel to $P\,1$, and $c\,d$ parallel to $P\,2$, between F_2 and the right support. Through P draw a line parallel to the closing line $a\,d$. $O\,1 = 1\text{-}2$; hence, the reactions of the supports are equal, and are each equal to 1 ton. The shear between the left reaction and F_1 is negative, and equal to $F_1 = 1$ ton. Between the left and the right support it is 0, and between the latter and F_2 it is positive and equal to 1 ton. The bending moment is constant and a maximum between the supports. To the scale of forces $P\,1 = 2$ tons $= 4{,}000$ lb., and to the scale of distances $a\,f = 30$ in. Hence, the maximum bending is $4{,}000 \times 30 = 120{,}000$ in.-lb. Ans.

(*b*) Using formula **117**,

$$M = \frac{S_4}{f}\frac{I}{c} = 120{,}000. \quad S_4 = 38{,}000;\ f = 6.$$

$$\text{Then, } \frac{I}{c} = \frac{120{,}000 \times 6}{38{,}000} = \frac{360}{19} = 19, \text{ nearly.}$$

$$\text{But, } \frac{I}{c} = \frac{\frac{\pi\,d^4}{64}}{\frac{d}{2}} = \frac{\pi\,d^3}{32}.$$

$$\text{Hence, } \frac{\pi\,d^3}{32} = 19, \text{ or } d^3 = \frac{32 \times 19}{3.1416}.$$

$$d = \sqrt[3]{\frac{32 \times 19}{3.1416}} = 5.784''. \quad \text{Ans.}$$

(743) Since the deflections are directly as the cubes of the lengths, and inversely as the breadths and the cubes of the depths, their ratio in this case is

$$\frac{18^3}{2 \times 6^3} : \frac{12^3}{3 \times 8^3}, \text{ or } \frac{27}{2} : \frac{9}{8} = 12.$$

That is, the first beam deflects 12 times as much as the second. Hence, the required deflection of the second beam is $.3 \div 12 = .025''$. Ans.

(744) The key has a shearing stress exerted on two sections; hence, each section must withstand a stress of $\frac{20,000}{2} = 10,000$ pounds.

Using formula **108,** with a factor of safety of 10,

$$P = \frac{A S_s}{10}, \text{ or } A = \frac{10 P}{S_s} = \frac{10 \times 10,000}{50,000} = 2 \text{ sq. in.}$$

Let b = width of key;

t = thickness.

Then, $b t = A = 2$ sq. in. But, from the conditions of the problem,

$$t = \frac{1}{4} b.$$

$$\left.\begin{aligned} \text{Hence, } b t = \frac{1}{4} b^2 = 2;\ b^2 = 8;\ b = 2.828''. \\ t = \frac{2.828}{4} = .707''. \end{aligned}\right\} \text{Ans.}$$

(745) From formula **118,** the deflection $s = a \frac{W l^3}{E I}$, and, from the table of Bending Moments, the coefficient a for the beam in question is $\frac{1}{48}$.

W = 30 tons = 60,000 lb.; l = 54 inches; E = 30,000,000; $I = \frac{\pi d^4}{64}$.

$$\text{Hence, } s = \frac{1 \times 60,000 \times 54^3}{48 \times 30,000,000 \times \frac{3.1416 \times 12^4}{64}} = .0064 \text{ in. Ans.}$$

(746) (*a*) The circumference of a 7-strand rope is 3 times the diameter; hence, $C = 1\frac{1}{4} \times 3 = 3\frac{3}{4}''$.

Using formula **129,** $P = 1,000\ C^2 = 1,000 \times (3\frac{3}{4})^2 =$ 14,062.5 lb. Ans.

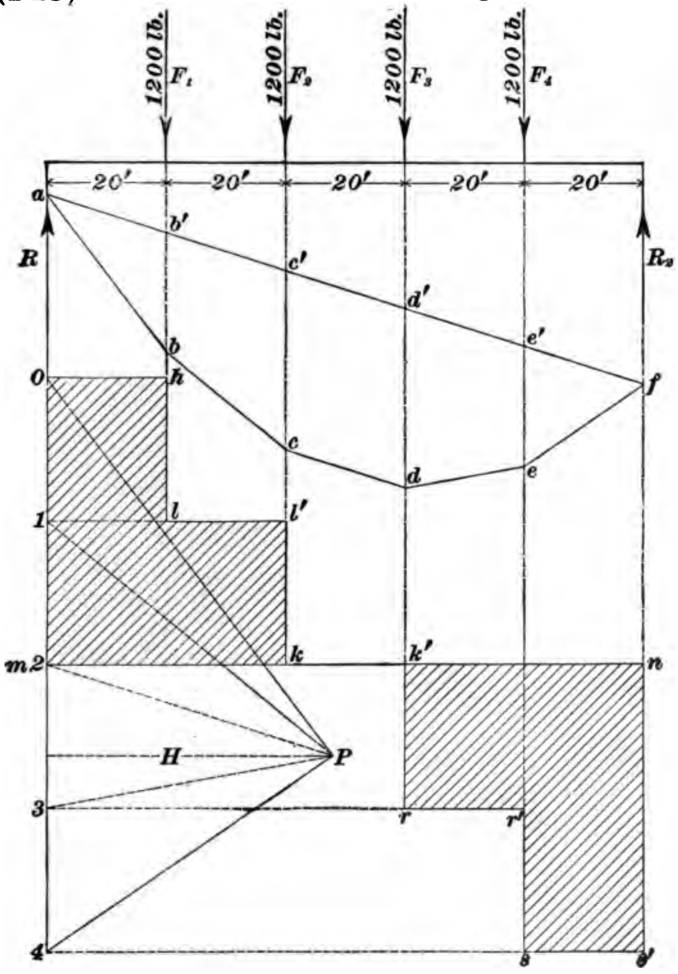
(*b*) Using formula **127,**

$$P = 100\, C^2, \text{ or } C = \sqrt{\frac{P}{100}} = \sqrt{\frac{1\frac{3}{4} \times 2,000}{100}} = 5.92''. \text{ Ans.}$$

(747) Using formula **113**,

$$p = \frac{S_1 t}{r + t} = \frac{120{,}000}{\frac{8}{2} + 6} = 12{,}000 \text{ lb. Ans.}$$

(748) The construction of the diagram of bending



Scale of forces 1″=1600 lb.
Scale of distance 1″=32′
FIG. 66.

moments and shear diagram is clearly shown in Fig. 66. It is so nearly like that of Fig. 58 that a detailed description is unnecessary. It will be noticed that between k and k' the shear is zero, and that since the reactions are equal the shear at either support $= \frac{1}{2}$ of the load $= 2,400$ lb. The greatest intercept is $c\,c' = d\,d' = 30$ ft. The pole distance $H = 2,400$ lb. Hence, the bending moment $= 2,400 \times 30 = 72,000$ ft.-lb. $= 72,000 \times 12 = 864,000$ in.-lb.

APPLIED MECHANICS.

(QUESTIONS 749–808.)

(749) See Fig. 341 in Art. **1434.** The construction is shown in Fig. 67.

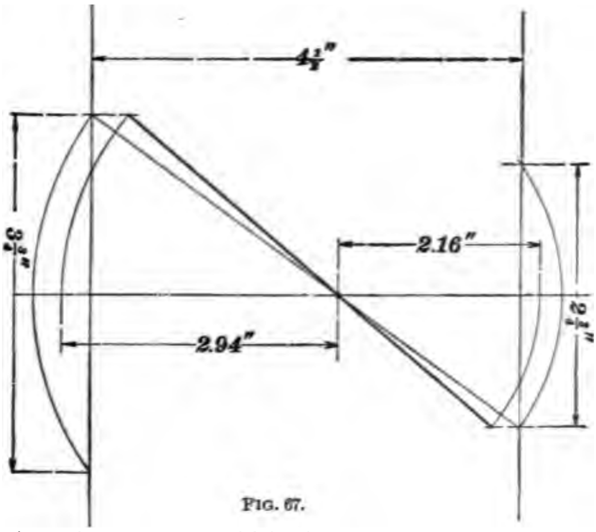


FIG. 67.

(750) The arms should be in the proportion 3 : 4. See Fig. 345 of Art. **1438.**

(751) In Fig. 68, let *A B* and *C D* be the two center lines of motion, making an angle of 10° with each other. Draw *a b* parallel to *A B*, and at a distance of 3″ from it; also, *c d* parallel to *C D*, and at a distance of 4″ from it. *O*, the point of intersection, will be the center of the lever, and *O E* and *O H* will be the center lines of the arms when in mid-position.

(752) The lengths of the lever arms will be as 3 × 12 : 3.5 :: 36 : 3.5. Either the motion shown in Fig. 343 or one of those shown in Fig. 344 may be used.

(753) Circumference of pinion $= 12 \times 3.1416 = 37.699'$.

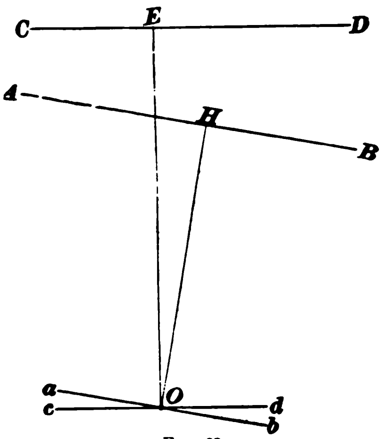


FIG. 68.

Distance each moves $= 37.699 \times \frac{6.5}{360} = .68'$. Ans.

(754) The construction is shown in Fig. 69. This, in connection with the principles explained in Arts. **1432** to **1438**, should enable the student to lay out the motion.

(755) See Art. **1441**.

(756) The forward stroke, in this case, is the stroke from left to right, while the crank-pin moves through the lower part of its circle. Cross-head A leads during the forward stroke, and

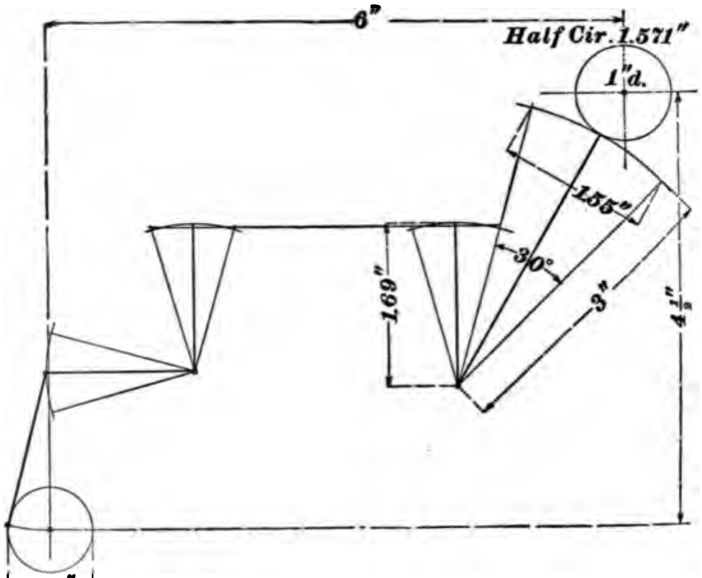


FIG. 69.

cross-head B during the return stroke. The construction to be used in determining this is shown in Fig. 349.

(757) The effect is the same as though one toggle-joint were used instead of two. First, find the pressure exerted by the screw. The distance that the power moves in one turn of the lever $= 20 \times 3.1416 = 62.832''$; distance that the nuts move $= \frac{1}{4}''$; velocity ratio $= \dfrac{62.832}{\frac{1}{4}} = 251.328$; force exerted $= 75 \times 251.328 = 18{,}849.6$ lb.

When distance $B = 16''$, the distance corresponding to h, in Fig. 352, $= \dfrac{16 - 12}{2} = 2''$. The distance corresponding to $O\,r$, therefore, equals $\sqrt{18^2 - 2^2} = \sqrt{320} = 17.888''$. Whence, applying formula **135,**

$$P = \frac{18{,}849.6 \times 17.888}{2 \times 2} = 84{,}295.4 \text{ lb. Ans.}$$

(758) In the diagram in Fig. 70, $H\,E$, $C\,E$, $E\,R$, and

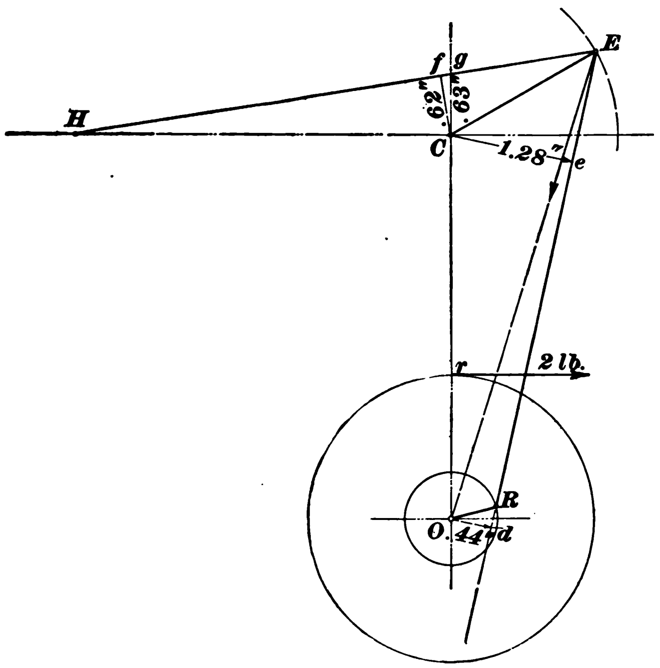


FIG. 70.

$O\,R$ are center lines of the parts which have the same letters in Fig. 22, question No. 758.

Draw Od perpendicular to ER produced; then by the principle of moments (see Art. **1447**), the thrust in the direction of $ER = \frac{2 \times Or}{Od} = \frac{2 \times 1.5}{.44} = 6.82$ lb. Erect the perpendicular Ce, and draw Cf perpendicular to HE and Cg perpendicular to HC. Then, the pull on the pin at H will equal $\frac{6.82 \times Ce}{Cf} = \frac{6.82 \times 1.28}{.62} = 14.08$ lb. Ans. Horizontal thrust of cross-head $= \frac{6.82 \times Ce}{Cg} = \frac{6.82 \times 1.28}{.63} = 13.86$ lb. Ans.

(759) See Art. **1457.**

(760) The construction is shown in Fig. 71. The various points are lettered in the same manner as in Fig. 358, and the description in Art. **1452** will apply to Fig. 71 in all respects, except that the circle must be divided into five equal parts instead of three, and the arc $d\,c\,b$ must contain three of these parts in order to have the ratio 3:2.

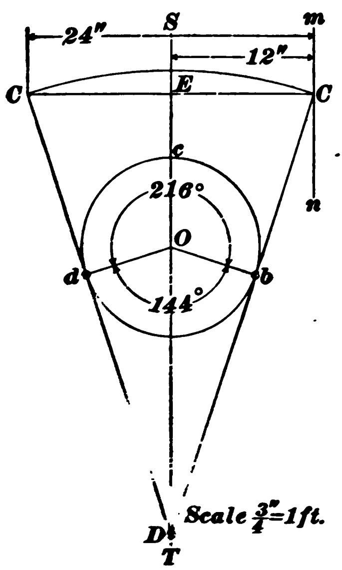


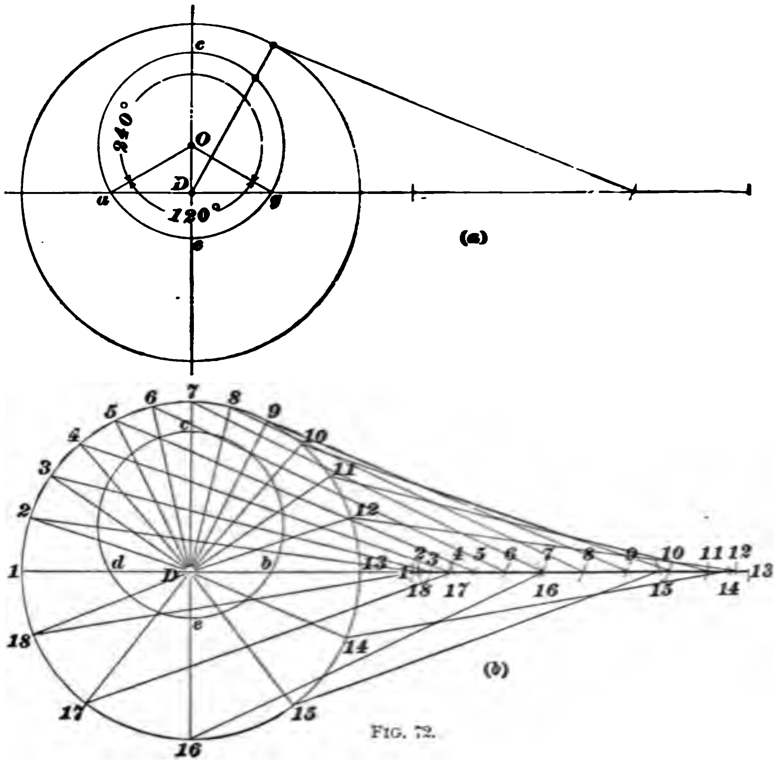
Fig. 71.

(761) (*a*) Fig. 72 (*a*) shows a diagram giving the proportions of the various parts; the arc $a\,e\,g$ is one-half as long as the arc $a\,c\,g$. See Arts. **1454** and **1455.**

(*b*) $360 \div 20 = 18$; hence, the crank-pin circle $d\,c\,b\,e$ must be divided into 18 equal parts. The motion is shown in Fig. 72 (*b*). See Art. **1455.**

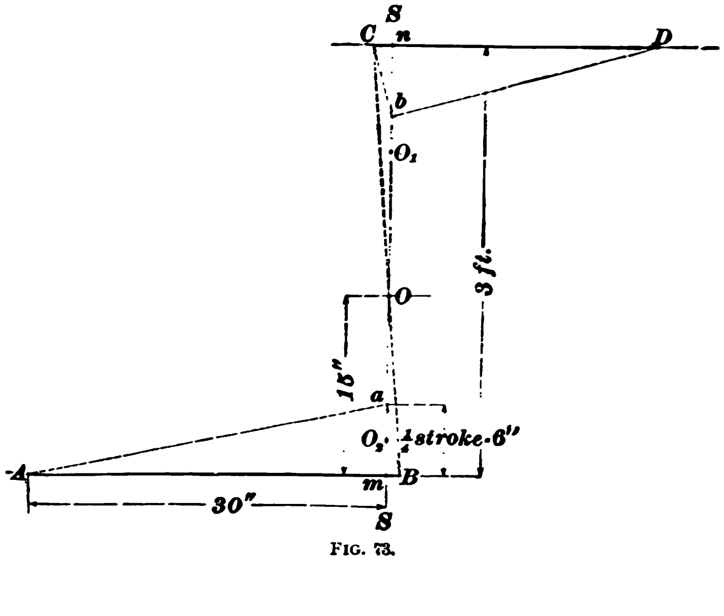
(762) See Art. **1459.** (*a*) Greatest rate $= 50 \times \frac{1}{\cos 28^\circ} = \frac{50}{.88295} = 56.63$ rev. Least rate $= 50 \times .88295 = 44.15$ rev.

(*b*) Connecting as described would double the variation



c
O
240°
D
a
g
120°
e
(a)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
c
d
D
b
e
(b)

FIG. 72.



S
C
n
D
b
O_1
3 ft.
O
15"
a
O_2
¼ stroke 6"
A
m
B
30"
S

FIG. 73.

(763) In Fig. 73, which is lettered the same as Fig. 364, draw the given center lines and locate points s and A from the data of the problem. Lay off $n\,s = \frac{1}{4}$ stroke; also $n\,b = \frac{1}{4}$ stroke. Connect A and s and draw $s\,B$ perpendicular to $A\,s$. Through O draw $B\,O\,C$; connect C and h, and through h draw $h\,D$ perpendicular to $C\,h$. Measure the lengths $C\,D$ and $A\,B$.

(764) The construction of the cam is shown in Fig. 74.

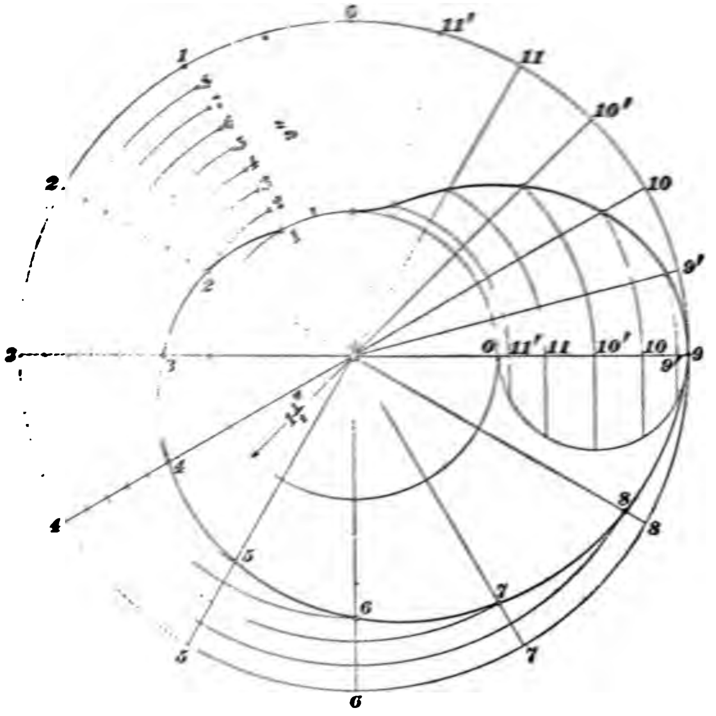


FIG. 74.

(765) The construction of the cam is shown in Fig. 75. It is similar in principle to the one shown in Fig. 366.

(766) The shape of the development of the groove is shown in Fig. 76. The method of laying it out is also indicated.

(767) This cam is shown in Fig. 77, and is like the one shown in Fig. 369 in all respects except that the dimensions are different. The description given in Art. **1474** will apply to this case.

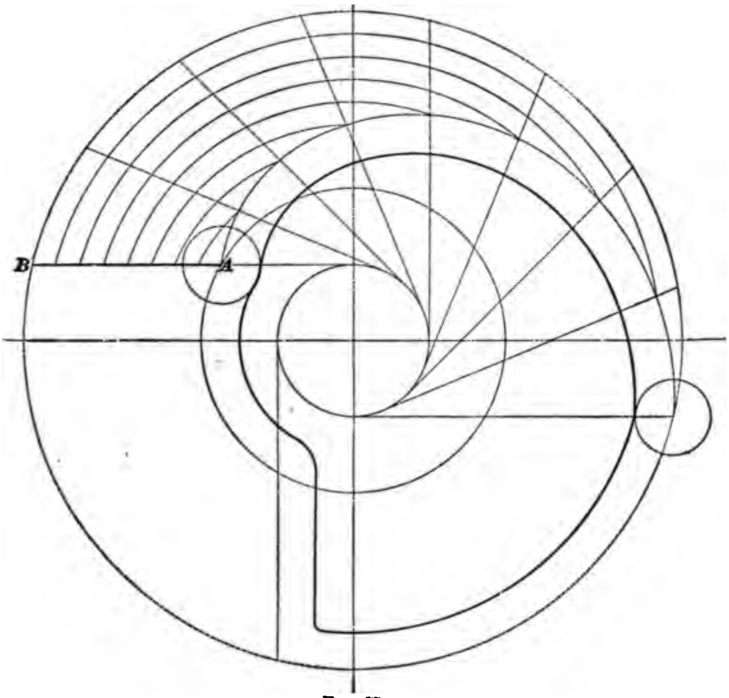


FIG. 75.

(768) See Art. **1478.** $\frac{52 \times 320}{48} = 346\frac{2}{3}$ rev. per min. Ans.

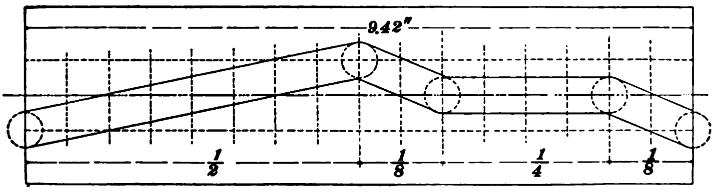


FIG. 76.

(769) $\frac{500 \times 10}{210} = 23.8'' = 23\frac{4}{5}''$. Ans.

(770) Substituting in formula **137**, $139 \times D_1 \times 20 = 1,800 \times 4 \times 3.5$, or

$$D_1 = \frac{1,800 \times 4 \times 3.5}{139 \times 20} = 10''. \quad \text{Ans.}$$

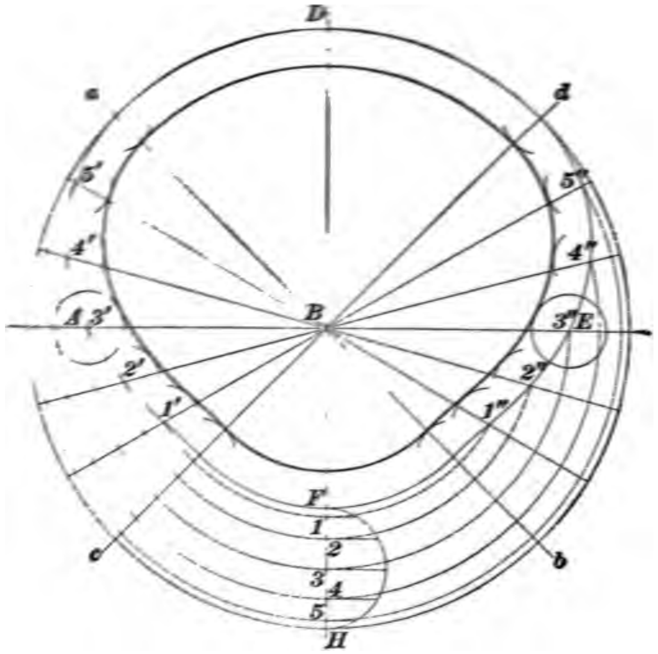


FIG. 77.

(771) See Art. **1481.** $900 \div 150 = 6$; $2 \times 3 = 6$, and $4 \times 1\frac{1}{2} = 6$. The ratios would then be 2 : 1 and 3 : 1; also 4 : 1 and $1\frac{1}{2}$: 1. Other ratios could be used.

(772) See Art. **1483.** (*a*) Effective pull in foot-pounds $= \dfrac{5 \times 33,000}{200 \times 3.1416 \times 3} = 87.54$ lb., nearly. Ans.

(*b*) Diameter in inches $= \dfrac{5 \times 33,000 \times 12}{50 \times 200 \times 3.1416} = 63$ inches. Ans.

(773) By laying out the two pulleys, the arc of contact is found to be about 150°. The allowable effective pull per inch in width, taken from the table is, therefore, 33.8 lb. Ans.

The effective pull of the belt $= \dfrac{6 \times 33{,}000 \times 12}{4 \times 3.1416 \times 1{,}500} = 126.05$ lb. $126.05 \div 33.8 = 3.7$ inches. A 4-inch belt would be used. Ans.

(774) The speed of the belt in feet per minute $= 14 \times 3.1416 \times 90$. Substituting in formula **142,**

$$W = \frac{630 \times 250}{14 \times 3.1416 \times 90} = 40'', \text{ nearly.}$$

The wheel should be 41 inches wide. Ans.

(775) Speed of belt $= 2 \times 205 \times 3.1416$. Substituting in formula **138,** $H = \dfrac{6 \times 2 \times 205 \times 3.1416}{900} = 8.6$ H. P., nearly. Ans.

(776) Calculating first for a double belt, we find that

$$H = \frac{6 \times 2 \times 205 \times 3.1416}{630} = 12.3, \text{ nearly.}$$

Next, calculating for a single belt with 30-inch pulley, we find that

$$H = \frac{6 \times 2.5 \times 205 \times 3.1416}{900} = 10.7, \text{ nearly.}$$

The double belt would, therefore, be the more effectual remedy. Ans.

(777) (*a*) Substituting in formula **147,**

$18 = d\sqrt{\dfrac{400}{70}}$; or $d = 18 \div \sqrt{5.7143} = 7.53$ inches. Ans.

(*b*) Substituting in formula **146,** $N = \sqrt{70 \times 400} = 167\frac{1}{3}$ rev. per min. Ans.

(778) (*a*) Substituting in formula **144,**

$$D = \frac{n_1 d}{N} = \frac{400 \times d}{225} = 1.78\,d.$$

Ratio, therefore, $= 1.78 : 1$. Ans.

(*b*) Substituting in formula **146,**

$225 = \sqrt{400 \times n_2}$, or $n_2 = \dfrac{225^2}{400} = 126.56$ rev. per min. Ans

(779) First calculate the middle diameter, using formula **148.**

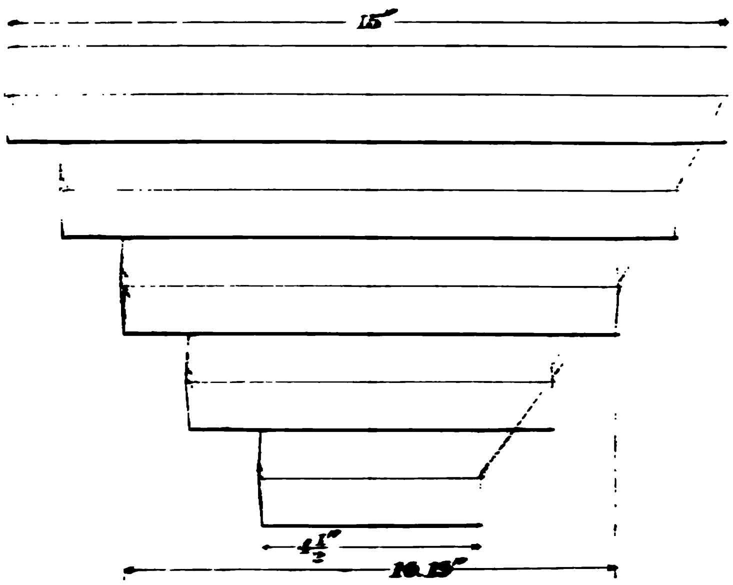


Fig. 78.

$$M = \frac{15 + 4.5}{2} + \frac{.08(15 - 4.5)^2}{20} = 9.75 + .44 = 10.19 \text{ inches}$$

One of the pulleys is shown in Fig. 78.

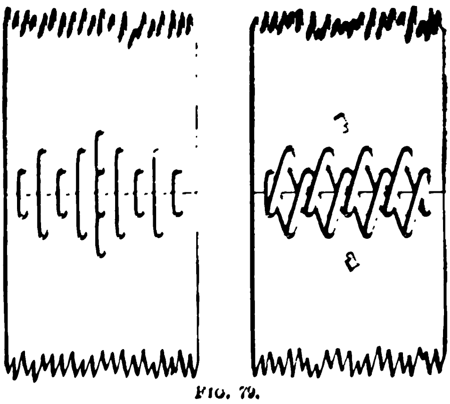


Fig. 79.

(780) See Fig. 79. An 8-inch belt should have a double row of holes, five in the row nearest the end. Space the holes equally.

(781) The arrangement of the pulleys is shown in Fig. 80. The belt leads from point a, on pulley A, into the plane

of pulley B; and from point b, on pulley B, into the plane of pulley A.

(782) Fig. 81 shows how the two shafts may be connected. The arrangement is entirely similar to that shown in Fig. 392.

(783) $\frac{60 \times 30 \times 60 \times 30}{15 \times 15 \times 10}$ = number of revolutions of last shaft = 1,440 rev. per min. Ans.

The crossed belt and gears each reverse the direction of rotation; the open belt does not change the direction. Hence, the first and last shafts turn in the same direction.

(784) Gears B and C, being idlers, do not affect the number of rotations made by D. Hence, D makes $\frac{90}{90} = 1$ turn for every turn of A. As the number of axes is even, the first and last turn in opposite directions.

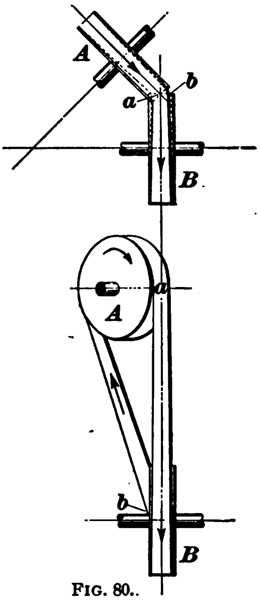


FIG. 80.

(785) Referring to Fig. 398, and applying formula **137**, we have $N \times d_1 \times d_2 = n \times f_1 \times f_2$, N and n being the number of revolutions of the spindle and lead screw, respectively. Solving for d_2,

$$d_2 = \frac{n \times f_1 \times f_2}{N \times d_1}.$$

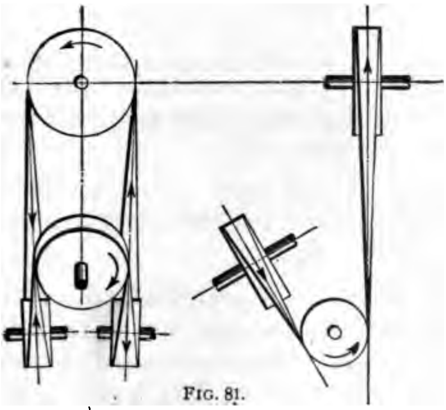


FIG. 81.

Now, to cut 4 threads per inch, the spindle must turn 4 times while the lead screw turns 8 times. Hence, sub-

stituting in the formula, we have $n = 8, f_1 = 2, N = 4, d_1 = 1$, and $d_2 = \frac{8 \times 2 \times f_2}{4 \times 1} = \frac{16 f_2}{4} = 4 f_2$.

Since $16 f_2$ remains constant for this lathe, we may hereafter write the formula as $d_2 = \frac{16 f_2}{N}$.

For 5 threads, $d_2 = \frac{16 f_2}{5} = \frac{16}{5} f_2$.

For 6 threads, $d_2 = \frac{16 f_2}{6} = \frac{8}{3} f_2$.

For 7 threads, $d_2 = \frac{16 f_2}{7} = \frac{16}{7} f_2$.

For 8 threads, $d_2 = \frac{16 f_2}{8} = 2 f_2$, etc.

To make out the table, we should aim to use the same gears as many times as possible. Start with 96 teeth for the gear on stud. Then, $f_2 = \frac{96}{4} = 24$ teeth. For 5 threads, $f_2 = \frac{5}{16} \times 96 = 30$, when $d_2 = 96$; for 6 threads, $f_2 = \frac{3}{8} \times 96 = 36$, when $d_2 = 96$; for 7 threads, $f_2 = \frac{7}{16} \times 96 = 42$; for 8 threads, $f_2 = \frac{1}{2} \times 96 = 48$. The other gears are found in the same manner.

(786) The gear makes $\frac{1}{2}$ a turn in passing the center at each end of the rack, two turns on top of the rack, and two turns while traveling over the under side.

(*a*) The circumference of the gear $= 10 \times 3.1416 = 31.416''$. Distance that the rack travels uniformly $= 2 \times 31.416 = 62.832'' = 5$ ft. $2.832''$. Ans.

(*b*) The motion is harmonic as the gear passes over the ends, and the horizontal distance traveled by the rack at each end = the radius of the gear; distance traveled at both ends $= 2 \times$ radius, or 10 inches. Hence, 5 ft. $2.832'' + 10'' = 6$ ft. $.832''$, the total travel. Ans.

(787) When driving through the worm and worm-wheel, 40 turns of the worm produce one turn of the driven shaft T. When driving through the gearing, 40 turns of the shaft S produce $\frac{40 \times 80}{20} = 160$ turns of T. Hence, the ratio of the "quick return" is 160 : 1.

(788) Circumference of gear $D_2 = 3.5 \times 3.1416 = 10.9956 = 11'$, nearly. Hence, for every foot travel of the table, D_2 makes $\frac{12}{11} = 1\frac{1}{11}$ revolutions. To find the number of turns made by the pulley for each foot passed over by the table, we have, therefore, $N \times 3.5 = \frac{12}{11} + 26$, or $N = \frac{12 \times 26}{11 \times 3.5} = 8.103+$. This, multiplied by the circumference of the pulley, $= \frac{8.103 \times 30 \times 3.1416}{12} = 63.6$. Hence, the ratio is 63.6 : 1. Ans.

(789) (*a*)

	D	F	A
Wheels locked	$+10$	$+10$	$+10$
Arm stationary	-10	$-10 \times \frac{100}{99}$	0
	0	$+10 - 10 \times \frac{100}{99}$	$+10$

Number turns of $F = 10 - \frac{1{,}000}{99} = -.101$ turn.

The wheel I, being an idler, it is not considered.

(*b*)

	D	F	A
Wheels locked	$+10$	$+10$	$+10$
	-10	$-10 \times \frac{100}{101}$	0
	0	$+10 - 10 \times \frac{100}{101}$	$+10$

Number turns of $F = 10 - \frac{1{,}000}{101} = +.099$. Ans.

(790) (*a*)

	D	F	A
Wheels locked	$+4$	$+4$	$+4$
Arm stationary	-4	$-\frac{5}{4}\times 4$	0
Arm stationary	$+1$	$+1\frac{1}{4}$	0
	$+1$	$+\frac{1}{4}$	$+4$

$F = +4 - 5 + 1\frac{1}{4} = +\frac{1}{4}$. Ans.

(*b*)

	D	F	A
Wheels locked	$+5$	$+5$	$+5$
Arm stationary	-5	$-\frac{5}{4}\times 5$	0
Arm stationary	-1	$-1\frac{1}{4}$	0
	-1	$-2\frac{1}{2}$	$+5$

$F = +5 - 6\frac{1}{4} - 1\frac{1}{4} = -2\frac{1}{2}$. Ans.

(791)

	D	F, D_1	F_1	A
Train locked	$+6$	$+6$	$+6$	$+6$
Arm fixed	-6	-60	$+\frac{40}{20}\times 60$	
	0	-54	$+126$	$+6$

(792) 20 turns of shaft S will produce $\frac{20\times 12\times 10}{3\times 100} =$ 8 turns of D, L. H. Hence,

	E	H, K	D
Wheels locked	-8	-8	-8
Arm fixed	$+8$	-8	0
Arm fixed	$+20$	-20	0
	$+20$	-36	-8

Pulley K makes -36 revolutions. Ans.

(793) See Arts. **1550** and **1558.**

(794) (*a*) See Art. **1552.**

(*b*) Substituting in formula **150,**

$$C = \frac{3.1416}{4\frac{1}{2}} = \frac{3.1416\times 2}{9} = .698 \text{ inch. Ans.}$$

(*c*) Substituting in formula **149,** $P = \frac{3.1416}{1.1424} = 2\frac{3}{4}$. Ans.

(795) (*a*) From formula **151**,

$$D = \frac{N}{P} = \frac{30}{3} = 10 \text{ inches. Ans.}$$

(*b*) Substituting in formula **152**,

$$OD = \frac{30+2}{3} = 10.667 \text{ inches. Ans.}$$

(*c*) Addendum $= \frac{1}{3} = .3333''$; working depth $= 2 \times .3333 =$ 667 inch.

(*d*) Clearance $= .3333 \times \frac{1}{8} = .042''$; whole depth $= .667 + .042 = .709''$. Ans.

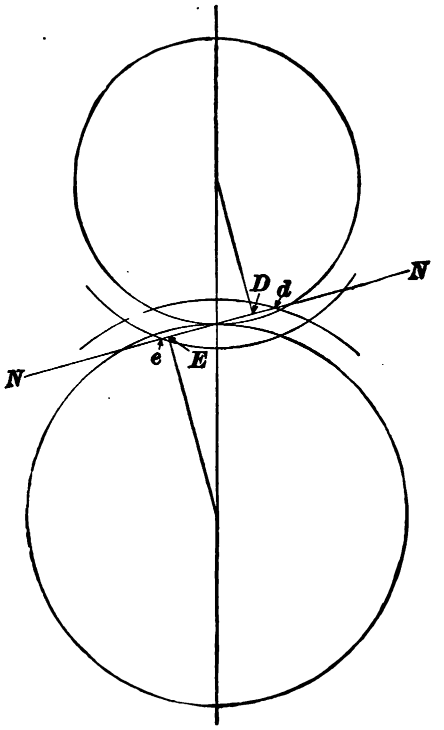


FIG. 82.

(796) See Fig. 414.

(797) (*a*) Number of teeth $= 48 \times 10 = 480$. Ans

(*b*) Number of teeth $= (14 - 2 \times \frac{1}{6}) \times 6 = 82$. Ans.

(798) From formula **153**, $D = \frac{1.25 \times 70}{3.1416} = 27.852$ inches. Ans.

(799) Substituting in formula **154**,

$$d = \frac{2AV}{V+v} = \frac{2 \times 15.5 \times 2}{2+3} = \frac{62}{5} = 12\tfrac{2}{5} \text{ inches. Ans.}$$

Substituting in formula **155**, $D = \frac{2 \times 15.5 \times 3}{3+2} = \frac{93}{5} = 18\tfrac{3}{5}$ inches. Ans.

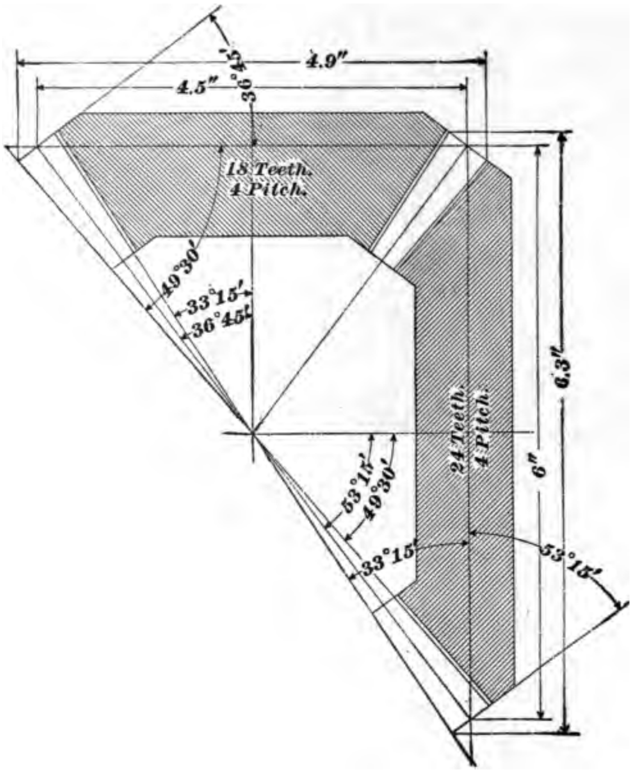


FIG. 83.

(800) (*a*) See Art. **1560.**

(*b*) See Art. **1563.**

(801) See Art. **1573.**

(802) Diameter of internal gear = 30 ÷ 3 = 10 inches. Diameter of pinion = 21 ÷ 3 = 7 inches.

Diameter of describing circle $= \frac{10 - 7}{2} = 1\frac{1}{2}$ inches.

See Art. **1565.**

(803) To determine whether two involute gears will interfere, use the construction shown in Fig. 425. In Fig. 82, it will be observed that the addendum circles of both gears cut the line of action $N\,N$, at points e and d, outside of points E and D. Hence, both gears will interfere.

(804) The solution to this example is shown in Fig. 83.

(805) Since the worm is triple-threaded, it will drive the wheel three teeth at every revolution; in 100 revolutions, it will drive it $100 \times 3 = 300$ teeth, or $\frac{300}{40} = 7.5$ revolutions. Ans.

(806) (a) The wheel must have the same circular pitch as the worm, or $\frac{3}{4}''$, and, as the worm is single-threaded, the number of teeth in the wheel is the same as the ratio of speeds, or 40. Hence, $D = \frac{\frac{3}{4} \times 40}{3.1416} = 9.55''$. From Table 34, the addendum $= .3 \times \frac{3}{4} = .23''$; outside diameter $= 9.55 + (2 \times .23) = 10.01$, or, say $10''$; whole depth, $.23 + (.35 \times \frac{3}{4}) = .23 + .26 = .49''$.

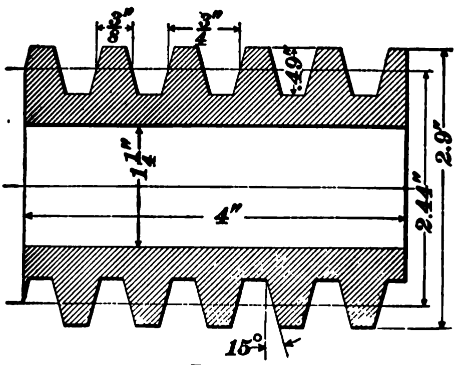


FIG. 84.

(b) The section of the worm is shown in Fig. 84.

(807) See Art. **1581.** Tangent of angle of thread $= \frac{\text{circum. of cylinder}}{\text{pitch}} = \frac{2 \times 3.1416}{\frac{1}{3}} = 2 \times 3 \times 3.1416 = 18.8496.$ The angle, therefore, equals 86° 58′, nearly. Ans.

(808) For every tooth moved, the screw would make $\frac{1}{72}$ of a turn. For every turn of the screw, the "feed" would be $\frac{1}{11\frac{1}{2}} = \frac{2}{23}$ inch. Hence, $\frac{1}{72} \times \frac{2}{23} = \frac{2}{1656} = \frac{1}{828}$ inch $=$ the required "feed." Ans.

APPLIED MECHANICS.

(QUESTIONS 809-858.)

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(809) Applying formula **156,** H. P. $= .0001904\, R\, W N$ $= .0001904 \times 4.5 \times 70 = 275 = 16.49$ H. P. Ans.

(810) From formula **156,**

$$R = \frac{\text{H. P.}}{.0001904\, W N} = \frac{6}{.0001904 \times 50 \times 200} = 3.15.$$

Might use an arm 3¼ or 3½ feet long. Ans.

(811) Substituting in formula **157,** H. P. $= .00001586\, r\, W N = .00001586\,(24 + .4) \times (315 - 8) \times 160 = 19.01$ H. P. Ans.

(812) (*a*) The angle of advance for a valve with neither lap nor lead is 0°.

(*b*) Cut-off would occur at the extreme end of the stroke when the valve was in mid-position.

(*d*) Adding lap would make the cut-off earlier, since it would necessitate turning the eccentric ahead in order to bring the lead right.

(813) Admission, cut-off, compression (on other end), release admission and cut-off (on other end), compression, release (on other end). See Figs. 452 and 453.

(814) (*a*) See Art. **1606.**

(*b*) See Art. **1606.**

(815) Increasing the lap without changing the angle of advance would delay admission, hasten cut-off, and decrease the port opening.

(816) First move the eccentric ahead. This will hasten the cut-off and also increase the lead. To diminish the lead

and make it the same as before, add lap to the valve, which will also have the effect of hastening the cut-off still more.

(817) Inside lap hastens compression and delays release.

(818) See Art. **1639.**

(819) Mark the position of the cross-head on the guides at the dead points. This gives the position of the ends of the stroke. With the engine on one center, say the forward center, set the valve with the proper lead. Now, move the crank forward until the valve cuts off, and measure the displacement of the cross-head. Move the crank forward until the valve cuts off on the return stroke, and measure the displacement of the cross-head. If the cut-off is earlier at the head than at the crank end, the valve spindle is too long, and *vice versa.* Suppose the valve spindle to be too long; then, to shorten it, turn the engine forward until it gives the head end lead, shorten the valve spindle, which will increase the lead; then, reduce the angle of advance until it gives the proper lead. Try the operation again. The lead at the other end of the cylinder will, of course, be too small, unless the engine is specially designed to give equal cut-off.

(820) Increasing the lead is accomplished by moving the eccentric ahead, which hastens the action of the valve, thus causing admission to occur earlier; when the crank reaches the center, therefore, the port will be open wider, or the lead will be greater. The only effect upon the port opening will be to make it occur earlier. It will not increase the port opening.

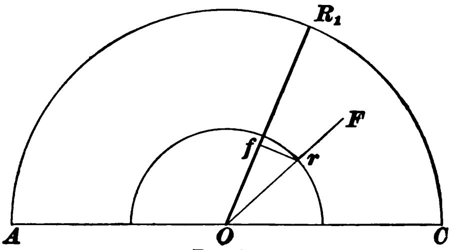

FIG. 85.

(821) (*a*) At mid-position, where the displacement is zero. The displacement of a valve is measured from mid-position.

(*b*) The diagram is shown in Fig. 85. The displacement of the valve for the crank position is equal to $r\ f$.

(822) (*a*) The port opening is always equal to the displacement minus the lap. See Art. **1611.**

(*b*) See Arts. **1613** and **1614** and Figs. 446, 447, and 448, etc.

(823) (*a*) See Art. **1622.** Since the steam should not flow over 6,000 feet per minute, we must have the area of the port $\times$ 6,000 = area of piston $\times$ piston speed in feet per minute.

$$\text{Area of piston} = 18^2 \times .7854 = 254.47 \text{ sq. in.}$$

$$\text{Piston speed} = \frac{40}{12} \times 2 \times 90 = 600.$$

Hence, area of port $\times$ 6,000 = 254.47 $\times$ 600, or

$$A = \frac{254.47 \times 600}{6{,}000} = 25.45 \text{ sq. in.}$$

$25.45 \div 18 = 1.41''$, or $1\frac{7}{16}''$, nearly. Ans.

(*b*) Assume a port opening of .8 of this, or $1.41 \times .8 = 1.128''$, say $1\frac{1}{8}''$. Ans.

(*c*) See Art. **1623.** The width of the exhaust port $= 1\frac{7}{16} + 3 + \frac{1}{8} - \frac{15}{16} = 3\frac{5}{8}''$. Ans.

(824) The diagram is shown in Fig. 86. Draw the valve circle with a radius of $2\frac{1}{2}''$, and the lap circle with a radius of $\frac{7}{8}''$, allowing $\frac{1}{16}''$ lead as shown. Maximum port

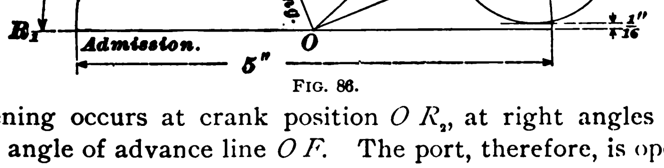

FIG. 86.

opening occurs at crank position $O\ R_2$, at right angles to the angle of advance line $O\ F$. The port, therefore, is open from crank position $O\ R_1$, at admission to $O\ R_2$, and is being closed from $O\ R_2$ to crank position at cut-off at $O\ R_3$.

(825) The diagrams are shown in Figs. 87 and 88. The crank positions at cut-off and release must be found by the

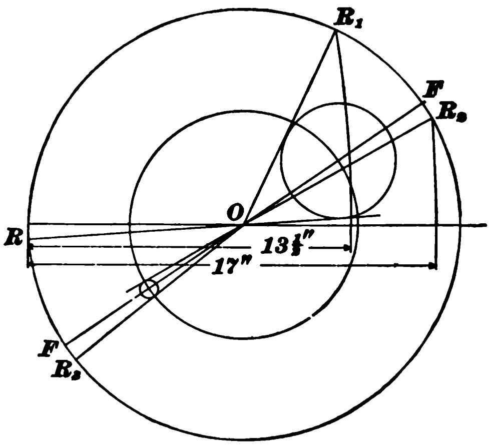

FIG. 87.

method explained in connection with Figs. 445 and 455. In Fig. 87, $O\,R$ is the point of admission; $O\,R_1$, of cut-off; $O\,R_2$, of release, and $O\,R_3$, of compression for the head end. In Fig. 88, the same letters refer to corresponding positions of the crank end. $F\,F$ is the angle of advance line in each case, and the diameters of the outside and inside lap circles are determined by drawing these circles tangent to the given crank positions for cut-off and release, respectively.

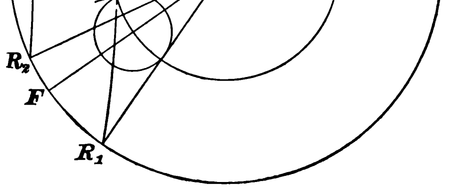

FIG. 88.

(826) The length of stroke of the piston must be known and the travel of the valve, both of which can be determined

by turning the engine through a revolution. The other dimensions easiest obtained would be the lead and laps, the former by measuring when the valve-chest cover is off and the engine is on a center, and the latter by removing the valve, taking its dimensions, and also the dimensions of the valve seat. Having these data, the diagram is easily drawn.

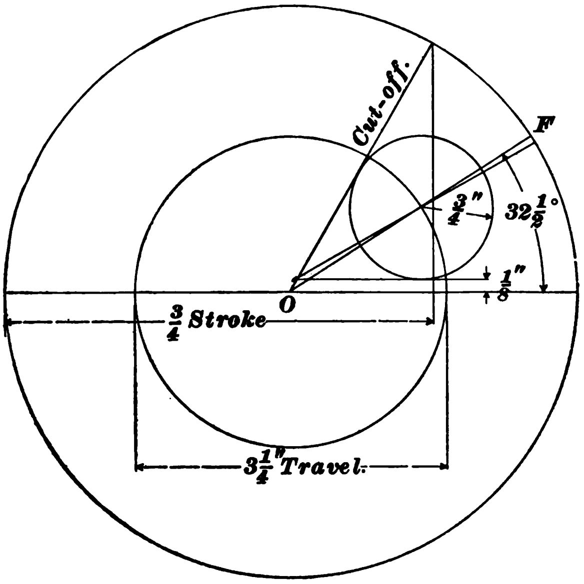

FIG. 89.

(827) The method of solving this problem is entirely similar to the method used in the general problem, Arts. **1627** to **1631.** The diagram is shown in Fig. 89; $O\ F$ is the angle of advance line.

(828) First calculate the width of steam ports and maximum port opening.

$$\text{Area of piston} = 17^2 \times .7854 = 226.98 \text{ sq. in.}$$

$$\text{Area of port} \times 6{,}000 = 226.98 \times \frac{26 \times 2 \times 150}{12} = 24.59 \text{ sq. in.}$$

This, divided by 17, which we will assume to be the length of port, gives $1.44 = 1\frac{7}{16}''$, nearly, as the width.

(849) See Art. **1650.**

(850) No. Changing the angle of advance varies the lead.

(851) See Fig. 93. First find the crank positions $O\ R$ and $O\ R'$ for the latest and earliest points of cut-off. Draw the lap circle about C tangent to $O\ R$ and draw the lead line $l\ l$. Through C draw the horizontal line $C\ h$, and with the compass point upon this line draw a lap circle of the same size as the former one, tangent to $O\ R'$. Its center will fall at D, and $C\ D$ is the distance that the eccentric must shift. The port opening for shortest cut-off is equal to $O\ d$, or $\frac{1}{8}$ inch.

(852) The valve diagram is laid out for full gear posi-

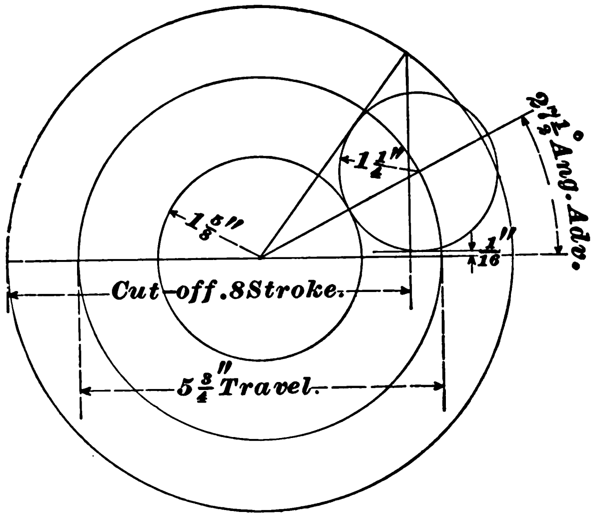

FIG. 94.

tion as shown in Fig. 94. This part of the process is like designing a plain slide valve, remembering that the valve is to over-travel $\frac{3}{8}$ of an inch.

In Fig. 95 are drawn the two link diagrams. In each, the eccentrics are drawn with the angle of advance found above. These diagrams are similar to and are lettered the same as the diagrams shown in Figs. 473 and 474, and further explanation seems to be unnecessary.

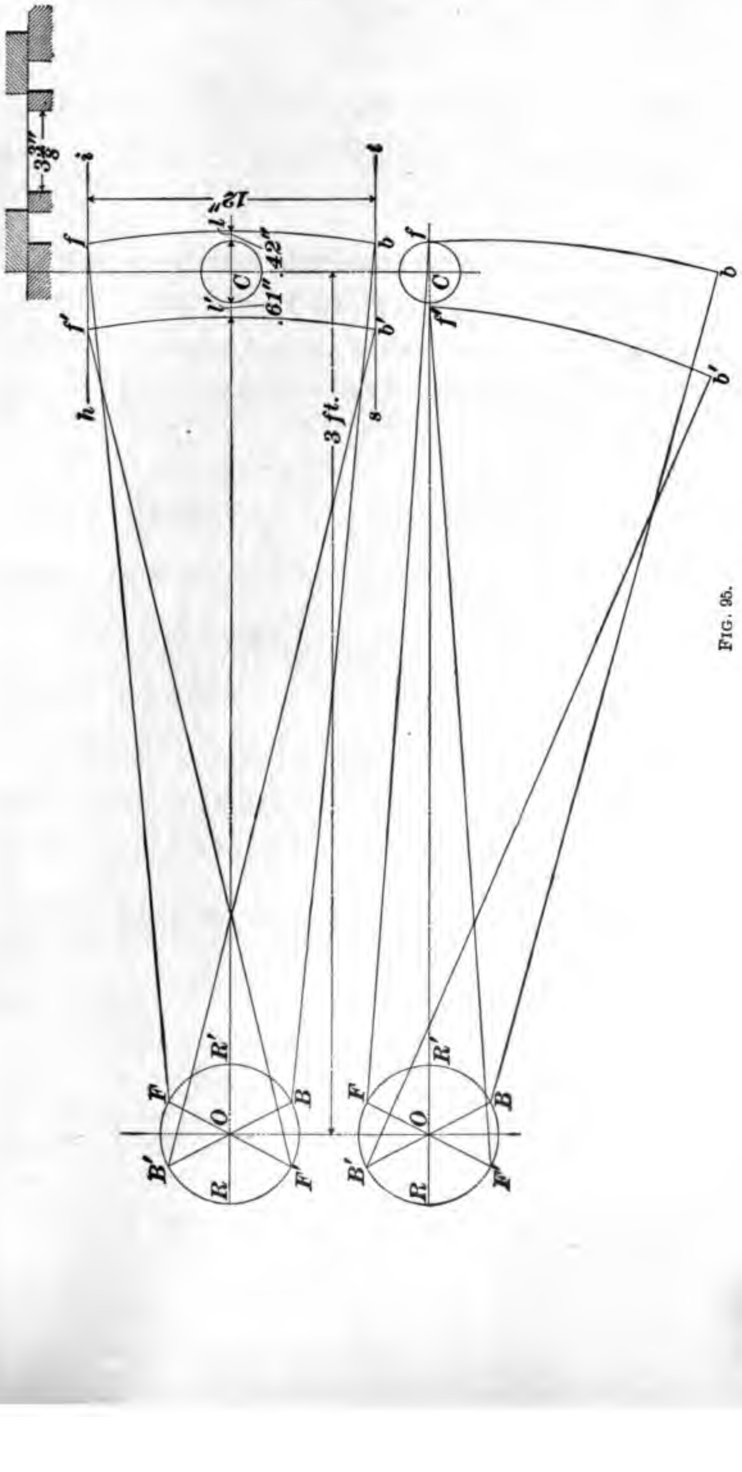

FIG. 95.

(853) (*a*) See Art. **1668.**
(*b*) See Art. **1675.**

(854) Use formula **158,** $N^2 = \frac{35,294}{h}$. For a height of 20″ we have $N^2 = 35,294 \div 20$, or $N = 42.008$ rev.; for a height of 18″, $N = 42.28$; for a height of 22″, $N = 40.053$ rev.

(*a*) $44.28 - 42.008 = 2.272$ rev. increase. Ans.
(*b*) $42.008 - 40.053 = 1.955$ rev. decrease. Ans.

(855) (*a*) See Art. **1674.**

(*b*) Compare formula **159** with formula **158,** Art. **1668.** Since W equals five times the weight of both balls together, it is equal to $10 \times B$ and

$$H = \left(1 + \frac{W}{B}\right) h = \left(1 + \frac{10}{1}\right) \times 1 = 1 + 10 = 11''. \quad \text{Ans.}$$

(856) See Art. **1676.**

(857) Substituting in formula **163,**

$$B = \frac{5}{.00000114 \times 10 \times 240^2} - \frac{4 \times 5.85 \times 6.5 + (3 \times 5.4 \times 14)}{9 \times 10} =$$
3.4045 lb.

Substituting in formula **162,**

$$W = \tfrac{1}{9}\,[.000028 \times 240^2 (4 \times 5.85 \times 6.5 + 3.4045 \times 9 \times 10 + 3 \times 5.4 \times 14) - (3.4045 \times 9 + 4 \times 5.85 + 3 \times 5.4)] = 115 \text{ lb.} \quad \text{Ans.}$$

(858) See Art. **1591.**

STEAM BOILERS.

(QUESTIONS 859–958.)

(859) See Arts. **1698** and **1702.**

(860) See Arts. **1764** to **1791.**

(861) Applying formula **168,**

$$t = \sqrt{\frac{p\,l\,d}{1,600,000}} = \sqrt{\frac{80 \times 15 \times 12 \times 3}{1,600,000}} = \frac{3}{16}'', \text{ nearly. Ans.}$$

(862) Applying formula **177,**

$$d = 1.13\sqrt{\frac{A\,p}{T}} = 1.13\sqrt{\frac{12 \times 13\frac{1}{2} \times 125}{6,000}} = 2\frac{1}{16}'', \text{ nearly. Ans.}$$

(863) Applying formula **184,**

$$t = h\sqrt{\frac{p}{k}} = 8\sqrt{\frac{150}{40,000}} = .49'', \text{ say } \frac{1}{2}''. \text{ Ans.}$$

(864) (*a*) Using formula **190,**

$$h = 14,500 \times .80 + 62,032\left(.08 - \frac{.06}{8}\right) = 16,097.32 \text{ B.T.U. Ans.}$$

(*b*) Using formula **189,**

$$W = 11.6 \times .80 + 34.8\left(.08 - \frac{.06}{8}\right) = 11.8 \text{ lb. Ans.}$$

(865) Since the number of B. T. U. contained in the gases is directly proportional to the temperature, 600° − 250° = 350° may be considered to represent the amount of heat saved. Then,

$$350 \div 2,800 = .125 = 12.5\%. \text{ Ans.}$$

(866) From Table 41, the factor of evaporation for 120° feed and 80 lb. pressure = 1.131; for 120° feed and 90 lb. pressure = 1.133; hence, the factor for 120° feed and 85 lb. pressure is evidently 1.132. In a similar manner it is found

that for 130° feed and 85 lb. pressure, the factor is 1.121. Consequently, for a difference of 10° in the feed, difference in the factors is 1.132 − 1.121 = .011 = .0011 for a difference of 1°. 127° − 120° = 7°, and .0011 × 7 = .0077 = difference for a difference of 7°. Hence, the factor for 127° feed and 85 lb. pressure = 1.132 − .0077 = 1.1243, and $\frac{28,930 \times 1.1243}{34.5 \times 10}$ = 94.3 H. P. Ans.

(**867**) From the table of the Properties of Saturated Steam, the latent heat of steam at 80 lb. gauge pressure is 886.59, and the temperature is 323.71. Applying formula **204**, and substituting,

$$Q = \frac{1}{886.59}\left[\frac{360}{22}(110 - 44) - (323.71 - 110)\right] = 97.7\%. \text{ Ans.}$$

(**868**) From the steam table, the volume of a pound of steam at 75 lb. gauge pressure is 4.811 cu. ft. Then, the area of the cross-section of the pipe must be

$$\frac{7,200 \times 4.811 \times 144}{4,000 \times 60} = 20.78 \text{ sq. in.},$$

and the diameter not less than $5\frac{3}{16}''$. Ans.

(**869**) This example might be worked by applying formula **186** for each pressure; but since there is a constant difference of 5 lb. between any two consecutive pressures, there will also be a constant difference between any two consecutive graduations, and the example may be worked as follows: Applying formula **186** for a pressure of 40 lb.,

$$d = \frac{4.5\,(40 \times 12.5664 - 8.5) - 21 \times 19}{107} = 17.05''. \text{ Ans.}$$

Applying again for 60 lb.,

$$d = \frac{4.5\,(60 \times 12.5664 - 8.5) - 21 \times 19}{107} = 27.62''. \text{ Ans.}$$

Then, $\frac{27.62 - 17.05}{4} = 2.64\frac{1}{4}''$ = distance which weight must be moved in order to produce a difference of 5 lb. per

sq. in. in the blow-off pressure. Hence, for a blow-off pressure of 45 lb.,

$$d = 17.05 + 2.64\frac{1}{4} = 19.69''. \quad \text{Ans.}$$

For 50 lb., $d = 17.05 + 2.64\frac{1}{4} \times 2 = 22.34''$. Ans.

And for 55 lb., $d = 17.05 + 2.64\frac{1}{4} \times 3 = 24.98''$. Ans.

(870) See Art. **1695**,

(871) (*a*) 4 ft. 3 in. = 4.25 ft. Total force exerted along one seam $= \frac{12 \times 4.25 \times 144 \times 80}{2} = 293{,}760$ lb. Ans.

(*b*) Total force along a girth seam $= 4.25^2 \times .7854 \times 80 \times 144 = 163{,}426$ lb. Ans.

(872) (*a*) to (*d*) See Arts. **1736** to **1739.**

(*e*) The double-riveted butt joint. (See formula **171.**)

(873) Applying formula **179**,

$$a = .885\, d\sqrt{\frac{T}{p}} = .885 \times 1\frac{3}{4}\sqrt{\frac{9{,}000}{140}} = 12.42'', \text{ say } 12\frac{1}{2}''. \quad \text{Ans.}$$

(874) (*a*) and (*b*) See Arts. **1764** and **1765.**

(*c*) See Art. **1770.**

(*d*) See Art. **1772.**

(875) (*a*) See Art. **1804.**

(*b*) See Art. **1805.** (*c*) $400 \times .4 = 160$ lb. Ans.

(*d*) See Art. **1805.**

(876) Using formula **193**,

$$p = H\left(\frac{7.6}{T_a} - \frac{7.9}{T_c}\right),$$

or $$H = \frac{p}{\frac{7.6}{T_a} - \frac{7.9}{T_c}} = \frac{.77}{\frac{7.6}{460 + 60} - \frac{7.9}{460 + 580}} = 110 \text{ ft.} \quad \text{Ans.}$$

(877) See Art. **1833.**

[illegible]

[illegible] evaporation for a [illegible] pressure [illegible] steam pressure [illegible]

[illegible] 1557. See Arts. 1532 and 1513.

[illegible] 1558. See Arts. 1559 to 1567.

[illegible] 1559. See Arts. 1597 and 1598.

[illegible] 1560. See Arts. 1707 to 1709 and Art. 1879.

(891) Applying formula **165** and taking $\frac{S_1}{f} = 9{,}500$,

$$t = \frac{fpd}{2 S_1 y} = \frac{1 \times 110 \times 38}{2 \times 9{,}500 \times .60} = .37'' = \frac{3}{8}''. \text{ Ans.}$$

(892) See Arts. **1740** and **1741.**

(893) Using formula **178,**

$$d = 1.13\, a \sqrt{\frac{p}{T}} = 1.13 \times 4.5 \sqrt{\frac{160}{8{,}000}} = .719'', \text{ say } \frac{3}{4}''. \text{ Ans.}$$

(894) (*a*) $120 \times 30 = 3{,}600$ lb. of steam per hour. Applying formula **188,**

$$a = \frac{.5\, w}{p + 10} = \frac{.5 \times 3{,}600}{70 + 10} = 22.5 \text{ sq. in. Ans.}$$

(*b*) $22.5 \times 70 = 1{,}575$ lb. Ans.

(895) See Art. **1802.**

(896) (*a*) Applying formulas **197** and **194,**

$$d = 13.54 \sqrt{E} + 4 = 13.54 \sqrt{\frac{.3 \text{ H. P.}}{\sqrt{H}}} + 4 =$$

$$13.54 \sqrt{\frac{.3 \times 700}{\sqrt{105}}} + 4 = 65.3''. \text{ Ans.}$$

(*b*) Applying formulas **196** and **194,**

$$S = 12 \sqrt{\frac{.3 \times 700}{\sqrt{105}}} + 4 = 58.3''. \text{ Ans.}$$

(897) Using formula **202,** in which $W = 150 \times 34.5$,

$$G = \frac{W}{Fe} = \frac{150 \times 34.5}{16 \times 10.25} = 31.55 \text{ sq. ft.} \quad 31.55 \times 36 = 1{,}136 \text{ sq. ft. Ans.}$$

(898) See Arts. **1868** to **1873.**

(899) Applying formula **200,**

$F = 2.25 \sqrt{H} = 2.25 \sqrt{135} = 26.14$ lb. of coal per sq. ft. of grate per hour. Then, $26.14 \times 3.5 \times 5 \times 10 = 4{,}575$ lb. Ans.

(900) See Arts. **1717** to **1721.**

(901) Applying formula **165**,

$$t = \frac{fpd}{2S_t y} = \frac{4 \times 140 \times 13 \times 12}{2 \times 64,000 \times .72} = \frac{15}{16}''. \quad \text{Ans.}$$

(902) Applying formula **170**,

$$h = \left(1 + \frac{7t}{4p}\right)d = \left(1 + \frac{7}{4} \times \frac{.75}{.375}\right) \times .75 = 3\frac{3}{8}''. \quad \text{Ans.}$$

(*b*) Applying formula **172**,

$$d = t + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} = \frac{3}{4}''. \quad \text{Ans.}$$

(*c*) Applying formula **173**,

$$y = \frac{h - d}{h} = \frac{3\frac{3}{8} - \frac{3}{4}}{3\frac{3}{8}} = .78 = 78\%. \quad \text{Ans.}$$

(903) Using formula **179**,

$$a = .885\, d \sqrt{\frac{T}{p}} = .885 \times 1 \sqrt{\frac{6,000}{150}} = 5.6''. \quad \text{Ans.}$$

(904) See Arts. **1773** to **1782.**

(905) 11.6 × .95 = 11.02 lb. of air required per pound of coal. Then, since 1 lb. of air at 62° has a volume of 13.14 cu. ft., the total number of cubic feet to be furnished per minute =

$$11.02 \times 1\frac{1}{2} \times 13.14 \times \frac{4 \times 6 \times 12}{60} = 1,042.6 \text{ cu. ft.} \quad \text{Ans.}$$

(906) (*a*) From Table 38, 632 H. P. Ans.

(*b*) Applying formulas **195** and **194**, we have, since 3 ft. 8" = 44",

$$\text{H. P.} = 3.33\, E \sqrt{H} = 3.33\,(A - .6\sqrt{A})\sqrt{H} =$$

$$3.33\left(\frac{44^2}{144} - .6\sqrt{\frac{44^2}{144}}\right)\sqrt{80} = 336 \text{ H. P.} \quad \text{Ans.}$$

(907) From Table 41, the factor of evaporation for a feed temperature of 120° and a steam pressure of 105 lb. gauge = 1.137; and for 200° feed and 105 lb. gauge = 1.054. Then,

$$\text{gain} = \frac{1.137 - 1.054}{1.137} = .073 = 7.3\%. \quad \text{Ans.}$$

(908) See Arts. **1876** and **1877.**

(909) (See Art. **1900.**) $300 \times 30 = 9{,}000$ lb. of water required per hour. Adding 15% for emergencies and 20% for leakage of water past the piston, etc., $9{,}000 + 9{,}000\,(.15 + .20) = 12{,}150$ lb. per hour $= \frac{12{,}150}{62.5} = 194.4$ cu. ft. per hour. Assuming the pump to make 40 strokes per minute, the displacement per stroke $= \frac{194.4 \times 1{,}728}{60 \times 40} = 140$ cu. in. Taking the stroke as twice the diameter,

$$d^2 \times .7854 \times 2\,d = 140, \text{ or } d = \sqrt[3]{\frac{140}{2 \times .7854}} = 4.47'', \text{ say } 4\tfrac{1}{2}''. \quad \text{Ans.}$$

Then, stroke $= 4\frac{1}{2} \times 2 = 9''$. Ans.

(910) See Art. **1716.**

(911) In Art. **1366** it is stated that a cylinder is twice as strong against transverse rupture as it is against longitudinal rupture. Since, in example 901, the efficiency of the joint is 72%, the relative strengths of the two seams are $\frac{2 \times .72}{1 \times .72} = \frac{2}{1}$, i.e., the girth seam is twice as strong as the longitudinal seam.

(912) See Arts. **1744** and **1745.**

(913) Applying formula **180,**

$$d = 1.13\sqrt{\frac{Ap}{T \cos B}} = 1.13\sqrt{\frac{35 \times 80}{6{,}000 \cos 18^\circ}} = .791'', \text{ say } \frac{13}{16}''. \quad \text{Ans.}$$

(914) Applying formula **185,**

$$W = \frac{a\,(pA - W_1) - W_2 c}{d} = \frac{4\,(110 \times 5^2 \times .7854 - 8) - 12 \times \frac{40}{2}}{40} =$$

209.2 lb. Ans.

(915) See Arts. **1806** and **1807.**

(916) The easiest and most accurate way to solve this example is as follows: Let H, T_a, and T_c represent the height

[illegible]

(917) [illegible]

(918) See Arts. 1878 and 1879.

(919) See Art. 1896.

(920) See Arts. 1722 to 1728.

(921) (a) and (b) See Art. 1750.

(c) The spherical vessel. Art. 1366.

(922) See Arts. 1748 and 1749.

(923) Using formula 181,

$$d = \sqrt[3]{\frac{p h l^2}{6,000}} = \sqrt[3]{\frac{120 \times 8 \times 33^2}{6,000}} = 5.585'', \text{ say } 5\tfrac{5}{8}''. \text{ Ans.}$$

(924) Applying formula **186,**

$$d = \frac{a(pA - W_1) - W_2 c}{W} = \frac{4(110 \times 5^2 \times .7854 - 8) - 12 \times 20}{265}$$

$= 31.6''$. Ans.

(925) See Art. **1808.**

(926) Applying formula **200,**

(*a*) $2.25\sqrt{H} = 2.25\sqrt{80} = 20\frac{1}{8}$ lb. per sq. ft. of grate per hour. Ans.

(*b*) $2.25\sqrt{115} = 24\frac{1}{8}$ lb. per sq. ft. of grate per hour. Ans.

(927) See Arts. **1840** to **1852.**

(928) See Art. **1880.**

(929) (*a*) At 65 lb. gauge pressure the latent heat of steam is (see steam table) 895.296 B. T. U., and the temperature is 311.605°. Applying formula **204,**

$$Q = \frac{1}{l}\left[\frac{W}{w}(t_2 - t_1) - (t - t_2)\right] =$$

$$\frac{1}{895.296}\left[\frac{327}{24}(120 - 40) - (311.605 - 120)\right] = 1.00346. \text{ Ans.}$$

(*b*) The answer to (*a*) shows that the steam is superheated, and the number of degrees of superheat can be determined by means of the expression given in Art. **1853.** Thus,

$$\frac{(Q-1)l}{.48} = \frac{(1.00346 - 1)\,895.296}{.48} = 6.5°, \text{ nearly. Ans.}$$

(930) See Arts. **1725, 1726,** and **1714.**

(931) Applying formula **167,** and substituting 12,000 for $\frac{S_1}{f}$,

$$p = \frac{2tS_1y}{fd} = \frac{2 \times \frac{5}{16} \times 12{,}000 \times .66}{42} = 118 \text{ lb. per sq. in. Ans.}$$

(932) Solving formula **174** for p, and substituting,

$$p = \frac{3\,t^2 S}{2\,r^2} = \frac{3 \times \left(\frac{1}{2}\right)^2 \times 10,000}{2 \times 24^2} = 6.5 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(933) Applying formula **182**,

$$p = \frac{6,000\,t^2}{[illegible]} = \frac{6,000 \times 7.5^2}{[illegible] \times 42^2} = 143.5 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(934) See Arts. **1783** to **1791.**

(935) See Arts. **1820, 1812,** and **1823.**

(936) Applying formula **199,**

$$F = 1.5\sqrt{H} - 1, \text{ or } H = \left(\frac{F+1}{1.5}\right)^2 = \left(\frac{14+1}{1.5}\right)^2 = 100 \text{ ft.}$$

Ans.

(937) (*a*) See Art. **1841.**

(*b*) Total heat of 1 lb. of steam at 60 lb. gauge pressure = 1,175.625 B. T. U. Heat units required to vaporize the water at 52° into steam at 60 lb. gauge = 1,175.625 − 52 + 32 = 1,155.625 B. T. U.

$$\text{Efficiency} = \frac{1,155.625 \times 175,000}{19,000 \times 13,500} = 78.8\%. \quad \text{Ans.}$$

(938) See Art. **1866.**

(939) See Art. **1861.**

(940) See Art. **1712.**

(941) See solution to (*b*), example 929.

$$\frac{(Q-1)\,l}{.48} = \frac{(1.0148-1)\,884.98}{.48} = 27.29°. \quad \text{Ans.}$$

(942) Solving formula **174** for t and applying,

$$t = \sqrt{\frac{2\,r^2 p}{3\,S}} = \sqrt{\frac{2 \times 17^2 \times 75}{3 \times 5,000}} = 1.7''. \quad \text{Ans.}$$

(943) Applying formula **183,**

$$p = \frac{k\,t^2}{h^2} = \frac{28,000 \times \left(\frac{5}{16}\right)^2}{4.5^2} = 135 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(944) See Arts. **1794** and **1800.**

(945) $\frac{175 \times 40}{60} = 116\frac{2}{3}$ lb. of coal per minute.

Applying formula **191,**

$$\text{H. P.} = \frac{p W V}{33{,}000\, y} = \frac{12 \times 116\frac{2}{3} \times 230}{33{,}000 \times .625} = 15.61 \text{ H.P.} \quad \text{Ans.}$$

(946) $\frac{3{,}000}{9\frac{1}{2} \times 15} = 21$ sq. ft. Ans.

(947) (*a*) $\frac{175{,}000 \times 1{,}155.625}{966.1} = 209{,}331$ lb. Ans.

(*b*) $209{,}331 \div 19{,}000 = 11.02$ lb. per pound of coal. Ans.

(*c*) $209{,}331 \div (19{,}000 \times .93) = 11.85$ lb. per pound of combustible. Ans.

(*d*) Horsepower $= \frac{209{,}331}{34.5 \times 22} = 275.8$ H.P. Ans.

(948) See Arts. **1881** to **1883** and **1698** to **1703.**

(949) Applying formula **190** to find the heat of combustion, $h = 14{,}500 \times .643 + 62{,}032 \left(.042 - \frac{.100}{8}\right) = 11{,}153.5$ B. T. U. Heat required to change 1 lb. of water at 100° into steam having a temperature of 240° is found from the steam table to be 1,087.14 B. T. U. Hence, $\frac{11{,}153.5}{1{,}087.14} = 10.26$ lb. Ans.

(950) From Table 41, the factor of evaporation for 55° feed and 40 lb. gauge pressure is 1.187. Consequently, $\frac{935 \times 1.187}{34.5} = 32.2$ H. P. nearly. Ans.

(951) Applying formula **166,**

$$p = \frac{2 t S_1}{f d} = \frac{2 \times \frac{3}{8} \times 12{,}000}{30} = 300 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(952) Applying formula **176**,

$$t = \frac{a}{2}\sqrt{\frac{p}{S}} = \frac{20}{2}\sqrt{\frac{30}{12,000}} = \frac{1}{2}''. \quad \text{Ans.}$$

(953) Applying formula **184**,

$$t = h\sqrt{\frac{p}{k}} = 4\sqrt{\frac{110}{24,000}} = .2712'', \text{ say } \frac{9}{32}''. \quad \text{Ans.}$$

(954) $18 - 11.8 = 6.2$ lb. free air. $8 + 6 = 14\%$ of hydrogen and oxygen. Then, $H - \frac{O}{8} = .08 - \frac{.06}{8} = .0725 =$ 7.25% free hydrogen. Therefore, percentage of water $=$ $14 - 7.25 = 6.75\%$, and the composition of the fuel may be written $C = 80\%$, $H = 7.25\%$, water 6.75%, and ash 6%. The products of combustion are:

Carbonic acid	$80 \times 3.67 =$	2.936 lb.
Water (steam) ..	$.0675 + .0725 \times 9 =$	0.72 lb.
Nitrogen	$11.8 \times .77 =$	9.086 lb.
Free air	$=$	6.2 lb.
Ash	$=$	0.06 lb.
Total	$=$	19.00 lb.

Then, as in Art. **1802**,

$$t = \frac{16,097.32 - .72 \times 966.1}{2.936 \times .217 + .72 \times .4805 + 9.086 \times .2438 + 6.2 \times .2375} =$$

3,297°. Ans.

(955) See Arts. **1816** and **1814**.

(956) $\frac{28,930 \times 1.1243}{3,120} = 10.43$ lb. of water per pound of coal from and at 212°. Ans.

(957) See Art. **1853**.

(958) See Arts. **1887** to **1893**.

Applying formula **206**,

$$t = 0.0001pd + \frac{1}{8} = 0.0001 \times 105 \times 4\frac{1}{2} + \frac{1}{8}'' = .172'',$$

$$\text{say } \frac{11}{64}''. \quad \text{Ans.}$$

MACHINE DESIGN.

(QUESTIONS 959-1023.)

(959) See Art. **1910.**

(960) See Art. **1911.**

(961) (*a*) See Art. **1915.**

(*b*) See Art. **1912.**

(962) See Arts. **1924, 1925, 1927,** and **1928.**

(963) From Table 43 the number of threads per inch corresponding to the diameters given are 13, 9, 7, and $4\frac{1}{2}$, respectively. Applying formula **211:**

$$d_1 = d - \frac{1.3}{n}'' = \frac{1}{2} - \frac{1.3}{13}'' = .4''. \text{ Ans.}$$

$$d_1 = d - \frac{1.3}{n}'' = \frac{7}{8} - \frac{1.3}{9}'' = .731''. \text{ Ans.}$$

$$d_1 = d - \frac{1.3}{n}'' = 1\frac{1}{4} - \frac{1.3}{7}'' = 1.064''. \text{ Ans.}$$

$$d_1 = d - \frac{1.3}{n}'' = 2 - \frac{1.3}{4\frac{1}{2}}'' = 1.711''. \text{ Ans.}$$

(964) (*a*) Apply formula **208.**

For $\frac{1}{2}''$ diameter, $p = .24\sqrt{.25 + .625} - .175'' = .0495''$

For $\frac{5}{8}''$ diameter, $p = .24\sqrt{.625 + .625} - .175'' = .0933''$

For $1''$ diameter, $p = .24\sqrt{1 + .625} - .175'' = .131''$.

For $1\frac{1}{2}''$ diameter, $p = .24\sqrt{1.5 + .625} - .175'' = .175''$.

Now, applying formula **210,**

$$n = \frac{1}{.0495} = 20.2, \text{ say 20 threads per inch. Ans}$$

$$n = \frac{1}{.0933} = 10.7, \text{ say 11 threads per inch. Ans.}$$

$n = \frac{1}{.131} = 7.6$, say 8 threads per inch. Ans.

$n = \frac{1}{.175} = 5.7$, say 6 threads per inch. Ans.

(*b*) By formula **210**, $n = \frac{1}{p}$; or, $p = \frac{1}{n}$. Hence,

$p = \frac{1}{20}'' =$ pitch for $\frac{1}{4}''$ screw. Ans.

$p = \frac{1}{11}'' =$ pitch for $\frac{5}{8}''$ screw. Ans.

$p = \frac{1}{8}'' =$ pitch for $1''$ screw. Ans.

$p = \frac{1}{6}'' =$ pitch for $1\frac{1}{4}''$ screw. Ans.

(965) (*a*) See Art. **1931.**

(*b*) $\frac{1}{2}'' \div 3 = \frac{1}{6}''$. Ans.

(966) See Art. **1930.**

(967) From Table 25, the ultimate tensile strength of wrought iron is 55,000 lb. per sq. in. From Table 28, the factor of safety for a varying stress is 6. Hence, the safe stress per square inch is $\frac{55,000}{6}$, and $11,800 \div \frac{55,000}{6} = \frac{11,800 \times 6}{55,000} = 1.2873$ sq. in. = area at bottom of thread. From Table 43, the nearest area in the last column is 1.2928 sq. in., corresponding to a nominal diameter of $1\frac{1}{2}''$. Hence, the bolt should have a diameter of $1\frac{1}{2}''$. Ans.

(968) Using formula **217**,

$$a = \frac{11,800}{6,000} = 1.967 \text{ sq. in.}$$

From Table 43, the nearest diameter corresponding to this area is $1\frac{7}{8}''$. Ans.

The reason that this diameter should be used instead of that calculated in the last example is that the bolt is greatly weakened by the sharp corners left by the thread cutting tool,

and the strength is not equal to that of a round bar whose diameter is the same as the diameter at the bottom of the thread.

(969) Using formula **218,**

$$d = .0228\sqrt{12,000} = 2\tfrac{1}{2}''. \quad \text{Ans.}$$

$$d_1 = 2.5 \times \frac{4}{5} = 2''. \quad \text{Ans.}$$

Using formula **219,**

$$n_1 = \frac{12,000}{300 \times 2^2} = 10 \text{ threads.} \quad \text{Ans.}$$

(970) The dimensions of both head and nut are the same. Hence, apply formulas **220-226.**

Height of head or nut =

$$h = \frac{7}{8}'' - \frac{1}{16}'' = \frac{13}{16}''. \quad \text{Ans.}$$

Diameter of nut or head across flats =

$$D = 1\frac{1}{2} \times \frac{7}{8}'' + \frac{1}{16}'' = 1\frac{3}{8}''. \quad \text{Ans.}$$

Diameter of nut or head across corners =

$$D_1 = 1.73 \times \frac{7}{8}'' + .07'' = 1.58''. \quad \text{Ans.}$$

Diameter of washer =

$$D_2 = 1\frac{1}{8} \times 1.58'' = 1.78'', \text{ say } 1\frac{25}{32}''. \quad \text{Ans.}$$

Thickness of washer =

$$t = .15 \times \frac{7}{8}'' = .13'', \text{ say } \frac{1}{8}''. \quad \text{Ans.}$$

Fig. 605 shows the manner of representing the bolt, except that a hexagonal head is to be shown instead of a square one, and the letters are to be replaced by the sizes calculated.

(971) In the last example, D was found to be $1\frac{3}{8}''$. The other dimensions are readily obtained from Fig. 606. The student should give the handle of the wrench a slight taper to improve the looks.

(972) From Table 28, Art. **1362,** the factor of safety for a varying stress is 6 for wrought iron. Hence, area of cross-section of eye bolt $= 2,500 \div \frac{55,000}{6} = .273$ sq. in. The corresponding diameter $= d = \sqrt{\frac{.273}{.7854}} = .59''$, say $\frac{5}{8}''$. Ans.

Applying formula **227,**

$$d_1 = .8 \times \frac{5}{8} = \frac{1}{2}''. \text{ Ans.}$$

Taking $a\,b$ equal to $\frac{1}{2}d^2$, $a\,b = \frac{1}{2} \times \left(\frac{5}{8}\right)^2 = .195$ sq. in.

If we assume that $a = \frac{1}{2}b$, $a\,b = \frac{1}{2}b^2$, and $b = \sqrt{.195 \times 2} = .624''$, say $\frac{5}{8}''$. Ans.

Whence, $$a = \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}''. \text{ Ans.}$$

(973) See Art. **1944.**

(974) See Fig. 623. Take height of thin nut as one-half that of outer or standard size nut.

(975) Both devices increase the friction between the threads of the screw and the threads in the nut; in this respect the principle is the same in both devices.

(976) Taking d as $1\frac{1}{4}''$, the various dimensions are readily obtained from Fig. 642. The student may supply any dimension that is omitted, taking such dimensions as he deems best.

(977) Applying formula **233,**

$$b = \frac{1}{4} \times 4 = 1''. \text{ Ans.}$$

$$t = \frac{1}{6} \times 4 = \frac{2}{3}'', \text{ say } \frac{11}{16}''. \text{ Ans.}$$

(978) Applying formula **233,**

$$b = \frac{1}{4} \times 12 = 3''. \text{ Ans.}$$

$$t = \frac{1}{6} \times 12 = 2''. \text{ Ans.}$$

(979) Applying formula **235**,

$$b = .2 \times 2\frac{7}{16} + .16 = .65'', \text{ say } \frac{11}{16}''. \quad \text{Ans.}$$

$$t = .1 \times 2\frac{7}{16} + .16 = .4'', \text{ say } \frac{7}{16}''. \quad \text{Ans.}$$

(980) Applying formula **237**,

$$b = \frac{1.5}{8}'' + \frac{1}{16}'' = \frac{1}{4}''. \quad \text{Ans.}$$

$$t = \frac{3 \times 1.5}{16}'' + \frac{1}{16}'' = \frac{5.5}{16}'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(981) Applying formula **238**,

$$d = 5\sqrt[3]{\frac{1.85}{110}} = 1.26''.$$

Using formulas **236**,

$$b = \frac{1.26}{3}'' = .42'', \text{ say } \frac{7}{16}''. \quad \text{Ans.}$$

$$t = \frac{1.26}{5}'' = \frac{1}{4}''. \quad \text{Ans.}$$

(982) Area of rod corresponding to diameter $d_1 = \frac{7,000}{6,000} = 1.167$ sq. in.

Hence, $d_1 = \sqrt{\frac{1.167}{.7854}} = 1.22''$, say $1\frac{1}{4}''$. Ans.

From formula **239**,

$$d_1 = .816\, d, \text{ or } d = \frac{d_1}{.816} = \frac{1.25}{.816} = 1.53'' = 1\frac{17}{32}''. \quad \text{Ans.}$$

The remaining dimensions are readily calculated by using formula **239**. The dimension b in Figs. 656 and 658 should extend to the top of the cotter instead of to the top of the slot, as shown. The student may take b as the depth of the cotter.

(983) Applying formula **241**,

$$d = 1.5\sqrt{\frac{10,000}{\sqrt{14,000 \times 400}}} = 3'', \text{ nearly.} \quad \text{Ans.}$$

Hence, $l = \frac{10,000}{3 \times 400} = 8.33''$, say $8\frac{3}{8}''$. Ans.

964. [illegible] **244.**

[illegible] $=$ [illegible] nearly. Ans.

[illegible] **245.**

$=$ [illegible] Ans.

965. [illegible] **24[illegible].**

[illegible] say [illegible]. Ans.

966. [illegible] formula **251.**

[illegible] say [illegible]. Ans.

Since [illegible] and [illegible] $= .4(10\frac{3}{8} - 8)$ [illegible]

[illegible] Ans.

967. Applying formula **249,** second column of Table [illegible],

$d = $ [illegible] $= 2.425''$, say $2\frac{[illegible]}{16}''$. Ans.

(968). (a) Apply formula **256.** Here, $n = \frac{4}{6}$; hence,

$$d_1 = 10\sqrt[3]{\frac{1}{1-(\frac{4}{6})^4}} = 10.38'', \text{ say } 10\frac{3}{8}''. \text{ Ans.}$$

(b) Diameter of hole $= 10.38'' \times \frac{4}{6} = 6''$, nearly. Ans.

(c) Weight of solid shaft per foot of length =

$$\frac{.7854 \times 10^2}{144} \times 490 = 267 \text{ lb., nearly.}$$

Weight of hollow shaft per foot of length =

$$\frac{.7854\,[(10\frac{3}{8})^2 - 6^2]}{144} \times 490 = 191 \text{ lb., nearly.}$$

Difference = 267 − 191 = 76 lb. per foot. Ans.

(989) (*a*) Using formula **208,**

$$p = .24\sqrt{3.25 + .625} - .175 = .3'', \text{ nearly.}$$

By formula **210,**

$$n = \frac{1}{.3} = 3.333, \text{ say } 3\frac{1}{2}.$$

Then, $p = \frac{1}{n} = \frac{1}{3.5} = \frac{2''}{7}$. Ans.

(*b*) Using formula **212,**

$$p = \frac{7}{5} = 1.4'', \text{ say } 1\frac{3}{8}''. \text{ Ans.}$$

(*c*) Using formula **215,**

$$p = \frac{2 \times 12}{15} = 1.6'', \text{ say } 1\frac{5}{8}''. \text{ Ans.}$$

(990) (*a*) Using formula **210** for each case,

$$n = 1 \div \frac{2}{7} = 3\frac{1}{2};\ n = 1 \div 1\frac{3}{8} = \frac{8}{11}, \text{ and } n = 1 \div 1\frac{5}{8} = \frac{8}{13}. \text{ Ans.}$$

(*b*) Using formula **211** for the first case,

$$d_1 = d - \frac{1.3''}{n} = 3.25'' - \frac{1.3''}{3.5} = 2.879''. \text{ Ans.}$$

For the second case, it will be remembered that $t = \frac{1}{2}p$; hence,

$$d_1 = d - 2\left(\frac{1}{2}p\right) = d - p = 7'' - 1\frac{3}{8}'' = 5\frac{5}{8}''. \text{ Ans.}$$

For the third case, $t_1 = \frac{3}{4}t = \frac{3}{4}p$; hence,

$$d = d - 2\left(\frac{3}{4}p\right) = d - 1.5p = 12 - 1.5 \times 1.625 = 9\frac{9}{16}''. \text{ Ans.}$$

(991) See Art. **1930.**

(992) See Art. **1935.** Using formula **217,**

$$a = \frac{15,600}{6,000} = 2.6 \text{ sq. in.}$$

Referring to Table 43, this area lies between the areas corresponding to diameters of 2″ and 2¼″; choosing the larger, the outside diameter should be 2¼″. Ans.

(993) From Table 43, the effective area of a 4-inch bolt is 9.993 sq. in. Referring to Art. **1935**, and using formula **217**,

$W = a\,S_t = 2 \times 9.993 \times 8{,}000 = 159{,}888$ lb., say 80 tons. Ans.

(994) (*a*) Using formula **218**,

$d = .0228\sqrt{3{,}600} = 1.368''$, say $1\frac{3}{8}''$. Ans.

(*b*) Using formula **219**,

$$n_1 = .0052 \times \frac{3{,}600}{1.375^2} = 9.89, \text{ say 10 threads. Ans.}$$

(995) Applying formula **217**, and taking S_t as 6,000 lb. per sq. in.,

$$a = \frac{3{,}200}{6{,}000} = .533 \text{ sq. in.}$$

From Table 43, the nearest diameter is 1″.

Applying formulas **220**, **221**, **224**, and **226**,

$$D = 1\frac{1}{2} \times 1'' + \frac{1}{16}'' = 1\frac{9}{16}''. \text{ Ans.}$$

$$h = h' = 1'' - \frac{1}{16}'' = \frac{15}{16}''. \text{ Ans.}$$

$$D_1 = 2.12 \times 1'' + .09'' = 2.21''. \text{ Ans.}$$

The student can readily make the drawing by referring to Fig. 605. Omit the washer.

(996) The dimensions are readily obtained by aid of formula **229**.

(997) (*a*) Using formula **233** for all three cases,

$$b = \frac{1}{4}d = \frac{1}{4} \times 3\frac{1}{2} = \frac{7}{8}''. \text{ Ans.}$$

$$t = \frac{1}{6}d = \frac{1}{6} \times 3\frac{1}{2} = \frac{7}{12}'', \text{ say } \frac{9}{16}''. \text{ Ans.}$$

(*b*) $$b = \frac{1}{4} \times 5 = 1\frac{1}{4}''. \text{ Ans.}$$

$$t = \frac{1}{6} \times 5 = \frac{5}{6}'', \text{ say } \frac{13}{16}''. \text{ Ans.}$$

(*c*) $b = \frac{1}{4} \times 1\frac{1}{2} = \frac{3}{8}''$. Ans.

$t = \frac{1}{6} \times 1\frac{1}{2} = \frac{1}{4}''$. Ans.

(998) Each key takes up 1,000 ÷ 2 = 500 H. P. Hence, applying formula **232**,

$$b = \frac{18\,H}{l\,d\,N} = \frac{18 \times 500}{10 \times 10 \times 50} = 1.8'', \text{ say } 2''. \quad \text{Ans.}$$

$$t = \frac{2}{3}\,b = \frac{2}{3} \times 2 = 1.333'', \text{ say } 1\frac{3}{8}''. \quad \text{Ans.}$$

(999) For the driven pulley, apply formula **234.**

$$b = \frac{2.75''}{7} + \frac{1}{4}'' = .64'', \text{ say } \frac{5}{8}''. \quad \text{Ans.}$$

$$t = \frac{2.75''}{12} + \frac{3}{16}'' = .42'', \text{ say } \frac{7}{16}''. \quad \text{Ans.}$$

For the driving pulley, apply formula **235.**

$$b = .2 \times 4.5'' + .16'' = 1.06'', \text{ say } 1\frac{1}{16}''. \quad \text{Ans.}$$

$$t = .1 \times 4.5'' + .16'' = .61'', \text{ say } \frac{5}{8}''. \quad \text{Ans.}$$

(1000) Using formulas **237,**

$$b = \frac{1.75}{8} + \frac{1}{16} = .28'', \text{ say } \frac{5}{16}''. \quad \text{Ans.}$$

$$t = \frac{3 \times 1.75}{16} + \frac{1}{16} = .39'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(1001) It is evident that a $3\frac{3}{4}''$ shaft will transmit a great deal more than 6 H. P. at 135 R. P. M. Hence, using formulas **238** and **236,**

$$d = 5\sqrt[3]{\frac{6}{135}} = 1.77''. \quad \text{Therefore,}$$

$$b = \frac{1.77}{3} = .59'', \text{ say } \frac{5}{8}''. \quad \text{Ans.}$$

$$t = \frac{1.77}{5} = .354'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(1002) The calculation of d, d_1, and the other dimensions is in all respects similar to that required in example 982. The various dimensions in the figure may be obtained by using formula **239.**

(1003) According to Art. **1969,** $b\,t$ should equal $\frac{5}{4}$ times the sectional area of the strap, and $t = \frac{1}{4}\,b$. Hence, $b \times \frac{1}{4}\,b = \frac{5}{4} \times \frac{3}{4} \times 3.5$; or, $b^2 = 13.125$ sq. in., and $b = \sqrt{13.125} = 3.623''$, say $3\frac{5}{8}''$. Ans.

$$t = \frac{1}{4}\,b = \frac{1}{4} \times 3\frac{5}{8} = \frac{29}{32}'', \text{ say } \frac{15}{16}''. \quad \text{Ans.}$$

The other dimensions are readily obtained from Fig. 661, and the drawing can be made by referring to it.

(1004) Art. **1969** states that b may be made equal to d when a steel cotter is used in a wrought-iron rod. Hence,

$$b = d = 2\frac{1}{2}''. \quad \text{Ans.}$$

Also, $$t = \frac{1}{4}\,b = \frac{1}{4} \times 2\frac{1}{2} = \frac{5}{8}''. \quad \text{Ans.}$$

(1005) From Table 43, the area at the bottom of the thread for a $3\frac{1}{2}''$ standard screw is 7.55 sq. in. Referring to Art. **1935,** it would not be advisable to take S_t greater than 4,000 lb. per sq. in. in this case. Hence, $W = a\,S_t = 7.55 \times 4{,}000 = 30{,}200$ lb. = load which rod may safely carry.

When held by a cotter, the rod is weakened owing to the cutting out of the slot, the equivalent diameter being (see formula **239**) $d_1 = .816\,d = .816 \times 3.5 = 2.856''$; that is, a rod having a diameter of $2.856''$ would be, theoretically, of the same strength as a rod having a slot in it for a cotter and $3\frac{1}{2}''$ in diameter. Hence, the equivalent area is $2.856^2 \times .7854 = 6.41$ sq. in. The area is less than in the first case, but the value of S_t may now be taken as $\frac{55{,}000}{10} = 5{,}500$ lb. per sq.

in., and the safe load is $6.41 \times 5,500 = 35,255$ lb. Hence, the rod and cotter are, apparently, slightly stronger than the rod and nut.

(1006) (*a*) Applying formula **240,**

$$d = 2.26\sqrt{\frac{5,000}{4,250}} \times 1.5 = 3''. \quad \text{Ans.}$$

$$l = 1\frac{1}{2}d = 1\frac{1}{2} \times 3 = 4\frac{1}{2}''. \quad \text{Ans.}$$

(*b*) Pressure per square inch of projected area =

$$p = \frac{5,000}{3 \times 4.5} = 370 \text{ lb.} \quad \text{Ans.}$$

(1007) Applying formula **240,**

$$d = l = 2.26\sqrt{\frac{10,000}{8,500}} \times 1 = 2.45'', \text{ say } 2\frac{1}{2}''. \quad \text{Ans.}$$

(1008) See example in Art. **1975,** and Table 46, Art. **1978.**

$$d = 1.5\sqrt{\frac{17,800}{\sqrt{12,000 \times 750}}} = 3.65'', \text{ say } 3\frac{5}{8}''. \quad \text{Ans.}$$

$$\text{Whence, } l = \frac{17,800}{3\frac{5}{8} \times 750} = 6.55'', \text{ say } 6\frac{1}{2}''. \quad \text{Ans.}$$

(1009) As in example 1008,

$$d = 1.5\sqrt{\frac{1,750}{\sqrt{8,500 \times 275}}} = 1.6'', \text{ say } 1\frac{5}{8}''. \quad \text{Ans.}$$

$$\text{Whence, } l = \frac{1,750}{1\frac{5}{8} \times 275} = 3.92'', \text{ say } 4''. \quad \text{Ans.}$$

(1010) Here, $l = 2d$. From formula **242,** $ld = \frac{P}{p}$.

Hence, $2d \times d = 2d^2 = \frac{3,750}{550}$; or, $d = 1.846''$, say $1\frac{7}{8}''$.
Ans

Also, $l = 2d = 2 \times 1\frac{7}{8} = 3\frac{3}{4}''$. Ans.

(1011) Circumference of wheel $= 2.75\pi$.

$$\text{R. P. M.} = N = \frac{5,280 \times 40}{2.75\pi \times 60} = 407.$$

Then, $p = \frac{220,000}{407} = 540$ lb. per sq. in.

Applying formula **241**, and getting S_f from Table 46,

$$d = 1.5\sqrt{\frac{10,000}{\sqrt{12,000 \times 540}}} = 2.97'', \text{ say } 3''. \quad \text{Ans.}$$

Hence, $l = \frac{10,000}{3 \times 540} = 6.17''$, say $6\frac{1}{4}''$. Ans.

The other dimensions may be obtained from Fig. 665 and formula **247.**

(1012) (*a*) Using formula **243** and Table 46,

$$d = 1.13\sqrt{\frac{9,600}{7,000}} \times 2.5 = 2.09'', \text{ say } 2\frac{1}{8}''. \quad \text{Ans.}$$

$$l = 2.5\,d = 2.5 \times 2\frac{1}{8} = 5\frac{5}{16}'', \text{ say } 5\frac{1}{4}''. \quad \text{Ans.}$$

(*b*) From formula **242,**

$$p = \frac{P}{l\,d} = \frac{9,600}{2\frac{1}{8} \times 5\frac{1}{4}} = 860 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1013) Using formula **250,**

$$d = .004\sqrt{PN} = .004\sqrt{25 \times 3,000} = 1.1'', \text{ say } 1\frac{1}{8}''. \quad \text{Ans.}$$

(1014) Using formula **249,**

$$d = .05\sqrt{P} = .05\sqrt{9,000} = 4.74'', \text{ say } 4\frac{3}{4}''. \quad \text{Ans.}$$

(1015) Applying formula **251,**

$$d_1 = \sqrt{6^2 + \frac{7,800}{15\pi \times 4}} = 8.8'', \text{ say } 8\frac{3}{4}''. \quad \text{Ans.}$$

$$e = \frac{1}{2}(d_1 - d) = \frac{1}{2}(8\frac{3}{4} - 6) = 1\frac{3}{8}''. \quad \text{Ans.}$$

$$t = .8\,e = .8 \times 1\frac{3}{8}'' = 1.1'', \text{ say } 1\frac{1}{8}''. \quad \text{Ans.}$$

Taking s as $2\,t$, $s = 2 \times 1\frac{1}{8}'' = 2\frac{1}{4}''$. Ans.

(See Fig. 668 for manner of representing the journal.)

(1016) This shaft should be calculated with the help of Table 49. To determine Z, we have, from the table of Bending Moments,

$$B = W\frac{l_1 l_2}{l} = 10 \times \frac{3 \times 5}{8} = 18\frac{3}{4} \text{ tons.}$$

The twisting moment $T = PR = 63{,}025 \times \frac{H}{N}$ in.-lb. (see formula **231**) =

$$\frac{63{,}025}{2{,}000 \times 12} \times \frac{H}{N} \text{ ft.-tons} = \frac{63{,}025}{2{,}000 \times 12} \times \frac{300}{60} = 13.13 \text{ ft.-tons.}$$

Then, $$Z = \frac{B}{T} = \frac{18.75}{13.13} = 1.4, \text{ nearly.}$$

From Table 49, $\sqrt[3]{k} = 1.461$ for $Z = 1.4$. Hence (see Table 32, Art. **1416**),

$$d_1 = d\sqrt[3]{k} = 1.461 \times 3.3\sqrt[3]{\frac{H}{N}} = 1.461 \times 3.3\sqrt[3]{\frac{300}{60}} = 8.25''.$$

Ans.

(1017) Applying formula **253**,

$$d = 68.5\sqrt[3]{\frac{90}{110 \times 7{,}000}} = 3.35''.$$

$$Z = \frac{.75\,T}{T} = .75.$$

From Table 49, $\sqrt[3]{k} = 1.209$ for $Z = .6$, and 1.277 for $Z = .8$. Hence, $1.277 - 1.209 : .8 - .6 = x : .75 - .6$; or, $x = .051$

Therefore, for $Z = .75$, $\sqrt[3]{k} = 1.209 + .051 = 1.26$, and $d_1 = 1.26 \times 3.35 = 4.22''$, say $4\frac{1}{4}''$. Ans.

(1018) Use formula **256.** In this case $m = \frac{6}{13}$.

Hence, $d = d_1\sqrt[3]{1 - m^4} = 13\sqrt[3]{1 - \left(\frac{6}{13}\right)^4} = 12.8''$, say $12\frac{13}{16}''$.

Ans.

(1019) Use formula **256.** Here $m = \frac{1}{2} = .5$. Hence,

$$d_1 = 16\frac{3}{4}\sqrt[3]{\frac{1}{1 - .5^4}} = 17.11'', \text{ say } 17\frac{1}{8}''. \quad \text{Ans.}$$

$$d_2 = 17\frac{1}{8} \div 2 = 8\frac{9}{16}''. \quad \text{Ans.}$$

(1020) The student should be able to draw the joint by referring to Art. **200** and Fig. 676. The keys may be proportioned by formula **233**. The student should use his own judgment regarding any dimensions not given.

(1021) Fig. 679 and formula **258** are a sufficient guide for the drawing of this coupling.

(1022) (*a*) From formula **260**,

$$n = \frac{1}{3}D + 2 = \frac{1}{3} \times 11 + 2 = 5\frac{2}{3}, \text{ say } 6. \quad \text{Ans.}$$

(*b*) From formula **259** and Table **50**,

$$d = kD = .239 \times 11 = 2.629'', \text{ say } 2\frac{5}{8}''. \quad \text{Ans.}$$

(*c*) From formula **260**,

$$B = D + 2d = 11 + 2 \times 2.625 = 16\frac{1}{4}''. \quad \text{Ans.}$$

(*d*) $C = D + 4.25\,d = 11 + 4.25 \times 2.625 = 22.16$, say $22\frac{3}{16}''$. Ans.

(*e*) $T = \dfrac{2D + 1}{7} = \dfrac{2 \times 11 + 1}{7} = 3\frac{2}{7}$, say $3\frac{1}{4}''$. Ans.

(1023) Using formula **250**,

$$d = .004\sqrt{PN} = .004\sqrt{100 \times 600} = .98'', \text{ say } 1''. \quad \text{Ans.}$$

MACHINE DESIGN.

(QUESTIONS 1024-1063.)

(1024) See Art. **2004.**

(1025) See Art. **2016.**

(1026) Radius of gear $= 15'' = 1\frac{1}{4}$ ft. $= R$. From formula **231,**

$$p = \frac{63{,}025\,H}{n\,R} = \frac{63{,}025 \times 20}{120 \times 15} = 700 \text{ lb., nearly.}$$

From formula **267,**

$$S = \frac{9{,}600{,}000}{v + 2{,}160} = \frac{9{,}600{,}000}{2.5 \times \pi \times 120 + 2{,}160} = 3{,}094 \text{ lb. per sq. in.,}$$

nearly.

Hence, from formula **266,**

$$b\,C = 16.8\,\frac{p}{S} = \frac{16.8 \times 700}{3{,}094} = 3.8 \text{ sq. in.}$$

Assuming that $b = 2\frac{1}{2}C$, $b\,C = 2.5\,C^2 = 3.8$ sq. in., or $C = \sqrt{\frac{3.8}{2.5}} = 1.2329''$. Hence, number of teeth $=$

$$\frac{\pi \times 30}{1.2329} = 76 +, \text{ say } 76. \quad \text{Ans.}$$

$$\text{Circular pitch} = \frac{\pi \times 30}{76} = 1.2401''. \quad \text{Ans.}$$

(1027) The velocity of a point on the pitch circle is $\pi \times 4\frac{2}{3} \times 60 = 880$ ft. per min., nearly. Hence, from formula **267,**

[illegible]

[illegible] 23.

[illegible]

[illegible]

[illegible] Ans.

[illegible] Ans.

[illegible] Ans.

[illegible] Ans.

[illegible] Ans.

[illegible] 105.

[illegible] Ans.

[illegible] Ans.

[illegible] Ans.

[illegible] Ans.

Formula 272 [illegible]

[illegible] = 5 arms. Ans.

By formula **270,** width of arm at center of wheel =

$$a = \sqrt{\frac{R\,b}{z}} = \sqrt{\frac{28 \times 13\frac{1}{2}}{5}} = 8.6948'', \text{ say } 8\frac{11}{16}''. \quad \text{Ans.}$$

Assuming the taper to be $\frac{1}{48}$ on each side, the width of arm at pitch line is

$$8\frac{11}{16} - \left(2 \times \frac{1}{48} \times 28\right) = 7\frac{1}{2}'', \text{ nearly.} \quad \text{Ans.}$$

Thickness of arm $= \frac{1}{2} C = \frac{1}{2} \times 4.511 = 2.255'' = 2\frac{1}{4}''$. Ans.

Thickness of stiffening rib $= .4\,C = .4 \times 4.511 = 1.804''$. Ans.

(1028) By formula **275,**

$$P = Qf = 280 \times \frac{1}{3} = \frac{280}{3}.$$

$$\text{Radius of wheel} = R = \frac{6 \times 12}{2} = 36''.$$

Then, by formula **231,**

$$H = \frac{PRN}{63{,}025} = \frac{280 \times 36 \times 110}{3 \times 63{,}025} = 5.864 \text{ H. P.} \quad \text{Ans.}$$

(1029) See Art. **2049.**

(1030) (*a*) By formula **282,**

$$d = \frac{D}{9} = \frac{1}{9}''. \quad \text{Ans.}$$

(*b*) By formula **283,** $w = 1.43\,D^2 = 1.43 \times 1^2 = 1.43$ lb. Ans.

(1031) By formula **291,**

$$P = 6{,}600\,d^2 = 6{,}600 \times 1.25^2 = 10{,}312.5 \text{ lb.} \quad \text{Ans.}$$

(1032) See Art. **2005.**

(1033) The values of the different dimensions, as determined from the table of proportions in Art. **2014,** are as follows:

$d = 14''$.

$a = 14 + 1 = 15''$.

$b = .5 \times 14 + 1 = 8''$.

$c = .66 \times 14 = 9.24''$, say $9\frac{1}{4}''$.

$e = .825 \times 14 - .25 = 11.3''$, say $11\frac{5}{16}''$.

$f = .6 \times 14 = 8.4''$, say $8\frac{13}{32}''$.

$g = .1 \times 14 + .5625 = 1.9625''$ say $1\frac{31}{32}''$.

$h = .1 \times 14 + .25 = 1.65''$, say $1\frac{21}{32}''$.

$k' = .08 \times 14 = 1.12''$, say $1\frac{1}{8}''$.

$i = .11 \times 14 = 1.54''$, say $1\frac{17}{32}''$.

$j = \frac{5}{8}''$.

$k = .5 \times 14 + 1.25 = 8\frac{1}{4}''$.

$l = \frac{3}{8}''$.

$m = .175 \times 14 + .3125 = 2.7625''$, say $2\frac{3}{4}''$.

$n = .25 \times 14 + .25 = 3\frac{3}{4}''$.

$n' = .1 \times 14 + .375 = 1.775''$, say $1\frac{3}{4}''$.

$o = 1''$.

$p = .25 \times 14 + .625 = 4\frac{1}{8}''$

$q = 1.75 \times 14 = 24\frac{1}{2}''$.

$q' = 1.5 \times 14 = 21''$.

$r = .15 \times 14 = 2.1''$, say $2\frac{1}{8}''$

$r' = .1 \times 14 = 1.4''$, say $1\frac{3}{8}''$.

$r_1 = 14''$.

$s = .9 \times 14 = 12.6''$, say $12\frac{5}{8}''$.

$t = .15 \times 14 + .375 = 2.475''$, say $2\frac{1}{2}''$.

$t' = .9 \times 14 = 12.6''$, say $12\frac{5}{8}''$.

$u = 1.5 \times 14 = 21''$.

$v = .25 \times 14 + .375 = 3\frac{7}{8}''$.

$w = 1.45 \times 14 = 20.3''$, say $20\frac{1}{4}''$.

$w' = 1.47 \times 14 = 20.58''$, say $20\frac{5}{8}''$.

$w_1 = 1.75 \times 14 = 24\frac{1}{2}''$.

$x = .1 \times 14 = 1.4''$, say $1\frac{3}{8}''$.

$y = .3 \times 14 + .75 = 4.95''$, say $5''$.

$y' = .2 \times 14 + .5 = 3.3''$, say $3\frac{1}{4}''$.

$z = .09 \times 14 = 1.26''$, say $1\frac{1}{4}''$.

$z' = 2\frac{1}{2}''$.

The student may use his own judgment to supply any dimensions not given.

(1034) See Art. **2023.**

(1035) Number of teeth $= N = 56 \times 1\frac{1}{4} = 70$.

$$\text{Circular pitch} = C = \frac{3.1416}{1\frac{1}{4}} = 2.5133''.$$

Applying formula **272,**

$s = .55\sqrt[4]{N^2 C} = .55\sqrt[4]{70^2 \times 2.5133} = 5.79$, or 6 arms. Ans.

(1036) See Art. **2037.**

(1037) See Art. **2057.**

(1038) See Art. **2066.**

(1039) See Art. **2074.**

(1040) See Arts. **2005** and **2006.**

(1041) See Art. **2022.**

(1042) The values of the different dimensions, as determined from the table of proportions in Art. **2011,** are:

$$d = 3\tfrac{1}{2}''.$$

$a = 3.25 \times 3.5 = 11\tfrac{3}{8}''.$

$b = 1.75 \times 3.5 = 6\tfrac{1}{8}''.$

$c = 3\tfrac{1}{2}''.$

$e = .5 \times 3.5 = 1\tfrac{3}{4}''.$

$f = .4375 \times 3.5 = 1.53125''$, say $1\tfrac{17}{32}''.$

$g = .09 \times 3.5 = .315''$, say $\tfrac{5}{16}''.$

$h = .3125 \times 3.5 = 1\tfrac{3}{32}''.$

$i = .25 \times 3.5 = \tfrac{7}{8}''.$

$j = .375 \times 3.5 = 1\tfrac{5}{16}''.$

$k = 1.0625 \times 3.5 = 3.72''$, say $3\tfrac{3}{4}''.$

$l = .875 \times 3.5 = 3\tfrac{1}{16}''.$

$m = 1.75 \times 3.5 = 6\tfrac{1}{8}''.$

$n = 1.25 \times 3.5 = 4\tfrac{3}{8}''.$

$o = \tfrac{1}{8}''.$

$p = \tfrac{7}{8}''.$

$q = .625 \times 3.5 = 2\tfrac{3}{16}''.$

$r = .25 \times 3.5 = \tfrac{7}{8}''.$

$s = .1875 \times 3.5 = \tfrac{21}{32}''$, say $\tfrac{11}{16}''.$

$t = .65 \times 3.5 = 2.275''$, say $2\tfrac{5}{16}''.$

$u = .75 \times 3.5 = 2\tfrac{5}{8}''.$

$v = 1.375 \times 3.5 = 4\tfrac{13}{16}''$, say $4\tfrac{7}{8}''.$

$x = .25 \times 3.5 = \tfrac{7}{8}''.$

$y = .5 \times 3.5 = 1\tfrac{3}{4}''.$

$z = .0625 \times 3.5 = \tfrac{7}{32}''$, say $\tfrac{1}{4}''.$

(1043) See Arts. **2020** and **2018.**

(1044) Applying formula **268,**

$$b = 4\tfrac{1}{2}\frac{H}{D} = \frac{4.5 \times 21}{44} = 2.148'', \text{ say } 2\frac{1}{8}''. \quad \text{Ans.}$$

(1045) Applying formula **274,**

$$e = \frac{Np_1}{Np_1 + R} = \frac{4 \times .5236}{4 \times .5236 + 1\tfrac{1}{4}} = 62.62\%. \quad \text{Ans.}$$

(1046) See Art. **2038.**

(1047) Applying formula **276,**

$$t = \frac{B + D}{200} + \frac{1}{16}'' = \frac{10 + 52}{200} + \frac{1}{16} = .3725'', \text{ say } \frac{3}{8}''. \quad \text{Ans.}$$

(1048) [illegible] formula 252.

[illegible] $= 1.94$ [illegible]

[illegible]

[illegible] formula 255.

[illegible] Ans.

(1049) See Art. [illegible]

(1050) See Art. [illegible]

(1051) Velocity = [illegible] = [illegible] feet per second.

Using formula 251.

[illegible] Ans.

(1052) By formula 253,

[illegible] = [illegible] per foot.

[illegible]

Applying formula 254,

[illegible] $- 2.23 \times 9.25 = 4.511$ lb. Ans.

(1053) See Art. 2070,

$n = .13\sqrt[3]{P} = .13\sqrt[3]{5,600} = 2.31$, say 2.

$d = .0115\sqrt{\frac{P}{\sqrt{n}}} = .0115\sqrt{\frac{5,600}{\sqrt{2}}} = .7237''$, say $\frac{3}{4}''$.

$d' = 1.2\,d = 1.2 \times \frac{3}{4} = .9''$, say $\frac{29}{32}''$.

Remaining dimensions are the same as those calculated in Art. 2070.

(1054) The values of the dimensions given in the table of proportions in Art. **2019** are:

$$d = 2\tfrac{1}{2}''.$$

$a = 3 \times 2.5 = 7\tfrac{1}{2}''.$	$m = .125 \times 2.5 = \tfrac{5}{16}''.$
$b = 7.5 \times 2.5 = 18\tfrac{3}{4}''.$	$n = .25 \times 2.5 = \tfrac{5}{8}''.$
$c = 5.375 \times 2.5 = 13\tfrac{7}{16}''.$	$o = .1875 \times 2.5 = \tfrac{15}{32}''$, say $\tfrac{1}{2}''.$
$e = 3 \times 2.5 = 7\tfrac{1}{2}''.$	$p = .375 \times 2.5 = \tfrac{15}{16}''$, say $1''.$
$f = 1.75 \times 2.5 = 4\tfrac{3}{8}''.$	$q = .625 \times 2.5 = 1\tfrac{9}{16}''.$
$g = 1.5 \times 2.5 = 3\tfrac{3}{4}''.$	$r = 1.25 \times 2.5 = 3\tfrac{1}{8}''.$
$h = 2.125 \times 2.5 = 5\tfrac{5}{16}''.$	$s = .25 \times 2.5 + .5 = 1\tfrac{1}{8}''.$
$i = .16 \times 2.5 = .4''$, say $\tfrac{7}{16}''.$	$v = 1.125 \times 2.5 + .1875 = 3''.$
$j = .25 \times 2.5 = \tfrac{5}{8}''.$	$w = 1.5 \times 2.5 = 3\tfrac{3}{4}''.$
$k = 1.875 \times 2.5 = 4\tfrac{11}{16}''.$	$y = .0625 \times 2.5 = \tfrac{5}{32}''.$
$l = 1.125 \times 2.5 = 2\tfrac{13}{16}''.$	$z = 2.5 - .0625 = 2\tfrac{7}{16}''.$

For the other dimensions necessary to draw the bracket, see Fig. 697.

(1055) Using formula **271**,

$$a = 1.75\sqrt[3]{\frac{6 \times 20}{1\tfrac{1}{2} \times 6}} = 4.15'', \text{ say } 4\frac{1}{8}''. \quad \text{Ans.}$$

$$\frac{1}{2}a = \frac{1}{2} \times 4\frac{1}{8} = 2\frac{1}{16}''. \quad \text{Ans.}$$

(1056) (*a*) The diameter of the wire composing the rope is, by formula **282**,

$$d = \frac{D}{9} = \frac{\tfrac{3}{4}}{9} = \frac{1}{12}''.$$

The weight of the rope per foot is, by formula **283**,

$$w = 1.43\ D^2 = 1.43 \times \left(\frac{3}{4}\right)^2 = .8044 \text{ lb.}$$

Stress due to bending is, by formula **287**,

$$S_b = \frac{E_1 d}{2R} = \frac{25{,}000{,}000 \times \tfrac{1}{12}}{13 \times 12} = 13{,}355 \text{ lb. per sq. in.}$$

Stress due to centrifugal force is, by formula **288**,

$$S_c = \frac{4\,w\,v^2}{\pi d^2 n g} = \frac{4 \times .8044 \times \left(\frac{5{,}000}{60}\right)^2}{3.1416 \times \left(\frac{1}{12}\right)^2 \times 42 \times 32.16} = 758 \text{ lb. per sq. in.}$$

Hence, $S_t = S - (S_b + S_c) = 25{,}000 - (13.355 + 758) =$ 10,887 lb. per sq. in.

T_1 = maximum tension on driving side =

$$\frac{\pi}{4} d^2 n S_t = \frac{\pi}{4} \times \left(\frac{1}{12}\right)^2 \times 42 \times 10{,}887 = 2{,}494 \text{ lb.}$$

$T_2 = \frac{1}{2} T_1 = 2{,}494 \times .5 = 1{,}247$ lb.

P = driving force $= T_1 - T_2 = 2{,}494 - 1{,}247 = 1{,}247$ lb.

$$\text{Horsepower} = H = \frac{PV}{33{,}000} = \frac{1{,}247 \times 5{,}000}{33{,}000} = 189 \text{ H. P.}$$

(*b*) The greatest deflection will, of course, be on the driven, or slack, side. Hence, applying formula **285**,

$$h = \frac{T_2}{2w} - \sqrt{\frac{T_2^2}{4w^2} - \frac{a^2}{2}} = \frac{1{,}247}{2 \times .8044} - \sqrt{\frac{1{,}247^2}{4 \times .8044^2} - \frac{212.5}{2}} =$$

14.7 ft. Ans.

(1057) Circular pitch $= C = \frac{\pi d}{84} = \frac{\pi \times 21}{84} = .7854''$.

Applying formula **272**,

$$z = .55 \sqrt[4]{84^2 \times .7854} = 4.74 +, \text{ say 5 arms. Ans.}$$

(1058) Using formula **277**,

$$a = \sqrt[3]{\frac{BR}{n}} = \sqrt[3]{\frac{8 \times 11}{4}} = 2.8'', \text{ say } 2\tfrac{7}{8}''. \text{ Ans.}$$

Assuming the taper to be $\frac{1}{48}$ on each side, the taper for both sides will be $\frac{1}{48} \times 2 = \frac{1}{24}$. The width of the arm at rim will be

$$2\tfrac{7}{8} - \left(\frac{1}{24} \times 11\right) = 2.417'', \text{ say } 2\tfrac{3}{8}''. \text{ Ans.}$$

$$\text{Thickness at center} = 2\tfrac{7}{8} \times \frac{1}{2} = 1\tfrac{7}{16}''. \text{ Ans.}$$

$$\text{Thickness at rim} = 2\tfrac{3}{8} \times \frac{1}{2} = 1\tfrac{3}{16}''. \text{ Ans.}$$

(1059) Using formula **279**,

$$S' = \frac{A'}{4} = \frac{7 \times 1\frac{3}{8}}{4} = 2.406 \text{ sq. in.}$$

Net section of one bolt $= 2.406 \div 6 = .401$ sq. in.

Hence, according to Table 43, $\frac{7}{8}''$ in. bolts will be used. Ans.

(1060) (1) The dimensions as taken from Table 52, are:

$$A = \frac{9}{32}''; B = \frac{7}{16}''; C = 2\frac{3}{4}''; D = 1\frac{3}{4}''; E = 8\frac{3}{4}''; F = 1\frac{1}{4}'';$$
$$G = \frac{7}{8}''; H = 1\frac{1}{2}'', \text{ and } I = \frac{3}{8}''.$$

(2) From the table of proportions in Art. **2021,**

$$d = 3''.$$

$a = 2 \times 3 = 6''$.	$j = .4 \times 3 = 1.2''$, say $1\frac{1}{4}''$.
$b = 1.5 \times 3 = 4\frac{1}{2}''$.	$k = .3 \times 3 = .9''$, say $\frac{15}{16}''$ or $1''$.
$c = .25 \times 3 + .375 = 1\frac{1}{8}''$.	$l = .15 \times 3 = .45''$, say $\frac{15}{32}''$ or $\frac{1}{2}''$.
$e = 1.25 \times 3 = 3\frac{3}{4}''$.	$m = 1.5 \times 3 = 4\frac{1}{2}''$.
$f = .2 \times 3 + .125 = .725$, say $\frac{3}{4}''$.	$n = .2 \times 3 + .25 = .85''$, say $\frac{7}{8}''$.
$g = 1.75 \times 3 = 5\frac{1}{4}''$.	$o = .15 \times 3 = .45''$, say $\frac{1}{2}''$.
$h = 1.4 \times 3 = 4.2''$, say $4\frac{1}{4}''$.	

(3) The various dimensions are readily obtained from Fig. 735.

(4) The dimensions, as obtained from Table 54, are:

$a = 12''$; $b = .79''$; $c = \frac{7}{8}''$; $d = 3\frac{3}{4}''$; $n = 12''$; $e = 1\frac{1}{4}''$; $f = 16\frac{1}{2}''$, and $g = 19''$.

(1061) Applying formula **280,**

$$w = .3\,D^2 = .3 \times \left(1\frac{3}{4}\right)^2 = .91875 \text{ lb. per ft.}$$

Hence, total weight $= 875 \times .91875 = 803.9$ lb. Ans.

(1062) (*a*) Using formula **281,**

$$H = \frac{v\,D^2}{825}\left(200 - \frac{v^2}{107.2}\right) = \frac{\frac{3,800}{60} \times \left(1\frac{3}{4}\right)^2}{825}\left(200 - \frac{3,800^2}{60^2 \times 107.2}\right) =$$

38.223 H. P. Ans.

(*b*) Referring to the diagram, Fig. 723, the horsepower appears to be about 38 H. P. Ans.

(1063) Applying the method described in Art. **2063**, and assuming that $\frac{R}{d} = 850$, we proceed as follows:

The driving force $= P = \frac{33{,}000\ H}{v} = \frac{33{,}000 \times 125}{4{,}900} = 842$ lb., nearly.

Then, $T_1 = 842 \times 2 = 1{,}684$ lb.

Stress due to bending is

$$S_b = \frac{E_1\, d}{2\,R} = \frac{25{,}000{,}000}{2 \times 850} = 14{,}706 \text{ lb. per sq. in.}$$

Hence, $S_t = 25{,}000 - 14{,}706 = 10{,}294$.

Cross-section of wires $= \frac{T_1}{S_t} = \frac{1}{4}\pi\, d^2\, n$, or

$$d = \sqrt{\frac{4\,T_1}{\pi\, n\, S_t}} = \sqrt{\frac{4 \times 1{,}684}{\pi \times 42 \times 10{,}294}} = .0704.$$

The diameter of the rope is, by formula **282**,

$$D = 9\,d = 9 \times .0704 = .6336, \text{ say } \frac{5}{8}''.$$

A recalculation taking into account the stress due to centrifugal force will not appreciably affect the result; hence, the diameter of the rope is $\frac{5}{8}''$. Ans. (*c*)

(*a*) Diameter of wires is, then,

$$\frac{5}{8} \div 9 = .625 \div 9 = .0694''. \quad \text{Ans.}$$

(*b*) Radius of pulley $= .0694 \times 850 = 58.99''$, say $59''$.

Diameter of pulley $= 59 \times 2 = 118'' = 9$ ft. $10''$, say 10 ft Ans.

MACHINE DESIGN.

(QUESTIONS 1064–1078.)

(1064) (*a*) See Art. **2076.**

(*b*) $100 - 8 = 92\%$. See Art. **2076.**

(1065) The values of the various dimensions, as determined from the table of proportions in Art. **2111**, are as follows:

$$D = 8''.$$

$$d = 1\frac{3}{8}''.$$

$$a = 2 \times 1\frac{3}{8} = 2\frac{3}{4}''.$$

$$b = 1.5 \times 1\frac{3}{8} = 2\frac{1}{16}''.$$

$$c = 1\frac{3}{8}''.$$

$$e = .75 \times 1\frac{5}{8}'' = 1\frac{7}{32}''.$$

$$f = .2 \times 8 = 1.6'', \text{ say } 1\frac{5}{8}''.$$

$$g = \frac{7.125}{6} = 1.1875'' = 1\frac{3}{16}''.$$

$$h = \sqrt{\frac{100 \times 8^2 \times .7854 \times 6}{16.5 \times 36}} = 7.125'' = 7\frac{1}{8}''$$

$$i = .26 \times 8 + .5'' = 2.58'', \text{ say } 2\frac{9}{16}''.$$

$$k = .075 \times 8 = .6'', \text{ say } \frac{5}{8}''.$$

$$l = \frac{7.125}{18} = .4'', \text{ say } \frac{3}{8}''.$$

$m = \frac{3}{8}''$.

$n = \frac{13}{16}''$.

o must be greater than $a + h - \frac{1}{2} d$, see wrist-pin end of connecting-rod in Fig. 760. But $a = .75 \times .2 \times 8 + .125 = 1.325''$, say $1\frac{5}{16}''$, and

$$h = \frac{.5 \times \pi \times (.155 \times 8 + .0625)^2}{4\left[1.3 \times (.155 \times 8 + .0625) - .25\sqrt{2.5 \times \frac{\pi}{4}(.155 \times 8 + .0625)^2}\right]}$$

$= .53''$, say $\frac{17}{32}''$. Then $o = 1\frac{5}{16} + \frac{17}{32} - \frac{1}{2} \times 1\frac{5}{8} + \frac{7}{32}$ (assumed) $= 1\frac{1}{4}''$.

$p = .5 \times 1\frac{3}{8} = \frac{11}{16}''$.

$q = .75 \times 1\frac{3}{8} = \frac{33}{32}''$, say $1''$.

The student may use his own judgment to supply any dimensions not given.

(**1066**) See Art. **2076.**

(**1067**) Using formula **303,** and substituting values given,

$d = .0091\sqrt{p\,L\,B} = .0091\sqrt{120 \times 9 \times 16} = 1.2''$, say $1\frac{1}{4}''$. Ans.

(**1068**) The values of the various dimensions, as obtained from the table of proportions in Art. **2093,** are as follows:

$D = 20''$.

$d = 3''$.

$a = .18\sqrt{2 \times 20} - .1875'' = .9509''$, say $1''$.

$b = .45 \times 1'' = .45''$, say $\frac{7}{16}''$.

$c = .65 \times 1'' = .65''$, say $\frac{21}{32}''$.

$d' = 2 \times 3 = 6''$.

$e = \frac{.06 \times 20}{4} = .3''$, say $\frac{5}{16}''$.

$f = 1' - \frac{1}{8} = \frac{7}{8}'$.

$g = .5 \times \frac{7}{8}' = \frac{7}{16}'$.

$h = .2 \times 20 + 1.5 = 5\frac{1}{2}'$.

$i = 1' - \frac{1}{8} = \frac{7}{8}'$.

$k = \frac{1.4 \times 20}{4} = 7'$.

$n = .08\,(20 + 34) = 4.32$, say 4.

The student may use his own judgment to supply any dimensions not given.

(1069) See Art. **2083.** 160 ft. per sec. Ans.

(1070) Using formula **303,** and substituting the values given,

$d = .01\sqrt{pLB} = .01\sqrt{100 \times 6 \times 8\frac{1}{4}} = .7'$, say $\frac{3}{4}'$. Ans.

(1071) See Art. **2083.**

(1072) (*a*) By formula **300,**

$d = .0118 \times 12\sqrt{100} = 1.416'$, say $1\frac{7}{16}'$. Ans.

(*b*) By formula **301,**

$d = .038\sqrt[4]{12^2 \times 32^2 \times 100} = 2.355'$, say $2\frac{3}{8}'$. Ans.

(*c*) By Art. **2103,**

$d = \frac{4}{25}D = \frac{4 \times 12}{25} = 1.92'$, say $1\frac{15}{16}'$. Ans.

In formula **300,** the assumption is made that the rod is subjected to tension and compression only. In deriving formula **301,** the rod is treated as though it were a long column. In the formula given in Art. **2103,** the length of the rod is assumed to be twice the cylinder diameter, and the steam pressure is assumed to be 80 lb. per sq. in., neither of which is true in this case. The value obtained by formula **301** would be used in practice.

(1073) See Art. **2081.** Take the initial pressure as .92 of the boiler pressure, or as $100 \times .92 = 92$ lb. per sq. in., gauge, and $92 + 14.7 = 106.7$ lb. per sq. in., absolute.

By formula **292,** the real cut-off $= k = \frac{22}{22 + 106.7} = .171'$.

By formula **293,** $p_c = \frac{106.7 + 16}{2} = 61.35$ lb. per sq. in. = absolute pressure at end of compression. Now draw the theoretical indicator diagram, as shown in Fig. 96. Assume

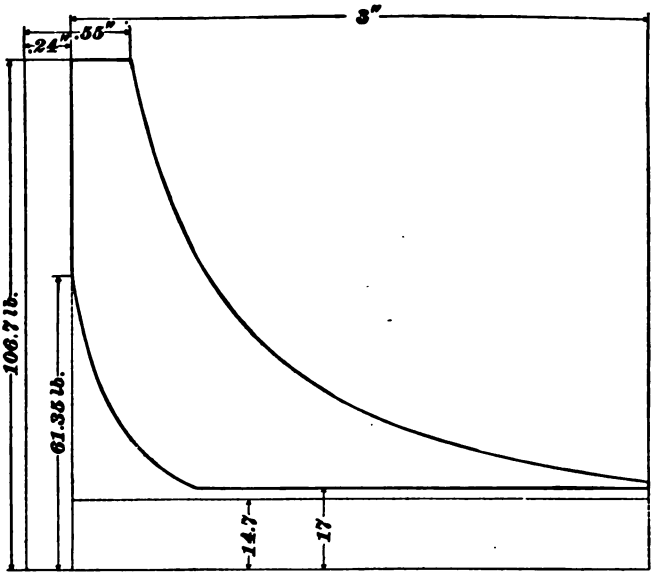

FIG. 96.

any convenient length (we have made the length 3"), also any convenient scale of pressures* (we have assumed 40 lb. = 1"), and then draw the diagram as described in Art. **2081.** Taking the clearance as 8%, the actual distance of the clearance line from the admission line $= 3 \times .08 = .24''$, as shown. The point of cut-off will, therefore, be $3.24 \times .171 = .554''$, say 55, from the clearance line. Since

* NOTE.—The student must use a different length of diagram, and a different scale of pressures, from that used here. We suggest that he use 4" for the length of his diagram, and 48 lb. = 1 in. or 50 lb. = 1 in., for his scale of pressures.

the engine is non-condensing, the back pressure line is taken 17 lb. above the vacuum line. The apparent cut-off $= \frac{.55 - .24}{3} = .1 = \frac{1}{10}$, very nearly.

If the diagram has been correctly drawn, the M. E. P. ought to be 26 lb. per sq. in. Assuming that the actual M. E. P. is but .85 of the theoretical M. E. P. just found, the actual M. E. P. may be taken as $26 \times .85 = 22.1$, say 22 lb. per sq. in. Then, area of cylinder

$$A = \frac{33,000 \times 200}{22 \times 600} = 500 \text{ sq. in.},$$

$$\text{and diameter} = \sqrt{\frac{500}{.7854}} = 25.23''.$$

To avoid fractions, take the diameter as 25″. Then, the area of the piston will be $25^2 \times .7854 = 490.875$ sq. in., and the piston speed, $\frac{33,000 \times 200}{22 \times 490.875} = 611$ ft. per min.

Taking the stroke as 30″, the revolutions per minute will be $\frac{611 \times 12}{2 \times 30} = 122.2$ R. P. M.

Hence, we have for our data the following:

Size of engine, 25″ × 30″.
Initial pressure, 92 lb., gauge.
Back pressure, 2.3 lb., gauge.
Pressure at end of compression, 46.65 lb., gauge.
Apparent cut-off, $\frac{1}{10}$.
Mean effective pressure, 22 lb.
Revolutions per minute, 122.2.
Piston speed, 611 ft. per min.
Clearance, 8%.
} Ans

(1074) Applying formula **296** (see Art. **2088**),

$d = .34D + 2\frac{1}{2}'' = .34 \times 30 + 2\frac{1}{2}'' = 12.7''$, say $12\frac{3}{4}''$. Ans.

(1075) (*a*) By formula **300**,

$d = .0167 D\sqrt{p} = .0167 \times 38\sqrt{100} = 6.346''$, say $6\frac{3}{8}''$. Ans.

(b) By formula **301**.

$$d = .4\sqrt{[illegible]} = .4\sqrt{26^2 \times 72^2 \times 100} = 5.928'', \text{ say } 5\tfrac{15}{16}''.$$

Ans.

(c) By Art. **2103**.

$$i = \frac{[illegible]}{5} = \frac{[illegible]}{5} = 5\tfrac{[illegible]}{5}'', \text{ say } 5\tfrac{5}{16}''. \quad \text{Ans.}$$

In absence of any further data in regard to the engine, the result obtained by formula **301** ought to be taken, for the reason that more conditions are included in the derivation of the formula.

(**1076**) See Art. **2079**.

(**1077**) See Art. **2080**.

(**1078**) The values of the various dimensions, as obtained from the table of proportions, in Art. **2090**, are as follows:

$$D = 24''.$$

$R = 52 \div 2 = 26''.$

$d = .5 \times 24 = 12''.$

$a = 12''.$

$b = 1.75 \times 12 = 21''.$

$c = .045 \times 12 - .0625 = .6025'', \text{ say } \frac{5}{8}''.$

$d' = .28 \times 24 = 6.72'', \text{ say } 6\frac{3}{4}''.$

$e = \frac{1}{4}''.$

$f = .375 \times 17\frac{1}{4} = 6\frac{15}{32}'', \text{ say } 6\frac{1}{2}''.$

$g = (\text{by construction})\ 17\frac{1}{4}''.$

$h = 1.35 \times 6\frac{3}{4} = 9.1125, \text{ say } 9\frac{1}{8}''.$

$i = 1.125 \times 12 = 13\frac{1}{2}''.$

$l = .26 \times 24 + .5 = 6.74'', \text{ say } 6\frac{3}{4}''.$

DYNAMOS AND MOTORS.

(QUESTIONS 1079–1138.)

(1079) The end b; because in looking at that end the current circulates around the helix in an opposite direction to the hands of a watch. (Art. **2160.**)

(1080) (a) Negative. (Art. **2138.**)
(b) Negative. (Art. **2138.**)
(c) Positive. (Art. **2138.**)

(1081) By formula **311**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total generated E. M. F. in volts, and R is the total resistance in ohms of the circuit. In this example, $E = 20$ volts and $R = 30 + 80 = 110$ ohms; hence, $C = \frac{E}{R} = \frac{20}{110} = .1818$ ampere. Ans.

(1082) Let A represent the first branch and B the second; then, $r_1 = 16.2$ ohms, $r_2 = 14.1$ ohms, and $C = 6.37$ amperes.

The current c_1, in branch A, is found by using formula **315**; substituting, gives $c_1 = \frac{C r_2}{r_1 + r_2} = \frac{6.37 \times 14.1}{16.2 + 14.1} = 2.9643$ amperes. Ans.

The current c_2, in branch B, is found by using formula **316**; substituting, gives

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{6.37 \times 16.2}{16.2 + 14.1} = 3.4057 \text{ amperes. Ans.}$$

(1083) From formula **328**, $W = \text{H. P.} \times 746$, where H. P. is the horsepower and W is the power in watts. In this example, H. P. = 2.33 horsepower; hence, $W = \text{H. P.} \times 746 = 2.33 \times 746 = 1{,}738.18$ watts. Ans.

(1084) (*a*) From Art. **2195** and formula **311,** $C = \frac{E'}{R'}$, where C is the current in amperes, E' is the difference of potential in volts between two points, and R' is the resistance in ohms between them. In this example, $E' = 58.4$ volts, and $R' = 2.3$ ohms; hence, $C = \frac{E'}{R'} = \frac{58.4}{2.3} =$ 25.3913 amperes. Ans.

(*b*) From formula **326,** $W = \frac{E^2}{R}$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and R is the resistance in ohms. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence,

$$W = \frac{E^2}{R} = \frac{58.4^2}{2.3} = \frac{3,410.56}{2.3} = 1,482.8521 \text{ watts. Ans.}$$

(*c*) By formula **327,** H. P. $= \frac{W}{746}$; by formula **326,** $W = \frac{E^2}{R}$; therefore (see Art. **2212**), H. P. $= \frac{E^2}{746\,R}$, where H. P. is the horsepower, E the E. M. F., or difference of potential in volts, and R the resistance in ohms.

Hence, H. P. $= \frac{58.4^2}{746 \times 2.3} = \frac{3,410.56}{1,715.8} = 1.9877$ horsepower. Ans.

(1085) Platinum, as it follows zinc in the list (Art. **2144**).

(1086) Towards the *east* (Art. **2157**).

(1087) By formula **313,** $E = C\,R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .75$ ampere, and $R = 17.2 + 8.2 + 11.3 = 36.7$ ohms; hence, $E = C\,R = .75 \times 36.7 = 27.525$ volts, the total E. M. F. developed in the battery.

By derivation from formula **313,** $E' = C\,R'$, where E' is the difference of potential in volts between two points, C is

the current in amperes flowing, and R' is the resistance in ohms between the two points.

Between c and b, $R' = 8.2$ ohms, and $C = .75$ ampere; hence, $E' = C R' = .75 \times 8.2 = 6.15$ volts. Ans.

Between b and a, $R' = 11.3$ ohms, and $C = .75$ ampere; hence, $E' = C R' = .75 \times 11.3 = 8.475$ volts. Ans.

Between a and c, the difference of potential is the difference of potential between a and b plus that between b and c, which is $8.475 + 6.15 = 14.625$ volts. Or, since the difference of potential between a and c is the available E. M. F. of the battery, when a current of .75 ampere is flowing, it can be calculated by using formula **314**, $E' = E - C r_i$, where E' is the available E. M. F., E is the total E. M. F. developed in the battery, C is the current flowing, and r_i is the internal resistance of the battery. In this case, $E = 27.525$ volts, $C = .75$ ampere, and $r_i = 17.2$ ohms; hence, $E' = E - C r_i = 27.525 - (.75 \times 17.2) = 14.625$ volts. Ans.

(1088) The sectional area of a wire .2 in. in diameter is $.7854 \times .2 \times .2 = .031416$ sq. in. or, nearly, .0314 sq. in.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000 (Art. **2175**), which gives $\frac{.5921}{1,000,000} = .0000005921$ ohm; or, in other words, the resistance of a piece of silver one inch long, and whose sectional area is one square inch, is .0000005921 ohm. Next, from this resistance and length calculate the resistance of 1,000 feet of the silver with a sectional area of 1 sq. in. by using formula **306**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, r_2 is the resistance after the length of the conductor is changed, l_1 is the original length of the conductor, and l_2 is the changed length. In this example, $r_1 = .0000005921$ ohm, $l_1 = 1$ inch, and $l_2 = 1,000$ feet, or 12,000 inches Hence, by substituting, $r_2 = \frac{r_1 l_2}{l_1} = .0000005921 \times \frac{12,000}{1} =$ .0071052 ohm; that is, the resistance of 1,000 feet of silver having a sectional area of 1 sq. in. is .0071052 ohm. From this result calculate the resistance of 1,000 feet of the

[illegible]

[illegible] resistance [illegible] times the [illegible] total resistance [illegible] the total resist[illegible] resistance. Hence [illegible]

[illegible] Ans.

[illegible] By formula [illegible] where [illegible] is the [illegible] seconds. [illegible]

[illegible] By formula [illegible] where [illegible] is the [illegible] resistance after [illegible] temperature in [illegible] temperature coefficient. In this [illegible] and [illegible] =

[illegible] = 2[illegible] ohms. Ans.

[illegible] By formula [illegible] the separate resistance of [illegible] is equal to the difference of [illegible] branches divide and where [illegible] current in that branch.

Hence the separate resistance of branch [illegible] is [illegible] = [illegible] ohms. Ans.

The separate resistance of branch [illegible] is [illegible] = 2.3673 ohms. Ans.

1101 By formula **312**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing in the circuit. In this example, $E = 22.4$

volts, and $C = .43$ ampere; hence, $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$ ohms, the total resistance of the circuit. Since the total resistance of a closed circuit is equal to the sum of the external and internal resistances, the external resistance must be the difference between the total resistance and the internal resistance. Hence, the external resistance $= 52.093 - 13.4 = 38.693$ ohms. Ans.

(1102) By transposition of terms in formula **319**, $C = \frac{Q}{t}$, where C is the current in amperes, Q is the quantity of electricity in coulombs, and t is the time in seconds. In this example, $Q = 368{,}422$ coulombs, and $t = 4.5 \times 60 \times 60 = 16{,}200$ seconds; hence, $C = \frac{Q}{t} = \frac{368{,}422}{16{,}200} = 22.7421$ amperes. Ans.

(1103) By formula **321**, $J = C^2 R t$, where J is the work done in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this example, $C = 2.4$ amperes, $R = 45$ ohms, and $t = 3{,}000$ seconds. Then the electrical work done $= 2.4 \times 2.4 \times 45 \times 3{,}000 = 777{,}600$ joules. By formula **323**, the mechanical work done $=$ F. P. $= .7373 J = .7373 \times 777{,}600 = 573{,}324.48$ foot-pounds. Ans.

(1104) By formula **327**, H. P. $= \frac{W}{746}$; by formula **324**, $W = CE$; therefore (see Art. **2212**), H. P. $= \frac{E\,C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 525$ volts, and $C = 12.5$ amperes; hence,

$$\text{H. P.} = \frac{E\,C}{746} = \frac{525 \times 12.5}{746} = 8.7969 \text{ horsepower. Ans.}$$

(1105) (*a*) By formula **325**, $W = C^2 R$, where W is the power in watts, C is the current in amperes, and R is the resistance in ohms. In this example, $C = 110$ amperes, and $R = 4.2$ ohms; hence, $W = C^2 R = 110^2 \times 4.2 = 50{,}820$ watts. Ans.

[illegible] $= \frac{P}{[illegible]}$, where [illegible] is the [illegible] and P is the power in watts. In this example, $P =$ [illegible] watts; hence,

$$[illegible] = \frac{P}{[illegible]} = \frac{[illegible]}{[illegible]} = [illegible] \text{ horsepower. Ans.}$$

(1106) The diagram, Fig. [illegible], shows the connections of the battery and galvanometer circuits in the circular type of resistance box for measuring unknown resistances by the Wheatstone's bridge method. The upper balance arm, Art. **2151**, of the bridge includes the resistance coils from c to a, the lower balance arm includes the coils from a to d, and the adjustable arm includes the coils from d to b. One pole of the battery B is connected to the junction of the two balance arms; the other to the junction of the adjustable arm and the unknown resistance X. One terminal of the galvanometer G is connected to the junction of the lower balance arm and the adjustable arm; the other to the junction of the upper balance arm and the unknown resistance.

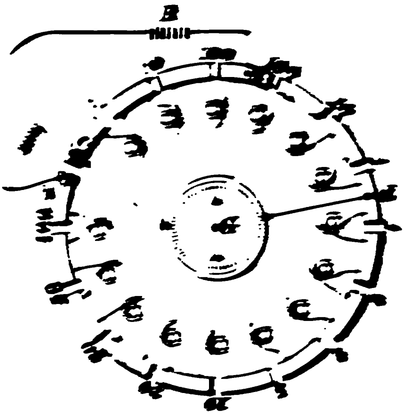

Fig. [illegible]

(1107) By formula **306**, the changed resistance for variation in length, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, l_1 is the original length, and l_2 is the changed length. In this case, $r_1 = 1$ ohm, $l_1 = 1{,}000$ feet, and $l_2 = 2{,}000$ feet. Then, the changed resistance $r_2 = \frac{1 \times 2{,}000}{1{,}000} = 2$ ohms. The next operation is to determine the resistance of the wire when its sectional area is changed. A round wire

$.1''$ in diameter has a sectional area of $.1^2 \times .7854 = .007854$ sq. in., and a square wire $.1''$ on a side has a sectional area of $.1 \times .1 = .01$ sq. in. By formula **307**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = 2$ ohms, $a_1 = .007854$ sq. in., and $a_2 = .01$ sq. in. Hence, $r_2 = \frac{r_1 a_1}{a_2} = \frac{2 \times .007854}{.01} = 1.5708$ ohms. Ans.

(1108) By formula **327**, H. P. $= \frac{W}{746}$, where H. P. is the horsepower, and W is the power in watts. In this example, $W = 54{,}200$ watts; hence, H. P. $= \frac{W}{746} = \frac{54{,}200}{746} =$ 72.6541 horsepower. Ans.

(1109) The sectional area of a round column .04 in. in diameter is $.04^2 \times .7854 = .00125664$ sq. in., or .001257 sq. in., nearly.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **2175**, which gives $\frac{37.15}{1{,}000{,}000} = .00003715$ ohm; or, in other words, the resistance of a quantity of mercury 1 in. long, and whose sectional area is 1 sq. in., is .00003715 ohm. Next, from this resistance and length, calculate the resistance of a column of mercury $72.3''$ high, with a sectional area of 1 sq. in. by using formula **306**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is its original length, and l_2 is its changed length. In this example, $r_1 = .00003715$ ohm, $l_1 = 1''$, and $l_2 = 72.3$ inches. Hence, $r_2 = \frac{r_1 l_2}{l_1} = \frac{.00003715 \times 72.3}{1} = .002685945$, or .002686 ohm, nearly; or,

in other words, the resistance of a column of mercury 72.3″ high, having a sectional area of 1 sq. in. is .002686 ohm. From this result calculate the resistance of the column when its sectional area is .001257 sq. in., by using formula **307**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance, r_2 is the resistance after the sectional area has been changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .002686$ ohm, $a_1 = 1$ sq. in., and $a_2 = .001257$ sq. in. Hence,

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.002686 \times 1}{.001257} = 2.1368 \text{ ohms. Ans.}$$

(1110) By formula **311**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts generated, and R is the total resistance in ohms of the circuit. Since the total resistance of a closed circuit is the sum of the external and internal circuits, $R = 33 + 30 = 63$ ohms, and $E = 45$ volts; hence, $C = \frac{E}{R} = \frac{45}{63} = .7143$ ampere. Ans.

(1111) In Fig. 28, question No. 1111, the reading of the voltmeter gives the total E. M. F. of the battery. Hence, after the connections are made, as shown in Fig. 29, question No. 1111, there is a closed circuit in which the total E. M. F. developed is 24.4 volts, and through which a current of .8 ampere is flowing. By formula **312**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed, and C is the current in amperes flowing. In this example, $E = 24.4$ volts, and $C = .8$ ampere; hence, $R = \frac{E}{C} = \frac{24.4}{.8} = 30.5$ ohms, the total resistance of the circuit.

From the reading of the voltmeter, after the connections are made as shown in Fig. 29, it will be seen that when a current of .8 ampere flows through the external resistance

from b to a, through R, there is a drop or loss of potential of 18 volts. By formula **312,** $R' = \frac{E'}{C}$, where R' is the resistance of a conductor, E' is the drop or loss of potential in that conductor, and C is the current in amperes flowing through it. In this case, $E' = 18$ volts, and $C = .8$ ampere; hence, $R' = \frac{E'}{C} = \frac{18}{.8} = 22.5$ ohms, the resistance of the external circuit from b to a, through the resistance R. Ans.

Since the total resistance of a closed circuit is the sum of the external and internal resistances, Art. **2191,** the internal resistance must be the difference between the total and external resistances. Hence, $30.5 - 22.5 = 8$ ohms, the internal resistance of the battery B. Ans.

(1112) By formula **324,** $W = C\,E$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and C is the current in amperes. In this example, $E = 510$ volts, and $C = 24.3$ amperes; hence, $W = 510 \times 24.3 = 12{,}393$ watts. Ans.

(1113) Referring to Art. **2187,** the total E. M. F. developed by connecting several cells in series is equal to the E. M. F. of one cell multiplied by the number of cells; hence, the E. M. F. of one of the groups of 6 cells is $6 \times 1.5 = 9$ volts. In the same article it is stated that connecting cells in multiple, or parallel, does not change the E. M. F. between the main conductors. In this case each group of six cells can be considered as one large cell developing an E. M. F. of 9 volts, and, consequently, the E. M. F. of the four groups connected in multiple, or parallel, is 9 volts, which would be the E. M. F. indicated by a voltmeter, connected to the main conductors C and C', as shown in Fig. 30, question No. 1113. Ans.

(1114) By formula **327,** H. P. $= \frac{W}{746}$; by formula **324,** $W = C E$; therefore (see Art. **2212**), H. P. $= \frac{E\,C}{746}$,

where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E =$ 250 volts, and $C = 65.7$ amperes; hence, H. P. $= \frac{E\,C}{746} = \frac{250 \times 65.7}{746} = \frac{16,425}{746} = 22.0174$ horsepower. Ans.

(1115) In Art. **2157** it is stated that when a compass is placed under a conductor in which an electric current is flowing from the south to the north, the north pole of the compass needle tends to point towards the west, and if the direction of the current in the conductor is reversed, the north pole will point towards the east. Since, in this example, the north pole of the needle tends to point towards the east, the current must be flowing *from* the north *to* the south.

(1116) End a; since (Art. **2160**), in looking at the face of the end a, the current circulates around the core in the same direction as the movement of the hands of a watch.

(1117) Attract one another; since (Art. **2138**), a positive charge is developed upon the ivory when rubbed with silk, and a negative charge upon sealing-wax when rubbed with fur; and from Art. **2137**, electrified bodies with dissimilar charges are mutually attractive.

(1118) The exposed end of the iron; since, from Art. **2144**, the iron forms the positive element of the cell, and, from Art. **2143**, the pole, or electrode, attached to the exposed end of a voltaic element is always of opposite sign to the element itself.

(1119) From Art. **2152**, iron and its alloys, nickel, cobalt, manganese, oxygen, cerium, and chromium.

(1120) Towards the south pole, since, from Art. **2151**, unlike poles attract one another.

(1121) Towards the north pole, since, from Art. **2151**, unlike poles attract one another.

(1122) *From* the north *to* the south, since, from Art. **2157**, the north pole of a compass needle tends to point to-

wards the east when the compass is placed over a conductor in which the current is flowing from the south to the north; and, by reversing the direction of the current in the conductor, the north pole of the needle tends to point towards the west, and the south pole towards the east.

(1123) The current should enter the wire at end b; since (Art. **2160**), in looking at the face of the south pole of the magnet, the current should circulate around the core in the direction of motion of the hands of a watch.

(1124) By formula **306**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length, and l_2 is its changed length. In this example, $r_1 = 100.8$ ohms, $l_1 = (112 \times 12) + 6 = 1,350$ inches, $l_2 = 11.7$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{100.8 \times 11.7}{1,350} = .8736 \text{ ohm. Ans.}$$

(1125) By formula **308**, $r_2 = \frac{r_1 D^2}{d^2}$, where r_1 is the original resistance of a round conductor, r_2 is the resistance after its diameter has been changed, D is its original diameter, and d is its changed diameter. In this example, $r_1 = 86.5$ ohms, $D = .1$ inch, and $d = .02$ inch; hence, $r_2 =$

$$\frac{r_1 D^2}{d^2} = \frac{86.5 \times .1^2}{.02^2} = \frac{86.5 \times .01}{.0004} = 2,162.5 \text{ ohms. Ans.}$$

(1126) By formula **309**, $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has risen, k is the temperature coefficient, and t is the number of degrees rise Fahrenheit. In this example, $r_1 = 91.8$ ohms, $t = 72 - 45 = 27$ degrees, and $k = .000244$, from Table 60. Hence, $r_2 = r_1 (1 + t k) = 91.8 (1 + 27 \times .000244) = 91.8 \times 1.006588 = 92.4048$ ohms. Ans.

(1127) By formula **310**, $r_2 = \frac{r_1}{1 + t k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the number of degrees

fall Fahrenheit, and a is the temperature coefficient of the material. In this example, $r_1 = .144$ ohm, $t = 87 - 41 = 46$ degrees Fahrenheit, and $a = .002155$, from Table 60. Hence,

$$r_2 = \frac{r_1}{1 + t\,a} = \frac{.144}{1 + 46 \times .002155} = \frac{.144}{1.09913} = .131 \text{ ohm.}$$

Ans.

(1128) First reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **2175,** which gives $\frac{3.565}{1,000,000} = .000003565$ ohm; or, in other words, the resistance of a block of platinum one inch long, and whose sectional area is one square inch, is .000003565 ohm. Next, from this resistance and length, calculate the resistance of 126 feet of platinum with a sectional area of 1 sq. in., by using formula **306,** $r_2 = \frac{r_1\,l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length of the conductor, and l_2 is its changed length. In this example, $r_1 = .000003565$ ohm, $l_1 = 1$ inch, and $l_2 = 126 \times 12 = 1,512$ inches. Hence,

$$r_2 = \frac{r_1\,l_2}{l_1} = \frac{.000003565 \times 1,512}{1} = .005390280 \text{ ohm;}$$

that is, the resistance of 126 feet of platinum having a sectional area of 1 sq. in. is .00539 ohm, nearly. From this result calculate the resistance of 126 feet of platinum when its sectional area $= .1^2 \times .7854 = .007854$ sq. in., by using formula **307,** $r_2 = \frac{r_1\,a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is its original sectional area, and a_2 is its changed sectional area. At this stage of the example, $r_1 = .00539$ ohm, $a_1 =$ 1 sq. in., $a_2 = .007854$ sq. in. Hence, $r_2 = \frac{.00539 \times 1}{.007854} =$ .6863 ohm. Ans.

(1129) From Art. **2184,** the fundamental equation of the Wheatstone's bridge is $X = \frac{M}{N} \times P$, where X is the

unknown resistance, M is the resistance of the upper balance arm, N is the resistance of the lower balance arm, and P is the resistance of the adjustable arm. It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and, hence, in this example, $M = 1$ ohm; the coils between a and d form the lower balance arm, and, hence, $N = 100$ ohms; the coils between d and b form the adjustable arm, and, hence, $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in the fundamental equation, gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms. Ans.}$$

(1130) By formula **311,** $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 36$ volts, and $R = 24 + 18 = 42$ ohms; since, Art. **2191,** the total resistance of a closed circuit is the sum of the internal and external resistances. Hence, $C = \frac{E}{R} = \frac{36}{42} = .8571$ ampere. Ans.

(1131) By formula **312,** $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E.M.F. in volts developed in the circuit, and C is the current in amperes flowing through the circuit. In this example, $E = 12.6$ volts, and $C = 2.7$ amperes; hence, $R = \frac{E}{C} = \frac{12.6}{2.7} = 4.6667$ ohms. Ans.

(1132) By formula **313,** $E = CR$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing through the circuit, and R is the total resistance of the circuit. In this example, $C = .8$ ampere, and $R = 31.5 + 11 = 42.5$ ohms, since, Art.

[illegible] [illegible] resistance [illegible] circuit is the sum of the [illegible] resistances. Hence, [illegible] Ans.

(1133) [illegible] formula **312**, $E = CR$, where E is the difference of [illegible] between two points in a circuit, C the [illegible] amperes flowing through the circuit, and R the resistance of the circuit between the two points. [illegible] example [illegible] amperes, and $R = 24$ ohms; hence, [illegible] $\times 24 =$ [illegible] volts. Ans.

(1134) [illegible] formula **312**, $R = \frac{E}{C}$, where R is the total resistance [illegible] between two points in a circuit, E is the [illegible] in volts between the two points, and C is the current in amperes flowing in the circuit. In [illegible] example the two conductors leading to and from the receptive device can be considered as in series, forming one single conductor 1,300 feet in length, in which the drop [illegible] of 250 volts, or .10 $\times$ 250 = 25 volts [illegible] 25 volts. Since $C =$ 80 amperes, then $R = \frac{25}{80} = .3125$ ohm; or, in other words, the sum of the resistances of [illegible] conductors which transmit a current of 80 amperes to and from the receptive device with a loss of 25 volts is .3125 ohm. Ans.

(*b*) The resistance per foot of any conductor is found by dividing its total resistance by its length in feet. Assume the two conductors leading to and from the receptive device to be one single conductor 1,300 feet in length, and offering a resistance of .3125 ohm; hence, its resistance per foot is

$$\frac{.3125}{1{,}300} = .00024 \text{ ohm. Ans.}$$

(1135) By formula **311**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 24$ volts, and $R = 8.1 + 15.9 = 24$ ohms, since, Art.

2191, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence,

$$C = \frac{E}{R} = \frac{24}{24} = 1 \text{ ampere.}$$

By formula **314,** $E' = E - C r_i$, where E' is the available, or external, E. M. F. in volts of a battery or other electric source in a closed circuit. E is the total E. M. F. in volts developed in the source. C is the current in amperes flowing through the circuit, and r_i is the internal resistance of the battery or electric source. In this example, $E = 24$ volts, $C = 1$ ampere, and $r_i = 8.1$ ohms. Hence, $E' = E - C r_i = 24 - (8.1 \times 1) = 15.9$ volts. Ans.

(1136) Let A represent the first branch, and B the second; then $r_1 = 1.2$ ohms, $r_2 = 2.2$ ohms, and $C = 45$ amperes.

The current c_1 in branch A will then be found by substituting these values in formula **315,** which gives

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{45 \times 2.2}{1.2 + 2.2} = \frac{99}{3.4} = 29.1176 \text{ amperes. Ans.}$$

Since the sum of the currents in the two branches is 45 amperes, the current in branch B is, therefore, the difference between 45 amperes and the current in branch A, or $45 - 29.1176 = 15.8824$ amperes. Ans.

(1137) By formula **317,** the joint resistance of two conductors connected in parallel is equal to the product of their separate resistances divided by their sum, or $\frac{r_1 r_2}{r_1 + r_2}$, where r_1 and r_2 are the separate resistances of the two branches. In this example, $r_1 = 45$ ohms, and $r_2 = 63$ ohms. Hence, $\frac{45 \times 63}{45 + 63} = 26.25$ ohms, the joint resistance of the two conductors connected in parallel.

(1138) From Art. **2203,** the joint resistance of three conductors connected in parallel is given by formula **318,** $R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 are the separate

[illegible] respectively. In this [illegible] and [illegible] ohms. Then the joint resistance of the three conductors [illegible] in parallel is

[illegible] ohms

DYNAMOS AND MOTORS.

(QUESTIONS 1139-1188.)

(1139) By formula **329,** $E = \frac{2 N S n}{10^8}$. In this example, $N = 6,250,000$ lines of force, $S = 100$ outside, or face, wires in series, for if 200 turns were wound around the core, there would be 200 outside, or face, wires, and, from Art. **2236,** one-half would be connected in series, and $n = \frac{1,200}{60}$ revolutions per second. Substituting these values in above formula gives $E = \frac{2 N S n}{10^8} = \frac{2 \times 6,250,000 \times 100 \times 1,200}{100,000,000 \times 60} =$ 250 volts. Ans.

(1140) From Art. **2249,** it will be seen that the current in the shunt field of a dynamo is equal to the difference of potential between the brushes divided by the resistance of the shunt field circuit, or $C_s = \frac{E}{R_s}$. In this example, $E =$ 220 volts, and $R_s = 440$ ohms; hence, $C_s = \frac{E}{R_s} = \frac{220}{440} = .5$ ampere. Ans.

(1141) See Arts. **2226** and **2227.**

(1142) In Art. **2214,** it is stated that a current will be induced in a closed coil or circuit when there is a change in the number of lines of force passing through that coil or circuit. In this case, as the magnetic field is uniform, there is no change in the number of lines of force passing through the coil C when it is moved from its original position to the position C', as shown by the dotted outlines; and, hence, no current will flow around the ring.

[illegible] resistance of the field cir- [illegible] between the brushes [illegible] of the dynamo [illegible] when the resistance [illegible] represents the resistance [illegible] of the rheostat then [illegible] represents the resis- [illegible] the resistance of the rheostat

has been cut out, or short-circuited, then $r_2 = \frac{E_2}{C_2} = \frac{360}{1.8} = 200$ ohms. Hence, the amount of resistance which was cut out, or short-circuited, in the rheostat is the difference between these two resistances, or $240 - 200 = 40$ ohms. Ans.

(1148) Use formula **332.** In this example, the input $= \frac{100 \times 65,000}{90.5} = 71,823.2044$ watts. Since one horsepower equals 746 watts, then 71,823.2044 watts equal $\frac{71,823.2044}{746} = 96.2778$ horsepower. Ans.

(1149) If the current circulates in the direction indicated by the arrow-heads, *neither* pole-piece will be a north pole; for, by applying the rule given in Art. **2160,** it will be seen that the north pole of one field coil is opposite the south pole of the other, and the lines of force circulate around the magnets without passing through the armature. If the winding of the right-hand coil were reversed, its top would be a north pole, and the top of the left-hand coil being also north, the pole-piece P would become a north consequent pole.

(1150) See Arts. **2232** and **2234.**

(1151) In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, then 10 horsepower are equivalent to $10 \times 746 = 7,460$ watts. Then, by formula **330,** the efficiency $\mathbf{E} = \frac{6,341 \times 100}{7,460} = 85$ per cent. Ans.

(1152) From b to a through the conductor; for, by applying the thumb-and-finger rule given in Art. **2221,** as indicated, it will be seen that the middle finger is pointing towards a from b.

(1153) In this example, it is first necessary to find the input in watts. In this example, the output = 11,900 watts, and the per cent. efficiency = 85. Then, by formula **332,** the input $= \frac{100 \times 11,900}{85} = 14,000$ watts input. As in Art.

2277, the watts lost are found by multiplying the input by the per cent. loss and dividing by 100. Hence, $\frac{14,000 \times 1.8}{100} = 252$ watts lost in core by eddy currents and hysteresis. Ans.

(1154) (*a*) See Art. **2241.** (*b*) See Art. **2238.**

(1155) According to Art. **2283** and formula **325,** $W_t = C^2 r_t$. In this example, $C = 120$ amperes, and $r_t = .040$ ohm; hence, $W_t = 120^2 \times .040 = 576$ watts. Ans.

(1156) From example 1155, the normal output from the dynamo is 120 amperes at 125 volts, or $120 \times 125 = 15,000$ watts. The next step is to determine the input at this output when the efficiency is 75%. By formula **332,** the input in this case is $\frac{100 \times 15,000}{75} = 20,000$ watts. From example 1155, there are 576 watts lost in the armature due to its resistance, and, from Art. **2285,** the loss in the armature due to its resistance is $\frac{576 \times 100}{20,000} = 2.88\%$. Ans.

(1157) See Arts. **2289** and **2290.**

(1158) Under Field Losses, in Art. **2279,** the watts lost in the series coils is found by using formula **325,** where $W = C^2 R$. In this example, $C = 40$ amperes, and $R = .04$ ohm; hence, $W = 40^2 \times .04 = 40 \times 40 \times .04 = 64$ watts, which represents the loss in the series coils. The watts lost in the shunt coil is given by formula **326,** where $W = \frac{E^2}{R}$. In this case, $E = 550$ volts, and $R = 550$ ohms; hence, $W = \frac{E^2}{R} = \frac{550 \times 550}{550} = 550$ watts, which is the loss in the shunt field. The total loss in the fields of a compound dynamo is equal to the sum of the losses in the series and shunt coils. Hence, the total loss in this case is $64 + 550 = 614$ watts. Ans.

(1159) P' is the south consequent pole of the field, since, from the rule in Art. **2160,** in looking through the coils c and d from a position near P', the current is circula-

ting around the field cores in the same direction as the movements of the hands of a watch; while, on the contrary, in looking through the coils *a* and *b* from a position near *P*, the current is circulating around the upper field cores in a direction opposite to the movements of the hands of a watch.

(1160) See Art. **2242.**

(1161) From Art. **2277,** the total loss in a dynamo is the sum of the separate losses; hence, in this example, the total loss in watts is $356 + 178 + 263 + 423 + 50 = 1{,}270$ watts. From Art. **2272,** the input to the dynamo in this case is $15{,}000 + 1{,}270 = 16{,}270$ watts. By formula **330** the efficiency $\mathsf{E} = \frac{15{,}000 \times 100}{16{,}270} = 92.1942\%$ at this output. Ans.

(1162) From example 1161, the loss in mechanical friction is 356 watts, and the input is 16,270 watts; hence (see Art. **2285**), the per cent. loss is $\frac{356 \times 100}{16{,}270} = 2.1881\%$. Ans. (*a*).

From example 1161, the loss in the core by eddy currents and hysteresis is 178 watts, and the input is 16,270 watts; hence (see Art. **2285**), the per cent. loss is $\frac{178 \times 100}{16{,}270} = 1.094\%$. Ans. (*b*).

From example 1161, the loss in the field coils is 263 watts, and the input is 16,270 watts; hence, the per cent. loss is $\frac{263 \times 100}{16{,}270} = 1.6165\%$. Ans. (*c*).

From example 1161, the loss in the armature $(C^2 r) =$ 423 watts, and the input is 16,270 watts; hence, the per cent. loss is $\frac{423 \times 100}{16{,}270} = 2.5999\%$. Ans. (*d*).

From example 1161, the sum of the separate losses is 1,270 watts, and this is the difference between the input and the output; the input is 16,270 watts; then, by formula **331,** the total per cent. loss $L = \frac{100 \times 1{,}270}{16{,}270} = 7.8058\%$. Ans. (*e*).

(1173) From Art. **2235**, it will be seen that the electromotive force generated in an armature is proportional to the number of lines of force passing through the core. Let E represent the electromotive force which is generated when [illegible] lines of force are passing through the core; then, by proportion, [illegible] : E :: [illegible] : [illegible], or E × [illegible] = [illegible] × [illegible]; therefore, E = [illegible] = [illegible] volts. Ans.

(1174) (*a*) See Art. **2278.** (*b*) See Art. **2283.** (*c*) See Art. **2279.**

(1175) Towards the side *a*; for by applying the thumb-and-finger rule given in Art. **2239**, and making the forefinger point in the direction of the lines of force and the middle finger in the direction of the current, the thumb will point towards the side *a*.

(1176) Use the formula given under Field Losses in Art. **2280**, $C_s = \frac{E_s}{r_s}$, which is a modification of formula **311.** In this example, $E_s = 525$ volts, and $r_s = 650$ ohms; hence, $C_s = \frac{E_s}{r_s} = \frac{525}{650} = .8076$ ampere. Ans.

(1177) The increase in voltage from no load to full load is $124.2 - 115 = 9.2$ volts, which is $\frac{9.2 \times 100}{115} = 8\%$ of the normal voltage. Therefore, the over-compounding is 8%. Ans.

(1178) See Art. **2233.**

(1179) First change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, 44 horsepower are equivalent to $44 \times 746 = 32,824$ watts. By formula **330,** the efficiency

$$\mathbf{E} = \frac{100 \times 29,820}{32,824} = 90.8481\%. \text{ Ans.}$$

(1180) In this example, the input $I = 20{,}000$ watts, and the output $O = 17{,}500$ watts. Then, by formula **331**, the per cent. loss $L = \frac{100 \times (20{,}000 - 17{,}500)}{20{,}000} = 12.5\%$. Ans.

(1181) In this example, the output is 12,500 watts, and the efficiency is 92.5%. Consequently, by formula **332**, the input $I = \frac{100 \times 12{,}500}{92.5} = 13{,}513.5135$ watts. Reducing this input from watts to horsepower, gives

$$\frac{13{,}513.5135}{746} = 18.1146 \text{ horsepower. Ans.}$$

(1182) See Art. **2286.**

(1183) In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, fifty-five horsepower are equivalent to $55 \times 746 = 41{,}030$ watts. The output of the dynamo, by formula **333**, $= \frac{41{,}030 \times 88.5}{100} = 36{,}311.55$ watts. Ans.

(1184) In the same manner as shown in Art. **2277**, it will be seen that the loss in watts in the field coils is equal to the input multiplied by the per cent. loss and divided by 100. In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, forty-five horsepower are equivalent to $45 \times 746 = 33{,}570$ watts. Consequently, the loss in the field coils is $\frac{33{,}570 \times 2}{100} = 671.4$ watts. Ans.

(1185) See Arts. **2247, 2249,** and **2252.**

(1186) From Art. **2285**, the per cent. loss in the core is found by dividing the watts lost in the core by the input, and multiplying by 100. Reducing 64 horsepower to watts gives $64 \times 746 = 47{,}744$ watts. Consequently, the loss in the core is $\frac{800 \times 100}{47{,}744} = 1.6756\%$. Ans.

(1187) See Art. **2289.**

(1188) See Art. **2256.**

(1222) See Arts. **2389** and **2407.**

(1223) See Art. **2309** and Fig. 897.

(1224) See Art. **2334.**

(1225) See Art. **2399.**

(1226) See Art. **2356.**

(1227) See Art. **2304.**

(1228) See Art. **2402.**

(1229) See Art. **2365.**

(1230) See Art. **2338.**

(1231) See Art. **2381.**

(1232) See Art. **2384.**

(1233) See Art. **2349.**

(1234) See Art. **2403.**

(1235) See Art. **2315.**

(1236) See Art. **2331.**

(1237) See Art. **2406.**

(1238) When in the position of least action, a coil is momentarily disconnected from the external circuit, then is thrown in parallel with the coil ahead of it, then in series with the other two coils which are then in parallel, then in parallel with the coil behind it, and then disconnected from the circuit again. See Art. **2312,** also, Fig. 898.

(1239) (*a*) See Art. **2352.** (*b*) See Arts. **2356** and **2359.**

(1240) (*a*) Of the 5 amperes input, by Ohm's law, $\frac{125}{62.5} = 2$ amperes go to the field, the loss being, therefore, $2 \times 125 = 250$ watts. The rest, or $3 \times 125 = 375$ watts, make up the friction and core losses of the machine (Art. **2349**).

When taking an input of 77 amperes at 125 volts, or 9,625 watts, there would still be required 250 watts for the field, and 375 watts for the core losses and friction. Of the 77 amperes, 75 flow through the armature, and as this has a resistance of .04 ohm, the armature $C^2 r$ would be $75^2 \times .04 = 225$ watts. The total losses would then be $250 + 375 + 225 = 850$ watts, and the output would, therefore, be $9,625 - 850 = 8,775$ watts, or $\frac{8,775}{746} = 11.76$ H. P. Ans.

(*b*) The output being 8,775 watts, and the input 9,625, the efficiency is, by formula **330**, $\frac{100 \times 8,775}{9,625} = 91.17$ per cent. Ans.

(1241) See Art. **2379.**

(1242) (*a*) and (*b*) See Art. **2317.**

(1243) See Art. **2302.**

(1244) See Art. **2338.**

(1245) (*a*) and (*b*) See Art. **2396**, and Figs. 928 and 929.

(1246) From Art. **2358**, the speed of the field would be $\frac{72}{5} = 14.4$ revolutions per second, or $14.4 \times 60 = 864$ revolutions per minute. With 2.5% slip, the speed of the armature would be $864 - (864 \times .025) = 842.4$ revolutions per minute. Ans.

(1247) See Art. **2367.**

(1248) (*a*) See Art. **2336.** (*b*) and (*c*) See Art. **2337.**

(1249) When the whole of the coil is directly under one pole piece. See Art. **2386.**

(1250) See Art. **2357.**

(1251) See Art. **2318.**

(1252) See Art. **2405.**

(1253) See Art. **2343.**

(1254) See Art. **2369.**

(1255) (*a*) and (*b*) See Art. **2340.** (*c*) See Arts. **2339** and **2340.**

(1256) See Arts. **2313** and **2314.**

(1257) See Art. **2371.**

(1258) See Art. **2355.**

(1259) See Art. **2336.**

(1260) See Art. **2331.**

(1261) See Art. **2356.**

(1262) There is no definite answer for this problem, as a great number of different arrangements is possible. See Art. **2404.** By comparing it with Fig. 931, Art. **2402,** it will be seen if the connections are correctly made and all the necessary instruments in place; the exact arrangement is a matter of taste and judgment.

(1263) See Art. **2347.**

(1264) See Art. **2319.**

(1265) The frequency being 132, and there being 11 pairs of poles, the motor will run at $\frac{132}{11} = 12$ revolutions per second, or $60 \times 12 = 720$ revolutions per minute. See Art. **2352.**

(1266) (*a*) and (*b*) See Arts. **2356** and **2357.**

(1267) See Art. **2298.**

(1268) See Art. **2341.**

(1269) See Art. **2372.**

(1270) See Art. **2298.**

(1271) The diameter of the driving pulley may be found from the rule given in Art. **1478.** In this example, the diameter of the driven pulley is 13 inches, its number of revolutions 700, and the number of revolutions of the driving pulley is 182. Then, $\frac{13 \times 700}{182} = 50$ inches, the diameter of the driving pulley. Adding 2% for belt slip (Art. **2361**) gives $50 + (50 \times .02) = 51$ inches. Ans.

(1272) (*a*) The strength of the direct current would be the same as the *effective* strength of the alternating current, which is .7 of its maximum strength. See Art. **2326.** Then, $.7 \times 12 = 8.4$ amperes. Ans.

(*b*) See Art. **2326.**

(1273) See Art. **2341.**